MODAL SPACE - IN OUR OWN LITTLE WORLD

by Pete Avitabile

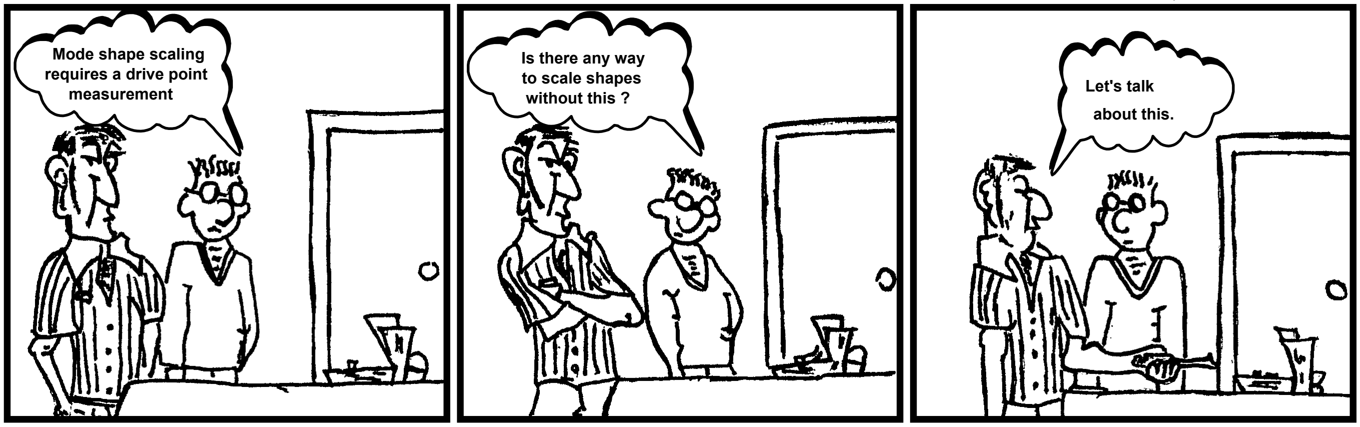

Illustration by Mike Avitabile

Mode shape scaling requires a drive point measurement.
Is there any way to scale shapes without this?
Let's talk about this

Mode shape scaling is an important item for the development of an accurate dynamic model that would be used for other structural dynamic studies. Some of these would be simulation and prediction, modification, and correlation, to name a few. While there may be some instances when scaling may not be critical, I will always recommend that this is done since this may be the only data ever acquired. Generally, a drive point measurement is required to scale mode shapes. However, there are alternate ways to collect measurements and obtain scaled mode shapes without a drive point measurement. Let's discuss this.

Recall that the poles and residues are the values that describe the measured frequency response function and can be written as

$$[H(s)] = \begin{matrix}\text{lower}\\ \text{residuals}\end{matrix} + \sum_{k=i}^{j} \frac{[A_k]}{(s-s_k)} + \frac{[A_k^*]}{(s-s_k^*)} + \begin{matrix}\text{upper}\\ \text{residuals}\end{matrix}$$

Now these residues can be shown to be related to the mode shapes. Without going through all the steps, the resulting relationship for the 'k' mode of the system can be written (with some terms expanded) as

$$[A(s)]_k = q_k \{u_k\}\{u_k\}^T$$

$$\begin{bmatrix} a_{11k} & a_{12k} & a_{13k} & \cdots \\ a_{21k} & a_{22k} & a_{23k} & \cdots \\ a_{31k} & a_{32k} & a_{33k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = q_k \begin{bmatrix} u_{1k}u_{1k} & u_{1k}u_{2k} & u_{1k}u_{3k} & \cdots \\ u_{2k}u_{1k} & u_{2k}u_{2k} & u_{2k}u_{3k} & \cdots \\ u_{3k}u_{1k} & u_{3k}u_{2k} & u_{3k}u_{3k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Now if we consider the 'r' column of these equations, then the residues can be related to the mode shapes using

$$\begin{Bmatrix} a_{1r} \\ a_{2r} \\ a_{3r} \\ \vdots \\ a_{rr} \\ \vdots \end{Bmatrix} = q\ \ u_r \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_r \\ \vdots \end{Bmatrix}$$

So for each measurement, a relationship between the residue and mode shape can be obtained as

$$\begin{aligned} h_{1r} &\Rightarrow a_{1r} = u_1 u_r \\ h_{2r} &\Rightarrow a_{2r} = u_2 u_r \\ h_{3r} &\Rightarrow a_{3r} = u_3 u_r \\ &\vdots \end{aligned}$$

but notice that there are more unknowns than equations and it doesn't matter how many extra equations are added to the list. The shapes cannot be determined unless one particular measurement is included - the drive point measurement which is given as

$$h_{rr} \Rightarrow a_{rr} = u_r u_r$$

With the drive point measurement, then the mode shape at the reference location can be obtained - thereby allowing all the other mode shape coefficients to be determined.

But what happens if a drive point measurement is not available or is very difficult to obtain. Is there any other way that the mode shapes can be scaled using other measurements that could possibly be made? Well, it turns out that the answer to this is YES. Let's describe a set of measurements that will enable an equivalent representation of the drive point scaling measurement.

Let's consider some terms in a frequency response matrix at arbitrary locations as shown. The 'r' subscript is the reference and the 'o', 'p', 'q', 's' and 't' are arbitrary measurements in that matrix. Most of the measurements are made relative to the 'r' reference but one measurement is not. We are assuming that the drive point measurement, h_{rr} , has not been measured but is shown in the matrix for illustration purposes. There are three particular measurements of interest that are needed to write some simple equations (these are shown with a double bar underscore in the matrix).

$$\begin{bmatrix} & & h_{or} & \\ \rightarrow & \underline{\underline{h_{pq}}} & \underline{\underline{h_{pr}}} & \leftarrow \\ & & \underline{\underline{h_{qr}}} & \leftarrow \\ & & \vdots & \\ & \Rightarrow & h_{rr} & \Leftarrow \\ & & h_{sr} & \\ & & h_{tr} & \end{bmatrix}$$

Recall that we can write the residue - mode shape relationship for a particular mode and for a particular measurement as

$$(1) \quad h_{pq} \Rightarrow a_{pq} = u_p u_q$$
$$(2) \quad h_{pr} \Rightarrow a_{pr} = u_p u_r$$
$$(3) \quad h_{qr} \Rightarrow a_{qr} = u_q u_r$$

(Note: For brevity, the scaling coefficient has been dropped) Three specific measurements have been selected here to illustrate the development of an alternate scaling mechanism.

Now, the first equation can be rewritten as

$$u_p = \frac{a_{pq}}{u_q}$$

and substituted into the second equation to give

$$u_r = \frac{a_{pr}}{a_{pq}} u_q$$

The third equation can be rewritten as

$$u_q = \frac{a_{qr}}{u_r}$$

and substituted into the modified second equation to give

$$u_r = \frac{a_{pr}}{a_{pq}} \frac{a_{qr}}{u_r}$$

And then, rearranging terms, gives the drive point equivalent as

$$u_r^2 = \frac{a_{pr}\, a_{qr}}{a_{pq}}$$

I know that I usually don't have this many equations to explain things but this only involved a few simple manipulations to reveal an alternate mechanism to obtain the mode shape coefficient for the reference degree of freedom. Remember that the drive point measurement was not used to obtain the mode shape coefficient for the reference point.

At times this can become a very useful approach especially when there is no access for a drive point measurement or it is inconvenient to obtain the drive point measurement. While I haven't used this often, it does come in handy when performing impact measurements and it is difficult to get the impact device into an area of the structure where access is restricted. It is also useful during shaker testing when it is difficult to make a drive point measurement.

I hope this clarifies your question regarding mode shape scaling and the need for a drive point measurement. If you have any other questions about modal analysis, just ask me.