MODAL SPACE - IN OUR OWN LITTLE WORLD

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What's the difference between operating deflection shapes and mode shapes? sometimes they look the same to me! Well . . . let's describe the differences.

This is a common stumbling point for many people. This is partly due to the words that we use. I would much rather call the data we receive from an operating condition, an *operating deflection pattern,* rather than use the word *shape*. But unfortunately, I can't change the nomenclature at this point.

Let's first recall how a structure responds, in general, due to any excitation

$$h(j\omega) \times f(j\omega) = y(j\omega) \qquad (1)$$

Of course, we realize that the input forcing function is actually applied in the time domain but we represent it in the frequency domain; also the response actually occurs in the time domain but it can also be represented it in the frequency domain.

So for a structure which is exposed to an arbitrary input excitation, the response can be computed using the frequency response function multiplied by the input forcing function. This is very simply shown in the schematic in Figure 1.

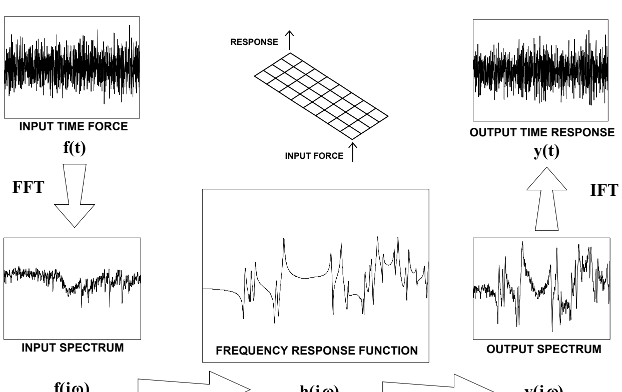

Figure 1 - Schematic Overviewing the Input-Output Structural Response Problem

The excitation shown is a random excitation that excites all frequencies. The most important thing to note is that the frequency response function acts as a filter on the input force which results in some output response. The excitation shown causes all the modes to be activated and therefore, the response is, in general, the linear superposition of all the modes that are activated by the input excitation. Now what would happen if the excitation did not contain all frequencies but rather only excited one particular frequency (which is normally what we are concerned about when evaluating operating conditions).

Let's consider a simple plate that is excited by an input force that is sinusoidal in nature. And let's also assume that the force is applied at one corner of the plate. For the example here, we are only going to consider the response of the plate assuming that there are only 2 modes that are activated by the input excitation. (Of course there are more modes, but let's keep it simple to start.) Now from figure 1 and equation 1 we realize that the key to determining the response is the FRF between the input and output locations. Also, we need to remember that when we collect operating data, we don't measure the input force on the system and we don't measure the system FRF - we only measure the response of the system.

First let's excite the system with a sinusoid that is right at the first natural frequency of the plate structure. The response of the system for one FRF is shown in Figure 2. So even though we excite the system at only one frequency, we know that the FRF is the filter that determines how the structure will respond. We can see that the FRF is made up of a contribution of both mode 1 and mode 2. We can also see that the majority of the response, whether it be in the time or frequency domain, is dominated by mode 1. Now if we were to measure the response only at that one frequency and measure the response at many points on the structure, then the operating deflection pattern would look very much like mode 1 - but there is a small contribution due to mode 2. Remember that with operating data, we never measure the input force or the FRF - we only measure the output response. So that the deformations that are measured are the *actual response* of the structure due to the input excitation - whatever it may be.

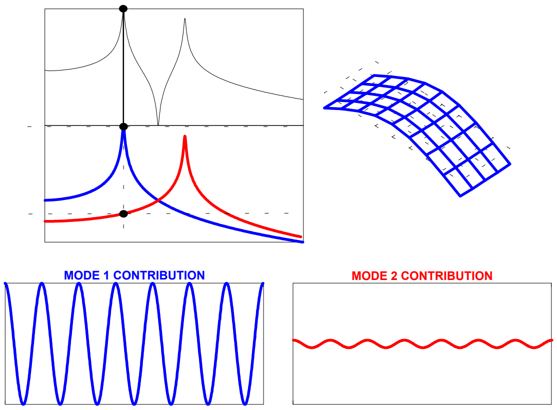

Figure 2 - Excitation Close to Mode 1

When we measure FRFs and estimate modal parameters, we actually determine the contribution to the total FRF soley due to the effects of mode 1 acting alone, as shown in blue, and mode 2 acting alone, as shown in red, and so on for all the other modes of the system. Notice that with operating data, we only look at the response of the structure at one particular frequency - which is the linear combination of all the modes that contribute to the total response of the system. So we can now see that the operating deflection pattern will look very much like the first mode shape if the excitation primarily excites mode one.

Now let's excite the system right at the second natural frequency. Figure 3 shows the same information as just discussed for mode 1. But now we see that we primarily excite the second mode of the system. Again, we must realize that the response looks like mode 2 - but there is a small contribution due to mode 1.

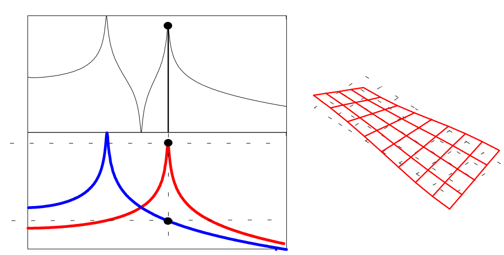

Figure 3 - Excitation Close to Mode 2

But what happens when we excite the system away from a resonant frequency. Let's excite the system at a frequency midway between mode 1 and mode 2. Now here is where we see the real difference between modal data and operating data. Figure 4 shows the deformation shape of the structure. At first glance, it appears that the deformation doesn't look like anything that we recognize. But if we look at the deformation pattern long enough, we can actually see a little bit of first bending and a little bit of first torsion in the deformation. So the operating data is primarily some combination of the first and second mode shapes. (Yes, there will actually be other modes but primarily mode 1 and 2 will be the major participants in the response of the system.)

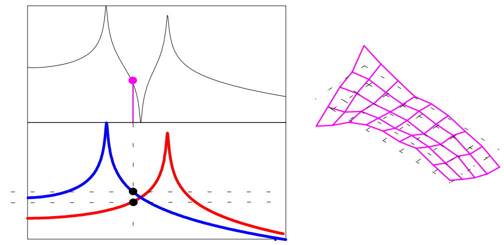

Figure 4 - Excitation Somewhere Between Mode 1 and Mode 2

Now, we have discussed all of this by understanding the FRF contribution on a mode by mode basis. When we actually collect operating data, we don't collect FRFs but rather we collect output spectrums. If we looked at those, it would not have been very clear at to why the operating data looked like mode shapes. Figure 5 shows a resulting output spectrum that would be measured at one location on the plate structure. Now the input applied to the structure is much broader in frequency and many modes are excited. But, by understanding how each of the modes contributes to the operating data, it is much easier to see how the modes all contribute to the total response of the system. So actually, there is a big difference between operating deflections and mode shapes - *we can now see that the modes shapes are summed together in some linear fashion to form the operating deflection patterns*. I hope that this helps to clear up the mystery as to the differences between operating deflection patterns and mode shapes.

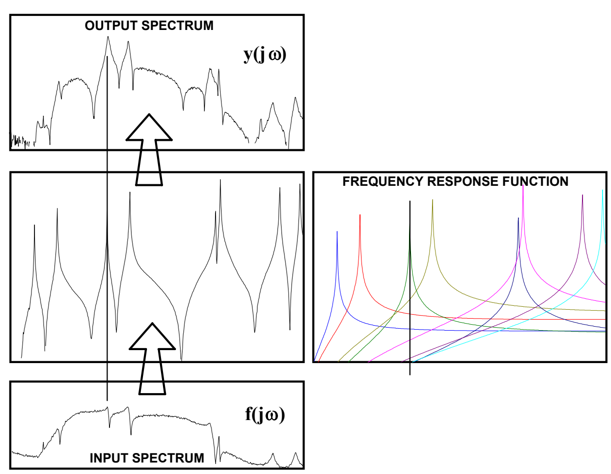

Figure 5 - Broadband Plate Excitation

Think about it and if you have any more questions about modal analysis, just ask me.