Dual Motor Problem Assignment

INTRODUCTION

This exercise is designed to combine the concepts of two degree of freedom systems with rotating unbalance forcing function. It is demonstrated in the Labview GUI "MotorProb.vi".

The problem statement is given as follows: A certain machine can be modeled as a two degree of freedom system as shown in the diagram. Two motors must be mounted on the machine but can be placed on either of the masses. You are the engineering team's vibration expert and you must determine the following:

A. Which locations are best for each motor in order to minimize the movement of mass M1?
B. Which locations are best for each motor in order to minimize the movement of mass M2?
C. Which locations are best to minimize the amount of force the machine imparts on the ground?

USING THE PROGRAM

The Labview GUI "MotorProb.vi" is designed to help solve this problem; it does all the basic mathematics of solving the system and displays the results graphically. Start out by opening the program and pressing the run button. Your screen should look like the following:

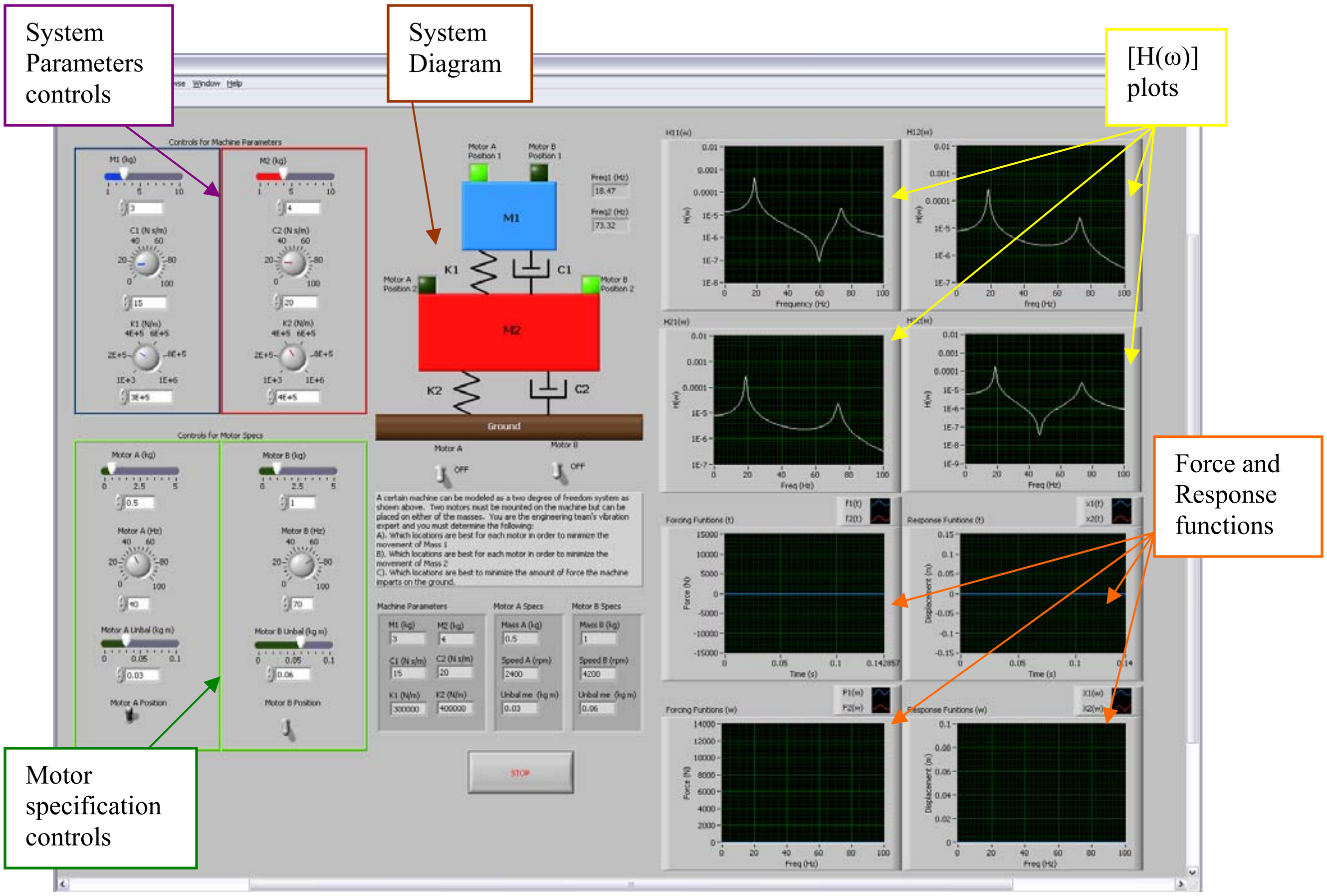

Fig. 1. Screenshot of the program MotorProb.vi when initially opened and run. This identifies some of the major parts of the program.

In the center of the program window is the block diagram describing the system. Directly below the diagram are two switches for turning the motors on and off, and below that is a restatement of the original problem. To the left are several controls that modify the system parameters, as well as the specifications of the two motors. On the right side of the screen are a set of graphs. The top four graphs represent the frequency response function matrix [H(ω)]. The bottom four graphs are the force and response functions in both the time domain and the frequency domain.

The LEDs on the diagram are used to indicate where each motor is mounted on the machine. Clicking on a dark LED will change the motor to that position. Each motor can be mounted on either of the two masses; therefore two LEDs are used for each motor. Try switching the motor positions around. This can also be done by clicking on the motor position switches at the bottom of the green box to the left.

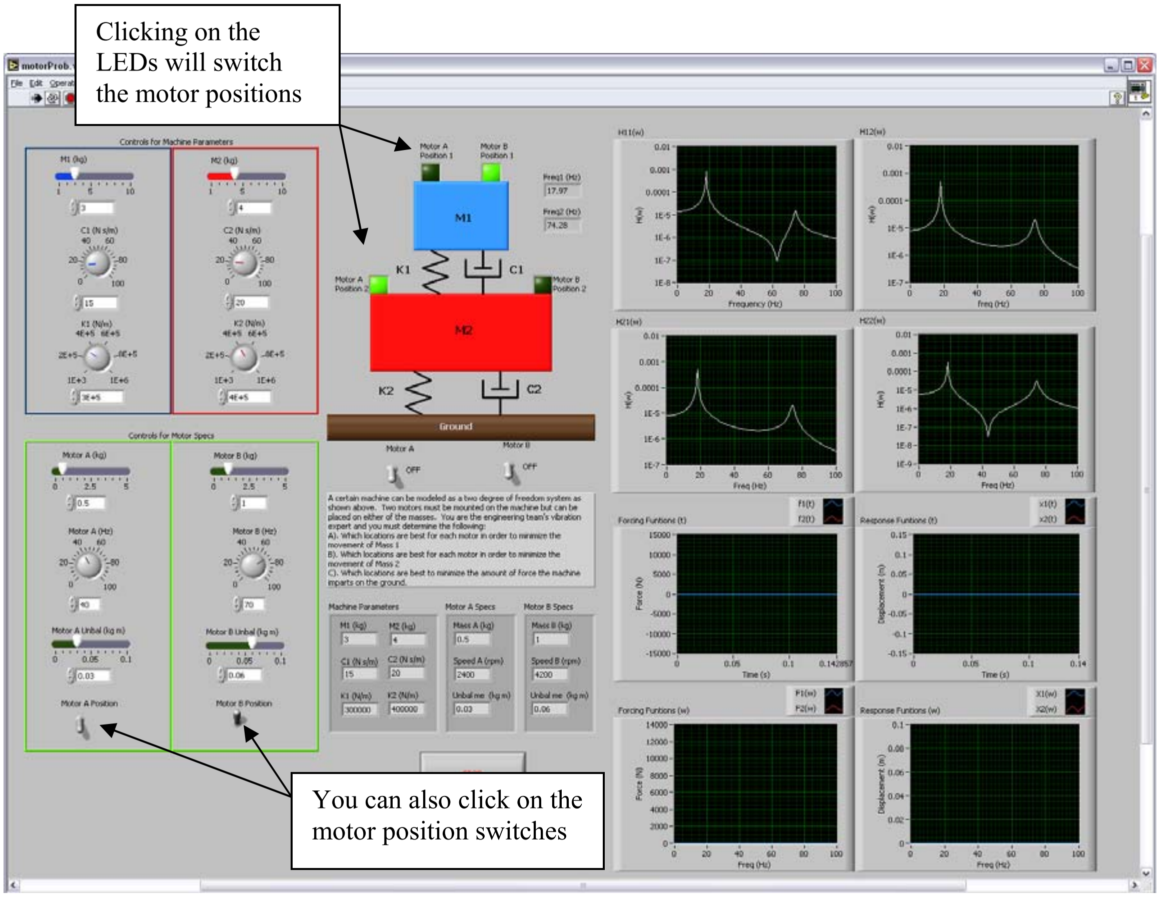

Fig. 2. Motor Position Switches. The motor positions can be changed by clicking on either the LEDs or the switches directly.

Note how the frequency response functions to the right of the diagram are automatically updated when a motor is moved. The mass of the motor is being removed from its previous position and being added to the mass of its new position. To determine what the effective masses are for each combination of motor positions see the system parameters shown in the lower left corner of the screen.

Just below the diagram and above the problem statement are the two switches that turn the motors on and off. Flip each of these to the 'on' position and note how the forcing function and response functions are automatically calculated.

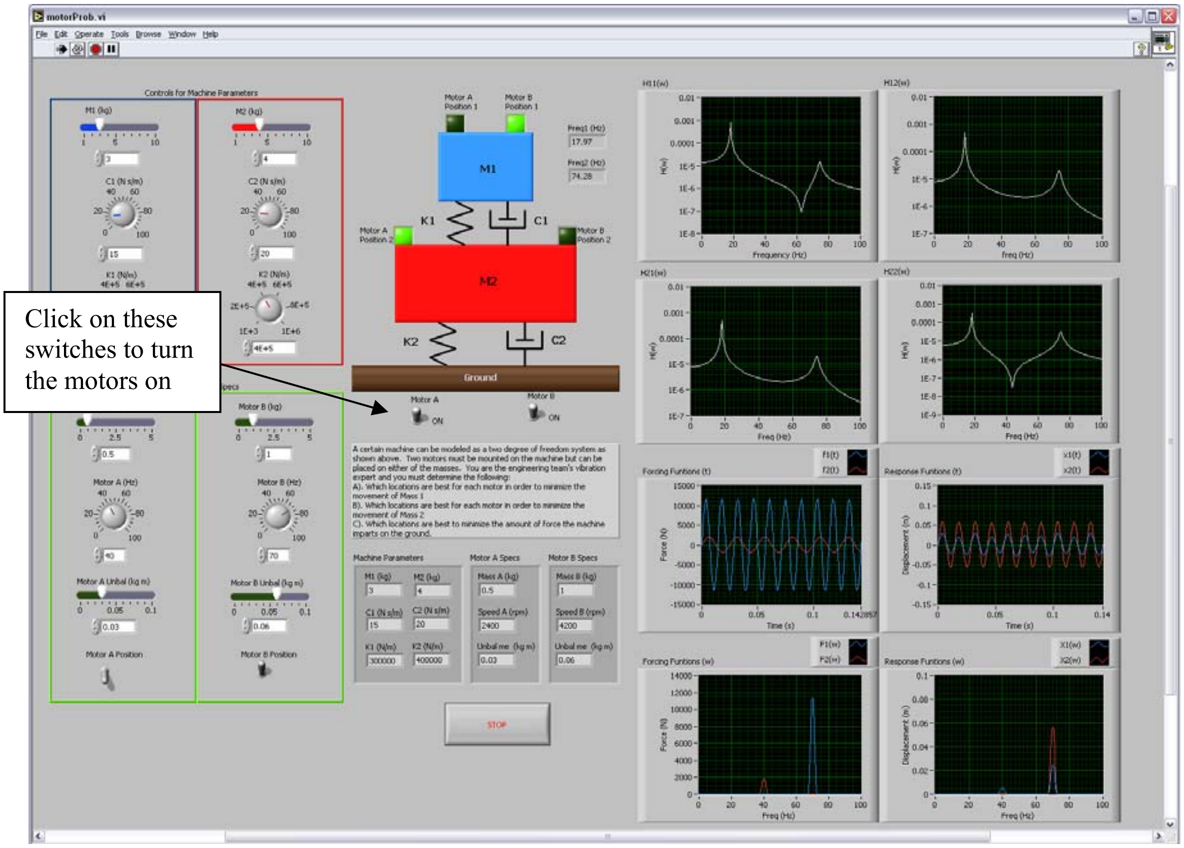

Fig. 3. Motor On/Off switches.

THEORY AND METHODS USED

The method that the program uses for calculating response is to use the equation of motion that defines the system and calculate the impedance matrix.

$$[Z(\omega)] = -\omega^2 [M] + i\omega [C] + [K]$$
$$[Z(\omega)]\{X(\omega)\} = \{F(\omega)\}$$

The impedance matrix, [Z(ω)], is then inverted to find the frequency response matrix, [H(ω)], which is then multiplied by the forcing functions to calculate the response.

$$\{X(\omega)\} = [Z(\omega)]^{-1}\{F(\omega)\} = [H(\omega)]\{F(\omega)\}$$
$$[H(\omega)] = [Z(\omega)]^{-1}$$

The forcing functions are calculated based on the rotating unbalance of each motor. Assuming that the machine is restrained from movement in the horizontal directions, it is clear that the vertical component of the input force from the motors is of the form:

$$F(t) = me\omega^2 \sin(\omega t)$$

These forces are added according to their respective positions on the machine to determine the functions $f_1(t)$ and $f_2(t)$. These are then transformed into the frequency domain using the Fourier transform, $F_1(\omega)$ and $F_2(\omega)$, and then multiplied by transfer function matrix to calculate the response functions, $X_1(\omega)$ and $X_2(\omega)$. Finally, the inverse Fourier transform is used to solve the time domain responses, $x_1(t)$ and $x_2(t)$.

Alternatively, an inverse Fourier transform could be performed on the frequency response function matrix to get $[h(t)]$ which can then be convolved with the time domain forcing functions, $\{f(t)\}$, to attain the response $\{x(t)\}$ directly.

In the interest of saving computation time, we can take advantage of the fact that we know the input frequencies of the two motors, and solve for the responses of the system just at those two frequencies.

For more information regarding the theory of two degree of freedom systems or convolution, see supporting documents.

SOLVING THE PROBLEM

Now that you understand how to use the program, it's time to try out different combinations of motor positions and see if we can answer the original questions. We will run through the problem with the initial settings as an example. Before we begin, make sure that if you've made any changes to the system parameters or the motor specifications that you return them to their original values. Table 1 summarizes the original settings.

Table 1. Original problem settings

M1 = 3 kg	**M2** = 4 kg
C1 = 15 (N s)/m	**C2** = 20 (N s)/m
K1 = 3e+5 N/m	**K2** = 4e+5 N/m

$\mathbf{M_A}$ = 0.5 kg	$\mathbf{M_B}$ = 1 kg
$\boldsymbol{f_A}$ = 40 Hz	$\boldsymbol{F_B}$ = 70 Hz
$\mathbf{(me)_A}$ = 0.03 kg m	$\mathbf{(me)_B}$ = 0.06 kg m

For each possible combination note the largest peak value for each of the masses. Your results should be fairly close to those summarized in Table 2.

Table 2. Summary of different combination results

Motor A Position	Motor B Position	X_1(40Hz)	X_1(70Hz)	X_2(40Hz)	X_2(70Hz)
1	1	0.005 m	**0.007 m**	0.005 m	**0.038 m**
1	2	0.006 m	**0.075 m**	0.006 m	**0.070 m**

2	1	0.005 m	**0.024 m**	0.001 m	**0.056 m**
2	2	0.006 m	**0.076 m**	0.002 m	**0.054 m**

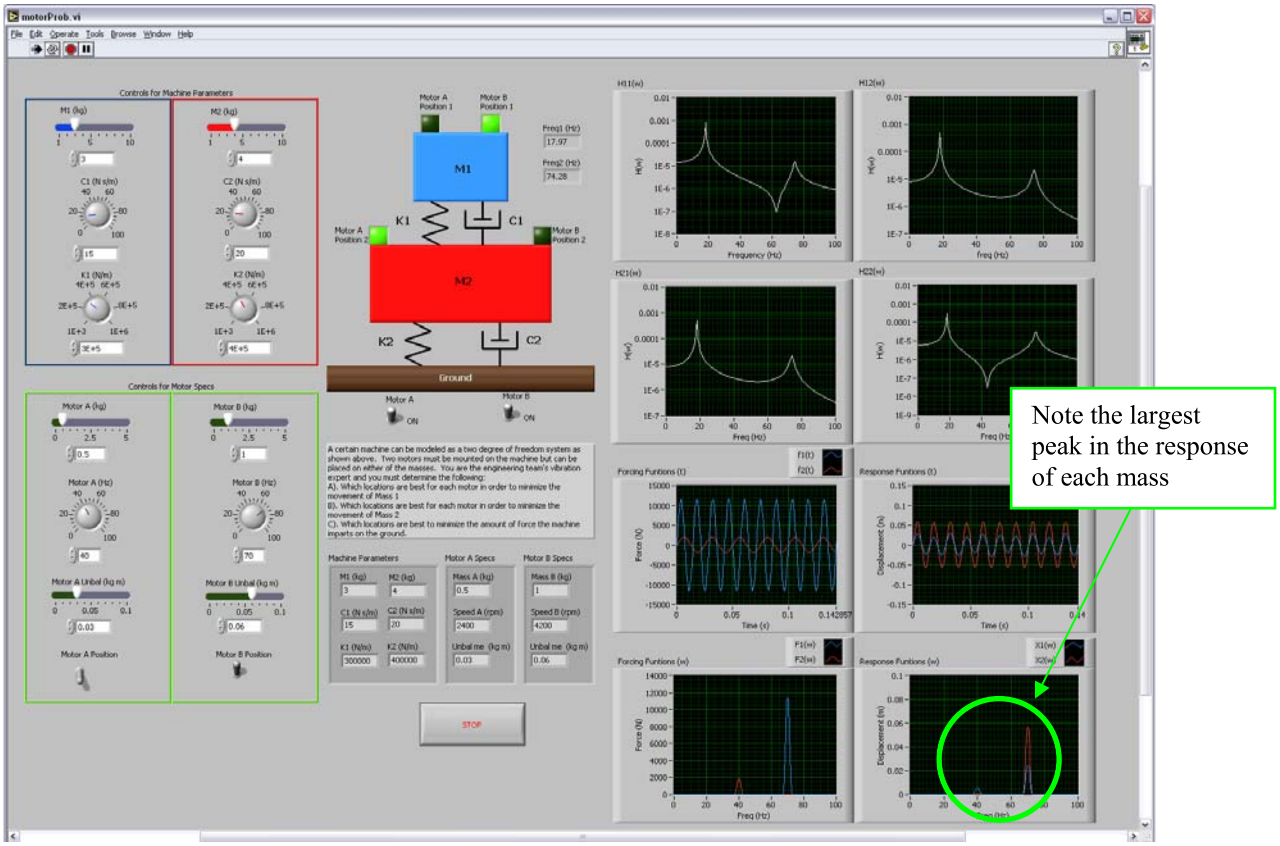

Fig. 4. Checking for the peak response at each motor position combination.

You should have noticed a couple of things from this exercise. First, the unbalance in motor B seems to be exerting a significantly larger force than that in motor A. This makes sense sine the total eccentricity $(me)_B$ is twice as much as eccentricity $(me)_A$, and motor B is running much faster. Recall that the force due to a rotating unbalance is proportional to angular velocity squared.

The second thing that should be noted is how the natural frequencies of the system move with each combination. The natural frequencies are in fact calculated for you, and are indicated as Freq1 and Freq2. When motor A is on the mass 1, and motor B is on mass 2, Freq2 is close to the input frequency of 70 Hz; and thus we observe the worst possible case.

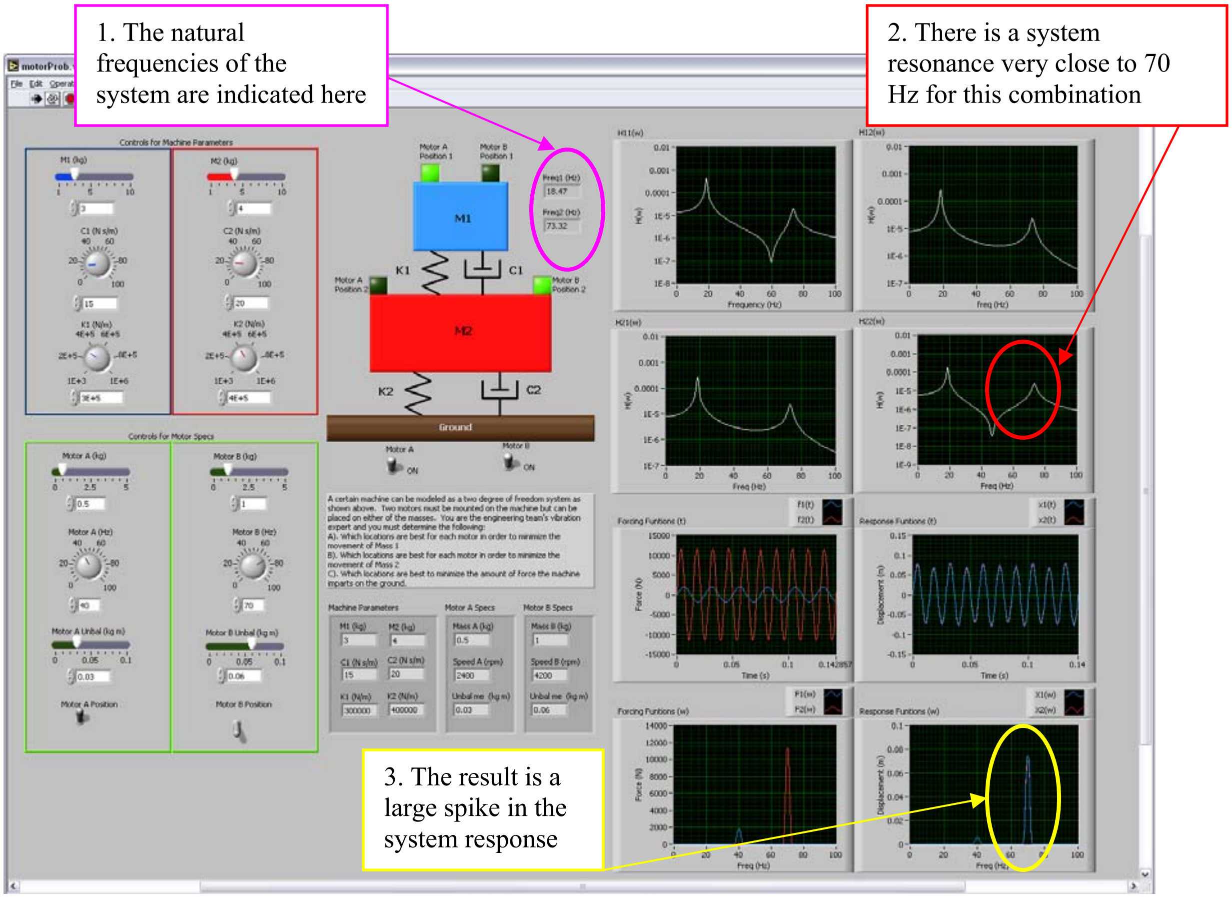

Fig. 5. Worst overall case near system resonance. At this combination there is a very large peak around 70 Hz in the system response.

However, when both motors are mounted on mass 1, the resonance of the system moves away from the 70 Hz input, and we observe our best case. In fact, there appears to be an anti-resonance in the $H_{11}(\omega)$ plot fairly around 66 Hz. And since the force at 70 Hz is being applied on mass 1 for this combination, the response is smaller as a result.

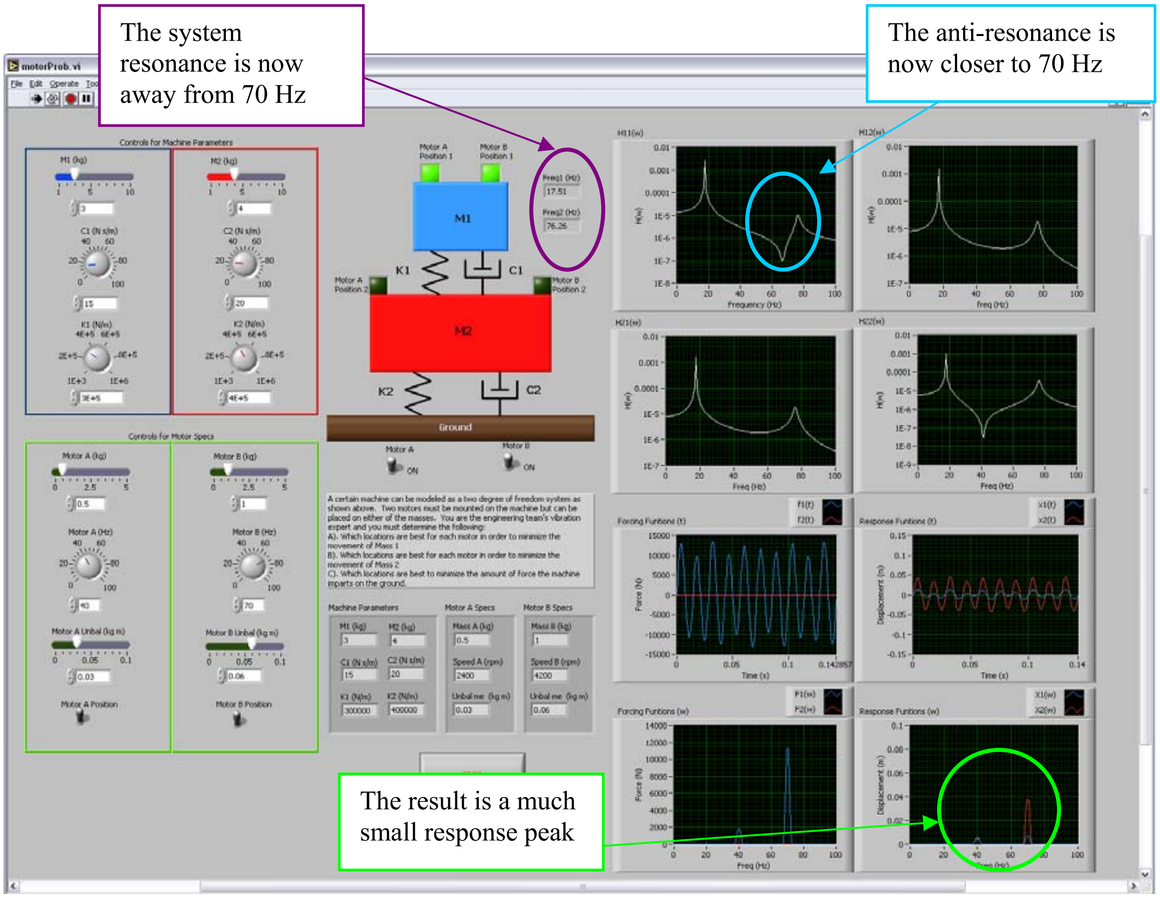

Fig. 6. Best overall case near anti-resonance. At this combination there is a anti-resonance near the 70 Hz input frequency, thus the response is much lower.

We now have enough information to answer the original question. Part A and B are simple; mounting both motors on mass 1 clearly minimizes the movement of both masses for this system. Part C may take a little more critical thinking. Remember that force acting on the ground comes from both the spring and the damper. The force from the spring is dictated by $K_2 x_2$, while the force from the damper is dictated by $C_2 \dot{x}_2$. Assuming that the displacement is of the form:

$$x_2 = X_2 \sin(\omega t)$$

Then the velocity is simply

$$\dot{x} = \omega X_2 \cos(\omega t)$$

The combined force then becomes

$$F_G = C_2 \omega X_2 \cos(\omega t) + K_2 X_2 \sin(\omega t)$$

It can be shown that the total RMS of two signals is given by the following

$$f(t) = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$

$$f_{RMS} = \sqrt{\frac{1}{T}\int_0^T \left[f(t)\right]^2 dt}$$

$$let \quad T = T_1 T_2, \quad \omega_1 = \frac{2\pi}{T_1}, \quad \omega_2 = \frac{2\pi}{T_2}$$

$$f_{RMS} = \sqrt{\frac{1}{2}\left(A_1^2 + A_2^2\right)}$$

Likewise, it can be shown that for multiple signals the total RMS is given by

$$f(t) = \sum_{i=1}^{n} A_i \sin\left(\frac{2\pi}{T_i}t\right) + B_i \cos\left(\frac{2\pi}{T_i}t\right)$$

$$f_{RMS} = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left(A_i^2 + B_i^2\right)}$$

The total amount of energy going into the ground is given by

$$F_G = C_2 \omega_A X_2 \cos(\omega_A t) + C_2 \omega_B X_2 \cos(\omega_B t) + K_2 X_2 \cos(\omega_A t) + K_2 X_2 \cos(\omega_B t)$$

$$(F_G)_{RMS} = \sqrt{\frac{1}{2}\left[\left(C_2 \omega_A X_2\right)^2 + \left(C_2 \omega_B X_2\right)^2 + \left(K_2 X_2\right)^2 + \left(K_2 X_2\right)^2\right]}$$

Based on this equation, we see that the total force that the machine is imparting into the ground is related to the displacement of mass 2, and thus the answer to part C is the same as that of part B.

IMPROVING THE SYSTEM

Now let's say we have a few more options, and it is possible to redesign the machine to have the properties listed in Table 3

Table 3. Properties of new machine

M1 = 3.1 kg	**M2** = 3.9 kg
C1 = 25 (N s)/m	**C2** = 10 (N s)/m
K1 = 4.2e+5 N/m	**K2** = 2.7e+5 N/m

How would this system change the answers to the given criteria? Enter the values into the controls to the left, and run through the exercise again. You'll find that for some combinations the natural resonances line up much closer to the input frequencies. You may have to adjust the y axis on some of the plots to be able to see the peaks. Clearly this is not a good design change unless we find a new pair of motors.

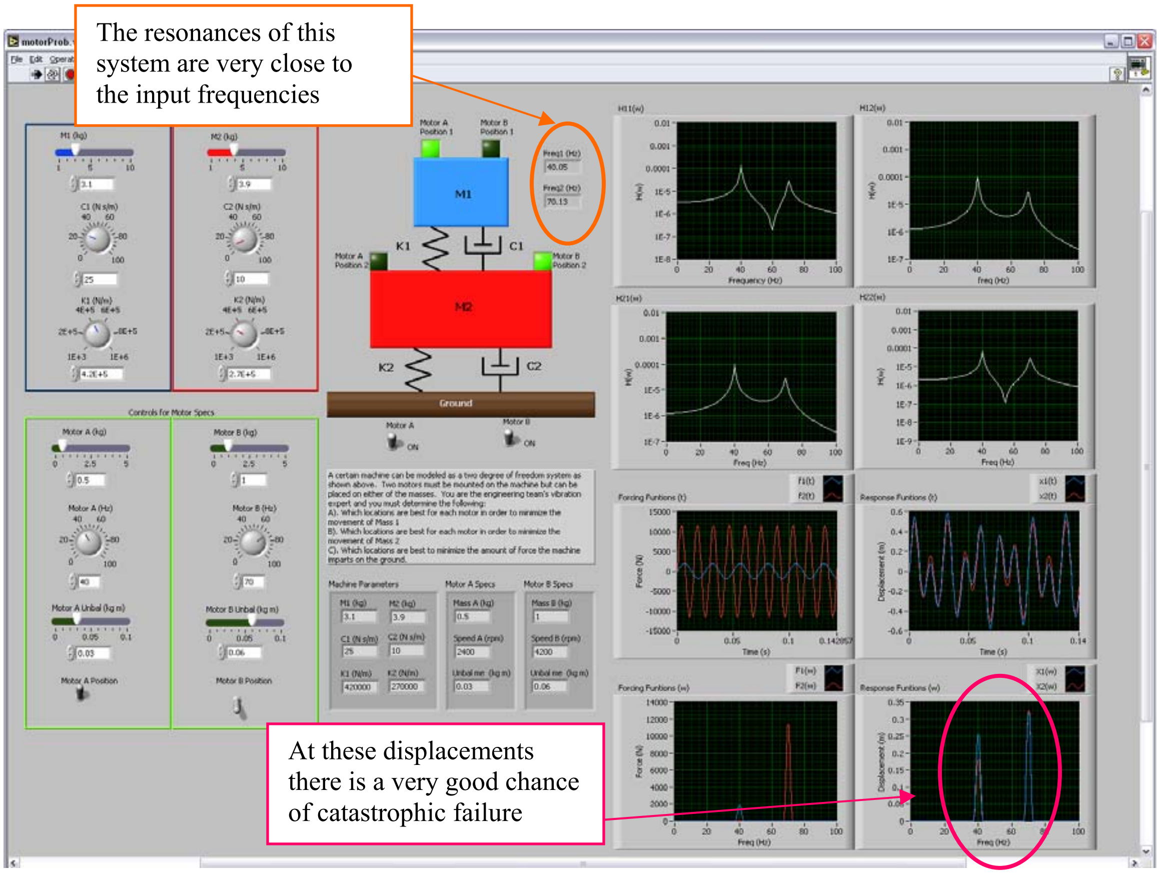

Fig. 7. This system performs very poorly with these motors. When the system resonances line up with both input frequencies the response increases dramatically.

Now let's say we also have the option to replace the motors with specifications shown in Table 4.

Table 4: Specifications of new motors

$\mathbf{M_A}$ = 0.7 kg	$\mathbf{M_B}$ = 1 kg
$\boldsymbol{f_A}$ = 45 Hz	$\boldsymbol{f_B}$ = 60 Hz
$\mathbf{(me)_A}$ = 0.05 kg m	$\mathbf{(me)_B}$ = 0.04 kg m

Do these motors perform better or worse for either of the given machine parameters? How do they compare to the previous motors?

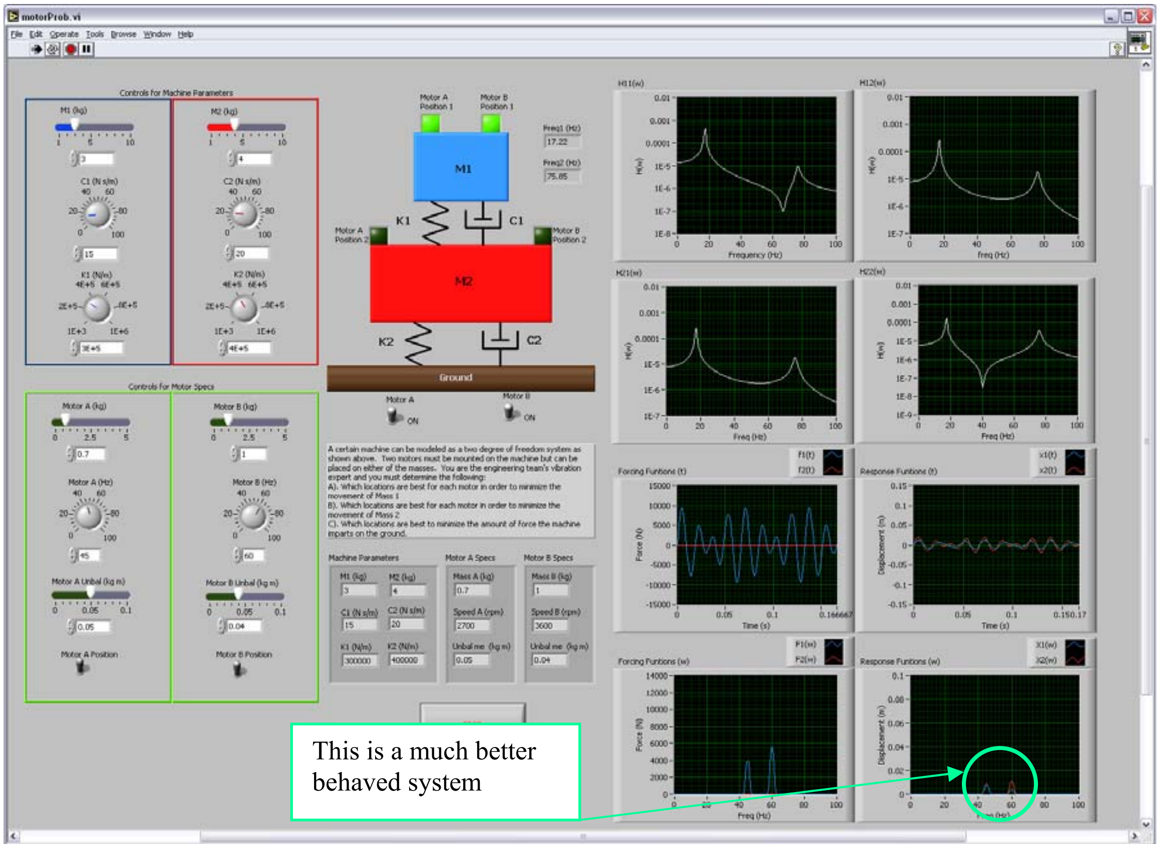

Fig. 8. The new motors have input energy at frequencies in between the system resonances. This way the response is minimal.

You may wish to continue to experiment with the program some more. Try to see what happens when the damping is changed in different areas. Or see how the resonances move when adjusting the stiffness or mass. Perhaps you can find an even better system. Keep in mind that though that in the real world you will always be limited on what modification you can make to a structure, and there are always other things to consider. Having thoroughly explored the program, if you were given this problem in the real world what final design would you recommend to the engineering team? Consider why you would make each decision carefully, and try to think of what other factors you may need to account for.