A Review of *The Art of Mathematics: Coffee Time in Memphis*

by Béla Bollobás. xvi + 359 pp. Cambridge University Press, 2006.

BY JAMES PROPP

The mathematician and puzzle-connoisseur Peter Winkler once joked, with a nod to Isaac Newton, "If I have seen farther than others, it is because I have stood on the shoulders of Hungarians." One of these Hungarians is the late Paul Erdös, famous within mathematics for his contributions to number theory and combinatorics and famed more broadly for his unique lifestyle and lingo (children are "epsilons," God is the "Supreme Fascist", God's collection of the best mathematical proofs is "The Book", and so forth). Many of Erdös' collaborators and successors are also Hungarian, and others have adopted what might be called "the Hungarian style," with an emphasis on snappy problems and clever solutions. I can think of no better way to get acquainted with these people and their work than to spend a few months periodically dipping into Béla Bollobás's new collection of mathematical puzzles, titled The Art of Mathematics: Coffee Time in Memphis.

Bollobás (the name is pronounced "bowl o' bosh") is one of the most ardent keepers of the Erdös flame. Since Erdös died - or, as Erdös would say, "left"—in 1996, Bollobás has organized a conference in his honor every year at the University of Memphis. (Full disclosure: I spoke at the 2006 conference.) Bollobás, a professor who divides his time between Trinity College (Cambridge) and the University of Memphis, has worked for decades in functional analysis, combinatorics and graph theory. In the course of years of teaching and research, he has devised (or learned of) many easily stated problems whose solutions possess one or more of the hallmarks that mathematicians prize, such as economy, surprise and fecundity.

Here is one of my favorites: Suppose 10 chairs are arranged in a circle, half of them occupied by students. Show that there exists some whole number n between 1 and 9 such that if each of the 5 students moves n chairs clockwise in the circle, or more of them will end up sitting in a previously occupied chair.

This is not how Bollobás actually poses the problem in his book; in problem 3 (in a series of 157 problems), he asks the reader to consider a more general situation. But the key idea that solves Bollobás' problem can be discovered by thinking about the special case I've described—and by following the clue that Bollobás helpfully provides in a separate section devoted to hints. (If you want to think about this problem on your own, now would be a good time to put aside this book review!) Bollobás's clue is a short question that at first seems like a non sequitur: "What about a random rotation?"

If we choose n randomly, then each student has a 4-out-of-9 chance of ending up in a previously occupied chair. So on average, the number of students sitting in previously occupied chairs will be 4/9 + 4/9 + 4/9 + 4/9 + 4/9, or 20/9, which is slightly greater than 2. Now comes the punch line: The only way the average value of an integer-valued quantity can exceed 2 is if it sometimes is 3 or greater.

This proof exhibits economy (the chief idea is contained in the five-word hint), surprise (who would think to bring randomness and probability into solving a problem like this?) and fecundity (the probabilistic method has been an enormously powerful tool in the hands of Erdös and others). Bollobás's book is full of tasty little morsels like this one, puzzles whose solution requires attacking them from some unexpected angle. The ability to come up with creative approaches to problems can be cultivated, but it cannot be taught; it is more of an art than a craft. Hence the first half of the book's title.

I prefer Bollobás's original title, Coffee Time in Memphis (which his editor convinced him to lengthen); it bears more of the stamp of Bollobás's personal style. This is a book that developed in the author's mind through the course of conversations with students and colleagues, sometimes in offices or classrooms but just as often in departmental lounges or cafes.

Mathematicians love to find elegant solutions to their research problems, but they can't always be sure that the challenges they set for themselves have sweet answers. It can be a great relief from these uncertainties to work on a problem secure in the knowledge that it has a pleasing solution, which the

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Thanks For the Contributions!

Our thanks to all who have contributed to the Department of Mathematical Sciences over the past few years. Your generosity has allowed us to make purchases, award scholarships, and engage in activities that would otherwise have been impossible.

Many of you have responded generously to UML phonathon and other fundraising contacts. These requests can benefit the Department of Mathematical Sciences directly if you specify that you wish to have your gift directed to Mathematics. Otherwise it will provide valuable assistance to the University at the College level.

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News from the Department of Mathematical Sciences
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Tangents

Fall 2007

2007 Alumni-Awards Banquet

Next Banquet set for April 25, 2008

The 2007 Alumni-Awards Banquet at the Brewery Exchange on April 20 was well attended by the college dean, current and emeritus faculty, adjuncts and students. Some forty people in all came to wine, dine and congratulate our 2007 scholarship winners, each of whom was given a book prize and certificate in addition to any stipend. The awardees were

- Arthur S. Zamankos Scholarship (for outstanding juniors and seniors):_Alan Bartlett, Keith McElroy, Michael Williams, Michelle Polys
- The Richardson Bedell Scholarship (for outstanding juniors and seniors):_Ryan Beaven
- The Mary Hall Prize (for the outstanding freshman): _Stephen Giardini
- Shapiro Scholar (outstanding math major):_ Jennifer Cole

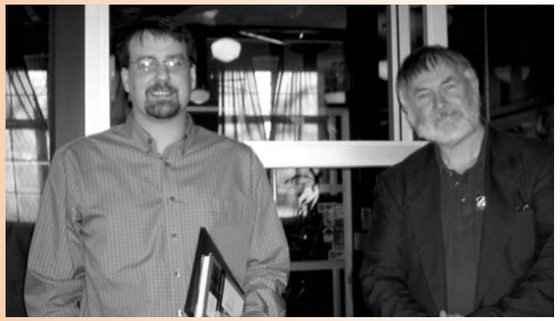
* Outstanding Graduate Student:_Drew Lazar

Prof. Tom Kudzma and Stephen Giardini.

This year was the first time the Shapiro Scholarship was awarded and we are very grateful to Bernie and Yana Shapiro for their dedication to the department and generous support of our students. Thank you both.

The Department of Mathematical Sciences entered a team in the Mathematical Competition in Modeling hosted by the Consortium for Mathematics and its Applications. The department has a strong tradition in this competition stretching back over a decade and once again our team performed extremely well, earning a Meritorious ranking which put them in the top 15% of the 800 or so participants worldwide. A copy of this year's problems and other information can be found at http://www.comap.com and their final report is at http://faculty.uml.edu/jgrahameagle/mcm2007.doc. A book prize was awarded to the three team members 2007 class valedictorian Chris Evans, Jason Percival and Michael Williams. Congratulations to the team members and their coach, Kiwi.

The Alumni-Awards Banquet is an annual event and we would be delighted to see more people there. If you have not been informed of this event in the past then please contact Prof Ken Levasseur (Kenneth_Levasseur@uml.edu) to make sure you are on our mailing list.



Drew Lazar and Prof. Charlie Byrne.

A review of *God Created the Integers*

by Stephen Hawking, 2005, 1162 pages, $29.95

BY GUNTRAM MUELLER

Do you mean that Stephen Hawking? Yes, that Stephen Hawking! What business is it of a physicist to edit a book of mathematics, selecting at his personal whim what he considers to be the brightest jewels of mathematical thought through these last 2500 years? The nerve! The gall! Where's the grievance committee? Well for one thing, he is the Lucasian Professor of Mathematics at Cambridge, Newton's old gig, and for another, he did just an absolutely excellent job of it. Did his interest in physics color his choice of pieces to include? Well, the selection is very strong in analysis, weak in algebra except for Gauss, nonexistent in topology, but it includes some 20th century foundations and logic.

Each of these pieces or excerpts is preceded by an interesting biographical sketch and several pages of mathematical mis-en-scene, explaining the context and the main results of that particular entry. Let's face it:

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Steve's ?

Fall 2007—From the Chair

Editorial Note: Although we are happy that Steve Pennell has agreed to serve as chair, his name doesn't lend it to as catchy a column title as "Kiwi's Korner." We've wrestled with ideas for Steve's but have decided to make it a contest. Winner gets a mathematics comic book. Send your entries to Stephen_Pennell@uml.edu and Steve will choose his favorite entry.

Kiwi recently finished his second term as math department chair. Despite our pleas, promises of good behavior, and attempted bribes, he decided not to run for a third term. I was elected to succeed him. The ceremonial change of offices took place on July 2.

Kiwi will be a tough act to follow. His low-key leadership style, his good judgment, his sense of humor, and his aversion to department meetings were greatly appreciated by all his colleagues during his six years as department chair. Thank you, Kiwi. Good on ya.

Thank you also to undergraduate coordinator Shelley Rasmussen and graduate coordinator Charlie Byrne, who keep our degree programs running. They spend many hours advising our students and helping them navigate the sometimes rocky road to the completion of their degrees.

Steve Pennell

Congratulations to Dan Klain, who was promoted from Associate Professor to Professor effective Fall 2007.

Three of our students gave presentations at conferences this summer. Graduate student Vincent Durante presented his work on "Development of a 2D RBF image reconstruction toolbox for sharp edged images" at the International Congress on Industrial and Applied Mathematics in Zurich, Switzerland. June graduates Jason Percival and Michael Williams gave presentations in San Jose, CA at the Pi Mu Epsilon National Conference, part of the Mathematical Association of America's summer Mathfest. Jason spoke on "Calculating the area of a spherical triangle," and Michael spoke on "Continued fractions with Gosper grids." Congratulations to Vinnie, Jason, and Michael.

We sadly say goodbye to three faculty members. As mentioned in the last issue of *Tangents*, Guntram Mueller retired after the spring 2007 semester. Mark Yin also retired after the spring 2007 semester, and Alina Stancu resigned to take a position at Concordia University in Montreal. Best wishes to all three. We will miss them.

In July, Vinnie Durante and I joined over 3,000 other applied mathematicians at the Sixth International Congress on Industrial & Applied Mathematics in Zurich. I am happy to report that applied math is thriving. New theoretical developments combined with rapid increases in computational power have led to exciting advances in many areas. To give just one example, Emmanuel Candès from Cal Tech gave a talk titled "Compressive sampling." According to his abstract, "One of the central tenets of signal processing and data acquisition is the Shannon–Nyquist sampling theory: the number of samples needed to capture a signal is dictated by its bandwidth. Here we introduce a novel sampling or sensing theory which goes against this conventional wisdom. This theory, now known as Compressed Sensing or Compressive Sampling, allows the faithful recovery of signals and images from… far fewer measurements or data bits than used by traditional methods." It was an exciting talk, especially now when we are confronted with a flood of data from so many sources. Despite all the excitement, rioting by math hooligans was kept to a minimum.

Please keep in touch, and stop in to visit the next time you are in the area. Remember to check out our web site, http://faculty.uml.edu/math, for items of interest.

What Are You Up To?

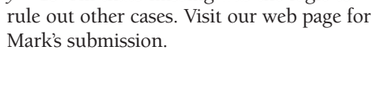
Want to keep your classmates up to date on what you're doing and where you are? Take a few moments to tell us where you are, and whatever else you might like to share. We'll add it to the UML Math Alumni page, http://www.uml.edu/dept/math/alumni.htm.

We can be contacted by mail at Department of Mathematical Sciences, North Campus, UMass Lowell, Lowell MA 01854. Telephone: (978) 934-2410. Email: mathematics@uml.edu

You might also wish to contact our Office of Alumni Relations, Southwick Hall 250, UML North, Lowell, MA 01854-3629. Toll free telephone: (877) UML-ALUM. Email: Alumni_Office@uml.edu

The *Tangents* Problem

Start with three line segments, A, B, and C. Connect an end of A to an end of B and the other end of B to an end of C. Using this system of line segments on a plane, how can you enclose maximum area using the system and a given line, as shown in the figure?



Up to four correct solutions from among all that are submitted by February 15, 2008 will earn the solver a "Math Challenge" T-shirt. You may submit your solution to mathematics@uml.edu or mail it to Ken Levasseur, Department of Mathematical Sciences, North Campus/Olney Hall, UMass Lowell, Lowell MA 01854.

Solution to the Spring 2007 problem

We didn't get a complete solution to the Spring 2007 problem.

Suppose x and y are two integers with $1 < x < y$ and $x + y \neq 100$. Jill knows their sum $x + y$ and Paul knows their product xy. The following conversation now takes place:

Paul: I do not know the two numbers.
Jill: I knew that already.
Paul: Now I know the two numbers.
Jill: Now I know them also.

What are the numbers?

All is not lost, however. Ornella Bascunan, one of our alumni who teaches at Lowell High School, gave the problem to her AP calculus class and one of her students, Mark Trout, successfully makes the case for $x = 4$ and $y = 13$. All that's missing was the logic to rule out other cases. Visit our web page for Mark's submission.

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A Review of *The Art of Mathematics: Coffee Time in Memphis*

problem-poser will, if pressed, reveal. Bollobás has included in the book the sorts of problems with which he loves to tease his colleagues and students, with pleasure on both sides.

Fans of books on recreational mathematics should be warned that this one is not for the faint-hearted or the mathematically unprepared. For instance, the first sentence of the solution of problem 84 reads, "This assertion is considerably trickier than the usual run-of-the-mill limit questions based on subadditivity or submultiplicativity; here we have to be a little more careful." Bollobás assumes his reader is already acquainted with (and perhaps a bit jaded by) large chunks of the advanced mathematics curriculum. If you do not come equipped with a knowledge of principles like continuity, compactness, contractibility and countability of the rational numbers (to mention just the ones that start with "c"), you will not find all of these problems to be fair challenges; you might want to read this book with a partner and take turns being the one to peek at the sections containing the hints and solutions. On the other hand, some of the puzzles require nothing more than elementary arithmetic and the right perspective on the problem. Some of my own favorites in this latter vein are problems 7, 13, 20, 21, 24, 31, 34, 40, 48, 55, 87, 102 and 119. Readers who find these more accessible problems fun would probably enjoy Peter Winkler's two collections of brain-teasers ("Mathematical Puzzles: A Connoisseur's Collection" and "Mathematical Mind-Benders") as well as "Proofs from THE BOOK" by Martin Aigner and Günter Ziegler.

Who are the right readers of this book? Mathematicians, certainly—especially younger ones who are still building up their mental toolkits. Corporate recruiters in Silicon Valley will probably find some of these problems to be good ways of assessing the mental athleticism of potential hires; in fact, I wouldn't be surprised to learn that some of these puzzles are already being used in this way. On the other side of the interview desk, job seekers might want to practice delivering the solutions to some of the more accessible puzzles, adding some pauses and brief false turns to make the whole thing sound unrehearsed. Likewise, mathematics professors administering oral examinations to Ph.D. students, and Ph.D. students seeking to pass those examinations, might want to go through this book. In addition, students at all levels (high school on up) may find in the notes a source of attractive unsolved research problems.

Erdös's collaborator Paul Turán once remarked that a mathematician is a machine for turning coffee into theorems. If so, then many of the theorems in Bollobás's book represent some extremely potent espresso. It would be a mistake to gulp them down too quickly. If enjoyed at a deliberate rate, alone or in conversation, these problems should have a stimulating effect on the prepared reader who takes the time to savor them.

This book review appeared in *The American Scientist* in Fall 2007.

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A review of *God Created the Integers*

reading all these introductions provides an excellent contextualization of a lot of mathematics. This is something that is sorely lacking in the usual mode of taking first this course, then that, with a strong emphasis on being able to solve various kinds of problems, but with little time available for seeing where these mathematical issues come from, nor the significance that these results have in the big picture. This book provides some of that larger picture.

What are some of the nuggets to look for? Firstly, the section on Archimedes, the account of the astounding devices that he built, his work on the estimation of pi—look Ma, no algebraic notation! Also significant is the fact that he had an estimate of the radius of the earth, and talked about the theory of the earth going around the sun, 1800 years before Galileo got into that little dustup with the church. Secondly, I recommend the section on Laplace. All of the known planets revolved about the sun in the same direction and in almost the same plane. The observed small perturbations of their orbits caused some concern. Do they build on each other? What holds the planets in place? Does God do it? Newton thought so. Laplace, 100 years later, in his 5-volume Celestial Mechanics, showed that Newton's three laws implied exactly the observed phenomena! Napoleon Bonaparte, looking through Laplace's work, remarked that he saw no mention of God. Laplace replied: "Sire, there is no need of that hypothesis."

For the record, here are the authors included: Euclid, Archimedes, Diophantus, Descartes, Newton, Laplace, Fourier, Gauss, Cauchy, Boole, Riemann, Weierstrass, Dedekind, Cantor, Lebesgue, Gödel, and Turing. In its 1162 pages, this book covers a lot of territory, including some pieces translated for the first time for this volume. I must say that reading some of these pieces is not easy, neither the ancient pieces nor the recent ones —I keep bumping my head on Gödel's work. But at the very least, even the introductions alone are excellent and well worth any student's, or professor's, time and effort.

Rating: ≡ ≡ ≡ ≡ (out of 4)

Steve Pennell, Chair

Writers: Ken Levasseur, Raj Prasad, Ann Marie Hurley, Dan Klain, Guntram Mueller, Marv Stick and Kiwi Graham-Eagle

Tangents is produced biannually by the Publications Office for the Department of Mathematical Sciences. Your comments are welcome.

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