Random Tilings and Random Surfaces

Jim Propp’s work on random tilings was featured in an article in Volume 7 of Dana Mackenzie’s “What’s Happening in the Mathematical Sciences” (American Mathematical Society, ISBN 0-8218-4478-4; $19.95). The 14-page article, entitled “Dominoes, Anyone?,” describes how “an innocent puzzle about tiling a checkerboard with dominos leads, ultimately, to a new model of random surfaces.” The figure below shows a tiling of the regular hexagon of side-length 20, using 1200 unit-rhombus tiles. This tiling was generated using Jim Propp and David Wilson’s “coupling from the past” method of Monte Carlo simulation, which guarantees that each of the possible tilings (of which there are over $10^{39}$) has exactly the same chance of being generated as every other. The tiles are shaded according to their orientation.

Two features of the picture leap to the eye: the tiling does not look random throughout the hexagon (tiles in each corner are lined up with their neighbors) and the tiling does not look flat (the eye readily supplies a third dimension). These two features are deeply related on a mathematical level: the relevance of three-dimensional geometry to what initially seems like a two-dimensional problem turns out to be one of the keys that permits one to explain why the visible randomness of a random tiling is pushed toward the middle.

Jim has written his own personal account of this work, available at http://jamespropp.org/whytile.html.

A Review of The Drunkard’s Walk: How Randomness Rules Our Lives


By RAVI MONTENEGRO

It is not often that a book about mathematics makes it on the New York Times bestseller list. But Leonard Mlodinow is not your typical academic, having quit college to work on a kibbutz, quit a faculty position at Caltech to work on TV and movie scripts, and having created Scholastic’s children’s games division.

Now that he has returned to Caltech, Mlodinow has written this excellent book about the role of randomness in our lives. One might say that The Drunkard’s Walk is a layman’s introduction to probability. This appears to be an oxymoron, achievable at best through a collection of anecdotes as in the ego trip of Nassim Taleb’s Fooled by Randomness, and yet, remarkably, Mlodinow manages to make the book highly readable while still being thorough.

A typical chapter in this book starts with an interesting tale about a topic in probability. An extensive discussion then ensues, mixing history, human psychology, and stories in which this topic played a role in people’s lives. When math is introduced it is generally through the guise of some easily accessible question involving randomness, with computations written out in prose and requiring no more than basic arithmetic.

One of the best of these discussions is the author’s illustration of Bayes Theorem. In 1989 he tested positive for HIV and was told there was a 99.9% chance he would be dead within a decade, as only 1 in 1,000 HIV tests produce a false positive. Mlodinow explains the error of this reasoning, and then leads the reader through a simple calculation which leads to the conclusion that among individuals with similar risk factors as himself 10 out of 11 positive tests will be false positives. Almost without noticing it the reader has learned about conditional probability, Bayes Theorem, and the need to consider an appropriate population when computing statistics.

This is not to say that everything is done rigorously. For instance, it is hard to imagine a calculation involving the full power of the Central Limit Theorem without stating a formula. Nevertheless, The Drunkard’s Walk communicates the

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Steve’s Sector

Spring 2009—From the Chair

For fans of winter (like Al Doerr), this has been a great year. We even had a couple of snow days to enjoy. It has been an active time in the math department, and there are many people to congratulate:

• One of our long-time adjunct faculty members, Arthur Forgetta, is the first member elected to the North Andover School District Educator Hall of Fame. One of his former colleagues wrote “Arthur worked tirelessly to help students, regardless of whether or not he had them in class. Always approachable, supportive and understanding, he nevertheless held students to high standards ... He cared first and foremost about what was best for the student.”

We are lucky to have him teaching with us. Congratulations, Art.

• Paul Bevillard, Anthony Giambalvo, Michelle Hanly, Dennis Holbrook, Matt McCreary, and Gerard Wilson completed their BS degrees in December. Best wishes to all of them for continued success.

• Sophomore math major Angus MacDonald ran in the NCAA Indoor Track and Field Championship on March 13 and 14. Angus is ranked 11th in the country in the mile. Congratulations, Angus.

Congratulations also to our math modeling team, Marco Bonett-Matiz, Bryan Crompton, and Alexander Frieden, and their coach Kiwi. The team spent the first weekend in February competing in the 2009 Mathematical Contest in Modeling. Teams have a choice of two problems to work on; our team chose to “to use a model to determine how best to control traffic flow in, around, and out of a [traffic] circle.” Results of the competition will be announced later this spring. For more information, go to http://www.comap.com/undergraduate/contests/mcm/ or Google “MCM competition.”

As you may have heard, careercauch.com recently rated the best and worst jobs in America. Not surprisingly to any of us, Mathematician led the list of best jobs, followed by Actuary (#2) and Statistician (#3)—a clean sweep for math this year. The study considers working environment, income, outlook for the future, and physical demands, and stress in determining rankings.

April is Math Awareness Month. This year’s theme is Mathematics and Climate, a hot topic to be sure. For more information, go to http://www.mathaware.org or Google Math Awareness Month.

I hope everyone had a happy Pi Day on 3.14. This year we had another holiday to celebrate: Square Root Day. 3/3/09 was Square Root Day because 3 is the square root of 9. I’ll leave it to you to figure out when the next Square Root Day will be.

Please keep in touch, and stop in to visit the next time you are in the area. Remember to check out our web site for items of interest. If you can, please join us for our annual Alumni Reception/Student Awards Ceremony on May 1.

Continued from page 1

A Review of The Drunkard’s Walk: How Randomness Rules Our Lives

The essence of the theorem and convinces the reader of its importance. Entertaining applications are taken from baseball (the chance a player of Roger Maris’ talent would beat Babe Ruth’s record for home runs in a season), from basketball (showing college basketball players are probably taking bribes), and from Wall Street (was Bill Miller’s once famed investment success convincing proof of his talent?), among others.

Mlodinow’s tales can also be quite damning for those who think math is just for “rocket scientists.” For instance, he leads us to wonder if O.J. Simpson’s trial would have ended differently had the prosecution had a stronger grasp of probability. The defense noted that only 1 in 2,500 battered women are later killed by the abusive partner. However, the relevant statistic here is a conditional probability: the chance that a battered wife who was murdered was murdered by her abuser. The answer: 90%!

If you have ever wanted to learn about the role of probability in our lives, and perhaps pick up some math in the process, then I highly recommend this book.

Alumni Update

Michael McGovern (B.S., 1986) writes: For the past 20 years I have been working in the Information Technology field. 15 years in high technology for companies like Telebit Corporation, 3 Com Corporation, and Unisphere Networks. For the past 5 years I have been an Assistant Vice President of Information Technology at the Cambridge Trust Company, a financial institution in Cambridge MA.

Patty (Valentine) Powers (B.S., 1983) writes: I still keep in touch with 2 other math majors (we were the 3 musketeers)—Peggy Donovan and Patty O’Donnell. Math was a very tough major but it really taught me in the “real” world about mathematical concepts which I have been using throughout my career. I think the most important quality I’ve learned is logic. Such a simple thing we say... but when working in SW design and development, you must create all types of scenarios that could occur in order to “break” the system.

This has helped me to design and develop large enterprise applications for many large companies (IBM, Nissan, Staples, etc...). They have all been pleased with my problem solving techniques. I currently work for IBM as a Sr. IT Application Architect using BMC Remedy (HelpDesk Suite) of Applications. It’s been a wonderful journey!!!
2009 mod 100

This year marks the 2nd centenary of the French mathematician Joseph Liouville (1809-1882) known for his work with complex variables, partial differential equations, and transcendental number theory. A real number $x$ is transcendental if it never appears as the root of any polynomial equation having integer coefficients, otherwise $x$ is said to be algebraic. Although most numbers we write down in everyday life are algebraic, a random number chosen between 0 and 1 is transcendental with probability 1. That is, real numbers are “almost always” transcendental. Liouville was the first to prove the existence of transcendental numbers, and also the first to exhibit a transcendental number. His first example was:

$$0.11001000000000000000010000...$$

where there is a 1 in each $n!$ position

(that is, 1st, 2nd, 6th, 24th, ...), and where 0 appears elsewhere in the expansion. It has since been shown that $e$ and $\pi$ are transcendental, as is $\sin(x)$ whenever $x$ is a non-zero algebraic number.

This year also holds the 100th anniversary of algebraists Claude Chevalley (1909-1984), Saunders MacLane (1909-2005), and of the topologist and mathematical physicist Stanislaw Ulam (1909-1984). Chevalley did foundational work in group theory and algebraic geometry, and was an original member of the Bourbaki group. MacLane is known for his work on cohomology and category theory.

Ulam was perhaps more remarkable and controversial. A childhood prodigy in Poland, Ulam initially made his mark in topology with what is now called the “Borsuk-Ulam theorem.” During WWII Ulam moved to the US, where he eventually joined the Manhattan Project. It was during this time that he invented the Monte-Carlo method of statistical sampling, now used throughout the physical sciences. Ulam was also the co-inventor, with Edward Teller, of the hydrogen bomb.

Ulam’s autobiography, “Adventures of a Mathematician,” is still in print and fun to read! Ulam was known for an uncanny ability to make accurate estimates, as well as for a paradoxical combination of genius and laziness that resulted in mathematical solutions of unusual brevity and brilliance. Gian-Carlo Rota, a friend and long-time colleague of Ulam, later wrote that “[Los Alamos physicists], together with Enrico Fermi and Richard Feynman,” but also that “...the traits of Stan Ulam’s personality that became dominant in his later years were laziness, generosity, considerateness, and most of all incisiveness of thought.”

References:

The Tangents Problem

Twenty-one balls of radii 20 mm, 21 mm, ..., 40 mm are packed into a pipe of internal radius 40 mm.

(a) If the balls are packed in order of size, from largest to smallest, how long must the pipe be in order to fully contain the balls?

(b) What is the shortest length of pipe that fully contains the balls if the balls can be packed in any order?

Give your answer in micrometers (10^-6 m) rounded to the nearest integer.

Up to four correct solutions from among all that are submitted by September 1, 2009 will earn the solver a “UML Math” T-shirt. You may submit your solution to mathematics@uml.edu or mail it to Ken Levasseur, Department of Mathematical Sciences, North Campus/Olney Hall, UMass Lowell, Lowell MA 01854.

Solution to the Fall 2008 Problem

The problem was taken from Project Euler (http://projecteuler.net).

Peter has nine four-sided (pyramidal) dice, each with faces numbered 1, 2, 3, 4

Colin has six six-sided (cubic) dice, each with faces numbered 1, 2, 3, 4, 5, 6.

Peter and Colin roll their dice and compare totals: the highest total wins. The result is a draw if the totals are equal. What is the probability that Pyramidal Pete beats Cubic Colin? Give your answer rounded to seven decimal places in the form 0.abcdefg.

Since we don’t want to give away the complete solution in case you want to try your hand at this series of problems, we’ll just say that the solution is 0.57cdefg. We had two correct solutions, submitted by Maureen Hamilton and Elliot Moy. They will both get one of our new UML Mathematical Sciences T-shirts.

The Mathematical Sciences Web Page

Have you visited Mathematical Sciences web page lately? Our new address is http://faculty.uml.edu/math

Have you lost your past issues of Tangents?
Go to the alumni section of the UML Math web page for links to back issues.

Steve Pennell, Chair
Writers: Ken Levasseur, Ann Marie Hurley, Dan Klain, Ravi Montenegro, Guntram Mueller (emeritus), Alex Olsen, Raj Prasad, Jim Propp, Marvin Stick

Tangents is produced biannually by the Publications Office for the Department of Mathematical Sciences. Your comments are welcome.
Teaching with Technology

Over the past few years, technology has gradually been creeping into the teaching of math courses at UML. Here is a sampling of some of that activity.

Homework. Online homework is becoming quite common in mathematics. At UML we have been using a few different systems, including WebWork. A demonstration course that gives a flavor of WebWork can be viewed at http://webwork.uml.edu/webwork2/UML_Demo.

Mathematica. Computer algebra systems such as Mathematica have developed into both a teaching and research tool. See the separate note on Jim Propp’s research in tiling. It is a good example of how computer algebra systems can enhance research.

One of the ways that Mathematica is used in teaching is through the Wolfram Demonstrations Project. Instructors can browse through thousands of demonstrations that illustrate concepts in their courses or they can develop their own demonstrations. Students can then view the demonstrations using a free “Player” application. For information on how to download the Player, and some demonstrations developed at UML, go to http://faculty.uml.edu/klevasseur/mathematicademos/.

Another example of how Mathematica is used in teaching is Exploring Abstract Algebra with Mathematica (EAAM), developed by UML’s Ken Levasseur and Al Hibbard (Central College). This project includes a microworld of abstract algebra in the form of a collection of Mathematica functions. It also includes a series of labs that guide students through concrete examples that illustrate abstract algebra. For more information on EAAM, go to http://central.edu/eaam/.

Ken is currently using these labs as part of an online course in discrete math, which leads into the next technology, online courses.

Online and Blended Courses. UML has been offering online courses, including mathematics courses, since 1996 and is part of the UMass Online System. Most recently, courses in precalculus, discrete math, statistics, and problem solving have been on the schedule.

We also offer an online course in applied mathematics as part of the multi-campus UMass Biotechnology Doctoral Program. Although the majority of math courses are face-to-face, some have an online component, and are referred to as a blended or hybrid courses. This format facilitates discussions and collaboration between student and the instructor outside of the normal class times.

Thanks For the Contributions!

Our thanks to all who have contributed to the Department of Mathematical Sciences over the past few years. Your generosity has allowed us to make purchases, award scholarships, and engage in activities that would otherwise have been impossible.

Many of you have responded generously to UML phonathon and other fundraising contacts. These requests can benefit the Department of Mathematical Sciences directly if you specify that you wish to have your gift directed to Mathematics. Otherwise it will provide valuable assistance to the University at the College level.

What Are You Up To?

Want to keep your classmates up to date on what you’re doing and where you are? Take a few moments to tell us where you are, and whatever else you might like to share.

We can be contacted by mail at Department of Mathematical Sciences, North Campus, UMass Lowell, Lowell MA 01854.

Telephone: (978) 934-2410.

Email: mathematics@uml.edu

You might also wish to contact our Office of Alumni Relations, Southwick Hall 250, UML North, Lowell MA 01854-3629. Toll free telephone: (877) UML-ALUM.

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