Tangents

News from the Department of Mathematical Sciences
University of Massachusetts Lowell

Winter 2004/2005

Faculty Research Spotlight

The UML Department of Mathematical Sciences has been active in a wide variety of funded research projects. Here is a sampling of some of the current activity.

Profs. Ken Levasseur, Alan Doerr, James (Kiwi) Graham-Eagle, and Marvin Stick are the UMass Lowell participants in Focus on Mathematics, a National Science Foundation Math and Science Partnership. A central goal of the project is to establish a community in which mathematicians, educators, and administrators work together with students to put rigorous mathematics at the core of their teaching and learning. The partnership, a five-year project running through 2008, is a collaboration with researchers from Boston University, WPI, Lesley College, and the Education Development Center. The school systems in the partnership are Lawrence, Chelsea, Waltham, Arlington and Watertown.

Prof. Steve Pennell is a co-principal investigator in an interdisciplinary, multi-semester project designed to lead students to appreciate the relevance and importance of basic STEM (Science, Technology, Engineering and Mathematics) material. This project, "Multi-Semester Interwoven Project for Teaching Basic Core STEM Material Critical to Solving Dynamic Systems Problems" is sponsored by the National Science Foundation. Pennell's collaborators include Assoc. Prof. Pete Avitabile of the Department of Mechanical Engineering and Prof. John White of the Department of Chemical Engineering. Starting in the sophomore level differential equations course and continuing in junior and senior level engineering courses, students analyze a dynamic system from various points of view, including mathematical modeling.

Since 1989 Prof. Charles Byrne has been collaborating with a research group at the UMass Medical School, Department of Radiology, including Drs. Michael King and Steve Glick. His work is mainly concerned with the development of mathematical algorithms for fast reconstruction of medical images from single photon and positron emission tomography (SPECT and PET). The applications are to tumor detection, weakness in the heart wall, breast cancer and other medical issues. The sponsors are the National Institutes of Health. Their current project involves detecting and correcting for patient motion during PET scans. Motion is a serious problem in a significant

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2004 Alumni Reception— A Great Success

Hoping for a repeat on April 22

The annual UML Mathematical Sciences Alumni Reception and Awards Ceremony was held last May 23. For the first time, it all took place in one location, the Brewery Exchange, and it was a great success. We had a nice mix of alumni, students and faculty, together with family members of all three constituents.

Mary and Russ Bedell congratulate Richardson-Bedell award winner Steven Ward.

Student awards were presented to

- Steven Ward, a Richardson-Bedell Scholarship
- Brian Krejca, a Richardson-Bedell Scholarship
- Ryan Hill, a Mary Hall Prize
- Sine Pol, a Mary Hall Prize
- Melissa Fay Bartos, for Outstanding Graduate Student
- Brian Francis, for Outstanding Graduate Student
- Vijaya Vanga,for Outstanding Undergraduate Student
- Kristin Jenkins, for Outstanding Undergraduate Student

COMAP Modeling Team members were also recognized.

- Jeremy Achin, Tim Owen and Brian Krejca
- Patrick Joyce, Brian Petrowicz and Alex Smith

For photos of the event, visit the UML Mathematical Sciences Web page.

We hope to have even more of you in attendance this year at the Brewery Exchange on Friday, May 22, when we present many of the same student awards and the first annual Arthur Zamanakos Scholarship. April

Kiwi presents Vijaya Vanga with the Outstanding Undergraduate Student award. Vijaya is a former Richardson-Bedell winner.

Prof. Mary Hall

Kiwi's Korner

It's fall 2004, and this is the seventh installment of Tangents. It is also the beginning of my second term as department chair.

We are all quite excited because the chancellor has given the green light to hire four new full time faculty in mathematics with the possibility of four more next year. After a substantial number of retirements two years ago the department has been forced to increase class sizes and rely more on part time faculty—a change in this trend is certainly most welcome. We have been extremely fortunate to have a group of outstanding part time faculty members who return semester after semester to help us out. I would like to take this opportunity to thank them for their contributions to the department and the education of our students.

James Graham-Eagle

The result of the Consortium for Mathematics and its Applications Competition in Modeling are in and one of our teams performed extremely well. Jeremy Achin, Tim Owen and Brian Krejca worked on devising a QuickPass system for an amusement park in which customers could decrease their time in line by "reserving" a space for a later time. Existing systems have met with only limited success for several reasons, among them that the QuickPass lines are often no shorter than the regular lines, and that the wait time for a QuickPass user can depend erratically on the time it was issued. A full description of the problem can be found at www.comap.com/undergraduate/contests/mcm/contests/2004/ problems. The contest this year attracted 599 teams from 11 countries. The UMass Lowell project was judged Meritorious which ranked in the top 11% of entries, ahead of teams from University of Michigan, Cornell, Northwestern, Worcester Polytechnic and Boston University. A complete summary of the participating institutions and the results is available in pdf format at http://www.comap.com/undergraduate/contests/mcm/2001Resultsreport.pdf.

This semester we were honored to have a brief visit from Arthur and Alice Zamanakos. Last year they donated $200,000 to set up the "Arthur Zamanakos Scholarship" for outstanding mathematics students in their junior or senior year. The couple visited a mathematics lecture, had lunch with the chancellor and other guests and a tour of the campus. There was much reminiscing during lunch about growing up in Lowell more years ago than anyone cared to admit. The following evening Arthur was awarded the 2004 Circle of Distinction Award, which honors benefactors who have demonstrated exceptional commitment to the University's mission of education, innovation and community outreach.

So that's what's been happening here. If you have news which you think others would enjoy hearing please let us know—we would be glad to include it in future issues of this newsletter.

Kiwi

Dr. Everett V. Olsen

In September, Dr. Everett V. Olsen passed away. Dr. Olsen had enjoyed a long tenure at the predecessor institutions on the University. He joined Lowell Technological Institute as assistant to the President in 1948; was named executive vice president in 1969 and acting president upon the retirement of President Martin J. Lydon. In 1974, LTI merged with Lowell State to become the University of Lowell. Dr. Olsen was acting president of the newly created institution for two years. When the trustees appointed Dr. John Duff as the president of ULowell, Dr. Olsen returned to the position of executive vice president where he remained until he retired in 1981. He maintained his connection with the University by acting as a special assistant to the president for development and fund-raising until 1988.

As you know the building in which the Mathematical Sciences department was housed for many years was named for Dr. Olsen. His son Alexander has been a member of the faculty of the Mathematical Sciences department for many years. We extend the sympathies of the department, faculty, students and alumni to the Olsen Family.

The *Tangents* Problem

A Knotty Problem

You are served a plate containing 100 spaghetti noodles. You randomly grab two ends from the pile and tie them together. Then you repeat this process until there are no ends left. What is the expected number of loops at the end?

Four correct solutions from among all that are submitted by March 1 will earn a "Math Challenge" t-shirt.

Solution to previous problem

Spring 2004: It is easy to use 12 matches to make a polygon of area 5 (see diagram in the Spring 2004 issue). Can you use 12 matches to make a polygon of area 4? To make it more interesting, each vertex of the polygon must be at a point with integer coordinates (where the matches have unit length). Note: It must be a true polygon, with no crossings.

Issam Badr '01, Andrew Miller '66 Physics, '72 Math, and Andrew Golay '03 all solved the problem. Issam and Andrew Miller will be getting a Math Challenge t-shirt. Andrew Golay has declined a second shirt, which is why we are giving away four shirts this time.

James Graham-Eagle, Chair

Writers: Ken Levasseur, Raj Prasad, Ann Marie Hurley, Dan Klain, Guntram Mueller, and Alex Olsen

Tangents is produced biannually by the Publications Office for the Department of Mathematical Sciences. Your comments are welcome.

Mathematics in the News

Shizuo Kakutani: A Mathematician for the Ages

A drunk man will find his way home, but a drunk bird may get lost forever[1]

Strolling along together after a meeting one summer in Amherst were, from left, S. Alpern of LSE (one of Raj Prasad's favorite collaborators); A. Hajian of Northeastern, a former student of Kakutani; Marshall Stone of Harvard; Alexandra Bellow of Northwestern, another of Kakutani's former students; and Shizuo Kakutani.

Shizuo Kakutani, mathematics professor at Yale University and one of the great mathematicians of the last century, died at the age of 92 this past August in Hamden, Conn. He is survived by his wife, Kay, and daughter, Michiko.

Widely known to most mathematicians, he has proved fundamental results in complex analysis, topological groups, functional analysis, probability theory, Brownian motion, topology and ergodic theory. Among economists and non-mathematicians, he is probably most famous for his fixed point theorem (*A generalization of Brouwer's fixed point theorem* [Duke Math J.,1941]), which is used extensively by mathematical economists. A 1982 survey by G. Debreu cites more than 350 instances where Kakutani's fixed point theorem is the main tool for proving the existence of an economic equilibrium. This theorem is the key step in the work of three researchers who received Nobel Prizes in economics.

John Nash (the subject of Sylvia Nassar's book *A Beautiful Mind*) used it in his original 1950 proof of the existence of Nash equilibria, work for which he received the Economics Nobel almost 40 years later in the 1990s. In 1954, the economists Arrow and Debreu used Kakutani's fixed point theorem to prove there are prices for goods that balance supply and demand in a complex economy. Arrow and Debreu's work was awarded the Economics Nobel prize in 1972.

[1] A joke by Shizuo Kakutani at a UCLA colloquium talk as attributed in Rick Durrett's book Probability: Theory and Examples. Kakutani's joke is an application of Polya's result that a simple random walk is recurrent in dimensions 1 or 2, but transient in 3 dimensions or higher.

A Personal Recollection by Raj Prasad

Much of what I have written above can be found on the Web. For more details about Kakutani's life, you may go to http//:faculty.uml.edu/vprasad/ KakutaniTangents.pdf.

I was fortunate, at the start of my career in the late'70s, to work with Kakutani when I was a postdoctoral fellow at Yale. Kakutani was a warm and gracious person who was genuinely concerned about his students and colleagues.

Contrary to the public perception that mathematical wizards are eccentric or crazy, Kakutani was a level-headed and delightful person to be around. His seminars and classes were exciting events in which to participate. I saw many theses come from these sessions.

Friends and former students of Shizuo would regularly meet with him in New Haven or nearby. The photo accompanying this article was taken at one such summer meeting in Amherst, Mass. After having lunch we would talk mathematics for the afternoon, and then all head back separately to our homes. Sessions with Kakutani were always fun, whether it involved discussing mathematics, mathematicians or if the Red Sox would ever finally win the World Series.

Fixed point theorems

Consider the following fixed point problems in different dimensions:

In dimension 1 (you may recall your Intermediate Value Theorem from Calculus I days)

One morning, exactly at sunrise, a monk began to climb a tall mountain. The narrow path, no more than a foot or two wide, spiraled

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Faculty Research Spotlight

fraction of heart patients and typically results in misdiagnosis. Most recently Byrne presented a paper at the annual medical imaging conference, held this year in Rome.

Assoc. Prof. Dan Klain and Asst. Prof. Konstantin Rybnikov are collaborating with Asst. Prof. Karen Daniels of the Department of Computer Science in a joint NSF-DARPA funded research program into the mathematics of 2D and 3D imaging. Their project, "Verification of Properties of Geometric Structures and Reconstruction of Geometric Objects from Partial Information," has three major themes: verification, reconstruction and approximation using partial information, and focuses on three major groups of problems: Finite containment, packing and covering problems; the verification of convexity and reconstruction of convex and locally convex bodies from partial data; and the verification of topological properties of a body, such as the Euler characteristic, from a finite sampling. Also related to this topic is the determination of the topology of a 3D body from projections data in various directions.

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Mathematics in the News

around the mountain to a glittering temple at the summit. The monk ascended the path at varying speeds, stopping many times along the way to rest and to eat the dried fruit he carried with him. He reached the temple shortly before sunset. After several days of fasting and meditation he began his journey back along the same path, starting at sunrise and again walking at variable speeds with many pauses along the way. His average speed descending was, of course, greater than his average climbing speed. Prove that there is a spot along the path that the monk will occupy on both trips at precisely the same time of day.[2]

Brouwer's fixed point theorem in dimension 2:

Take two pieces of 8 by 11 paper and lay them on top of one another so that every point on the top paper corresponds with a point on the bottom paper. Now crumple the top piece of paper in any way that you wish (without tearing) and place it back on top.

Brouwer's theorem tells us that there must be a point which has not moved laterally, i.e. which lies exactly above the same point that it did initially.

Brouwer's fixed point theorem in dimension 3: *You have a cup of coffee in front of you to which you give a quick stir. After the coffee has stopped moving, Brouwer's theorem tells you that there is a point which is in its original spot before the stirring.*

Brouwer's theorem in dimension n (guide: think of S as our coffee $x, f(x)$ is where the point x has moved to after the stirring).

If $x \rightarrow f(x)$ is a continuous point-to-point mapping of an n-dimensional closed simplex S into itself then there is a point x_0 such that $x_0 = f(x_0)$ (i.e., every continuous stirring leaves some point x_0 in its original location).

Kakutani's fixed point theorem, generalizing Brouwer's result:

Let S be an n-dimensional closed simplex and consider $C(S)$ the family of all non-empty closed convex subsets of S A point-to-set mapping $x \rightarrow F(x) \in C(S)$ of S into $C(S)$ is called upper semi-continuous if whenever $x_1 \rightarrow x$ and $y_1 \in F(x_1)$ and $y_1 \rightarrow y$ then $y \in F(x)$. A point-to-set map F is sometimes called a correspondence.

If $x \rightarrow F(x)$ is an upper semi-continuous point-to-set mapping of an n-dimensional closed simplex into $C(S)$, then there is a point x_0 such that x_0 is in $F(x_0)$. We call x_0 a fixed point for the correspondence F.

The proof is a typical Kakutani argument: clear, concise and aesthetic. He applies Brouwer's fixed point theorem to a sequence of point-to-point maps f_k to produce a sequence of points x_k (the f_k are related to F, via the k'th barycentric subdivision of the simplex, to get technical). The sequence x_k has a limit point x_0 which is the required fixed point. Furthermore this theorem is valid even if S is an arbitrary closed bounded convex set in Euclidean space. Brouwer's theorem is a special case when each $F(x)$ consists of only one point $f(x)$. In this case the upper semi-continuity of F is nothing but the continuity of f.

1 A joke by Shizuo Kakutani at a UCLA colloquium talk as attributed in Rick Durrett's book Probability: Theory and Examples. Kakutani's joke is an application of Polya's result that a simple random walk is recurrent in dimensions 1 or 2, but transient in 3 dimensions or higher.

2 Imagine there are two monks, one going down and one going up, each beginning on the same day at sunrise. At some the point in the day, the two monks must meet!

What Are You Up To?

Want to keep your classmates up to date on what you're doing and where you are? Take a few moments to tell us where you are, and whatever else you might like to share. We'll add it to the UML Math Alumni page on the Web:www.uml.edu/dept/math/alumni.htm.

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Thanks For the Contributions!

Our thanks to all who have contributed to the Department of Mathematical Sciences over the past few years. Your generosity has enabled us to make purchases, award scholarships, and engage in activities that would otherwise have been impossible.

Many of you have responded generously to the UML phonathon and other fundraising contacts. These contributions can benefit the Department of Mathematical Sciences directly if you specify that you wish to have your gift applied to Mathematics. Otherwise, it will provide valuable assistance to the University at the college level.

The Mathematical Sciences Web Page

Have you visited the Mathematical Sciences Web page lately? The address http://www.uml.edu/dept/math. You don't need to remember the address—just Google "uml math" and the first link should be ours.

Tired of typing in long addresses that you've see in Tangents? Go to the alumni section of the UML Math Web page for a list of past and present links.

Tangents
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