MODAL SPACE - IN OUR OWN LITTLE WORLD

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Is there any benefit to using multiple references?
I thought only one reference was needed
Let's discuss this.

This is a very good question. Its one that comes up often in terms of estimating modal parameters from test data. From modal analysis theory, we can easily show that only one reference is necessary in order to determine all of the modes of a system - at least from a theoretical standpoint! While theoretically this is true, from a practical standpoint, there is a strong need to have multiple references in many cases. Before we can understand this, let's take a look at some basic concepts that will help illustrate some of the problems that we might encounter.

Let's start this discussion with a simple structure that has mode shapes that are very directional in nature. We have used this structure before in other discussions (May/Jun 2000, Vol. 24, No. 3). The structure is shown in Figure 1 along with the first several modes.

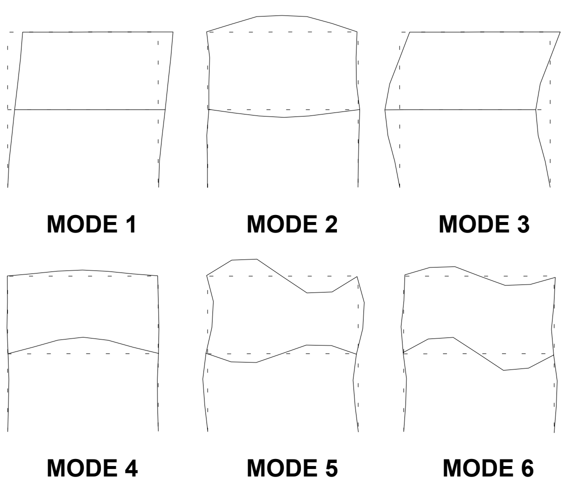

Figure 1

If we look at a reference point in the vertical direction (as shown in figure 2) over the bandwidth of the first six modes of the structure, we notice that there are only 2 peaks that are visible in the measured frequency response function. Yet we know that there are 6 modes in this frequency range. And if we took a reference point in the horizontal direction, we would also notice only 4 peaks. But upon closer examination of the measurement, we would notice that the first two frequencies of each of the measurements is different.

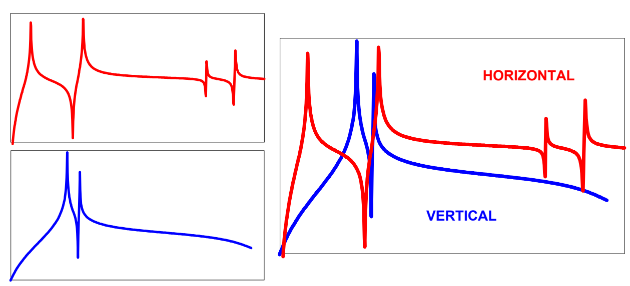

Figure 2

Let's recall the equation for the frequency response function

$$h_{ij}(j\omega) = \sum_{k=1}^{m} \frac{a_{ijk}}{(j\omega - p_k)} + \frac{a^*_{ijk}}{(j\omega - p^*_k)}$$

Basically, this equation is described by the residues (in the numerator) and the poles (in the denominator) for each of the modes of the system. We must remember that this frequency response function can be written for any one of the input-output combinations of interest. Now the interesting part of this equation is that while the residues change depending on which input-output combination is acquired, the poles do not change.

This implies that the poles of the system are global. They are independent of the particular input-output point. However, the residues do, in fact, change.

Now when a modal test is performed, typically all of the measurements are acquired relative to a particular reference. The reference location is typically either the fixed excitation location when performing a shaker excitation or the stationary accelerometer location when performing an impact test. So the measurements acquired will contain residues, relative to a particular reference as

$$\lfloor a_{11} \quad a_{12} \quad a_{13} \quad \cdots \rfloor_k$$

In this case, the reference is "1" since all of the residues are related to that DOF. The residues are a_{11}, a_{12}, a_{13} and so on. (Note that the "k" subscript is used to denote a particular mode of the system.)

We also need to remember that the residues are directly related to the mode shapes (and a scaling factor) as

$$a_{ijk} = q_k u_{ik} u_{jk}$$

This means that the residues are actually directly related to the mode shapes of the system as

$$\begin{Bmatrix} a_{11k} \\ a_{21k} \\ a_{31k} \\ \vdots \end{Bmatrix} = q_k \begin{Bmatrix} u_{1k} u_{1k} \\ u_{2k} u_{1k} \\ u_{3k} u_{1k} \\ \vdots \end{Bmatrix} = q_k u_{1k} \begin{Bmatrix} u_{1k} \\ u_{2k} \\ u_{3k} \\ \vdots \end{Bmatrix}$$

Notice that the reference DOF at point 1 can be factored out since it is common to all of the measurements. In doing this, it becomes very clear that the reference DOF carries a tremendous amount of weight regarding the magnitude of the residue; this is directly related to the magnitude of the frequency response function. If the reference point is associated with a very small mode shape response location on the structure for a particular mode, then the magnitude of the frequency response function will also be very small for that mode. On the other hand, if the reference point is associated with a very large mode shape response location then the magnitude of the frequency response function will be very large.

Of course, we can then also see that if the reference location is located at a DOF where the mode shape value is very large for one mode and very small for another mode, then the amplitude of the frequency response function will have the same attributes. This is a common problem in performing any modal test. We always try to locate the accelerometer at a location where all of the modes can be observed with the same strength across the desired frequencies of interest. However, this is often very difficult and, in many cases, almost impossible.

However, we can use some of the redundancy in the frequency response matrix to help with this situation. If we look at some of the terms of this matrix, then there are some interesting things to note. The residue matrix is shown along with some of the terms expanded for reference.

$$\left[A(s)\right]_k = q_k \,\{u_k\}\,\{u_k\}^T$$

$$\begin{bmatrix} a_{11k} & a_{12k} & a_{13k} & \cdots \\ a_{21k} & a_{22k} & a_{23k} & \cdots \\ a_{31k} & a_{32k} & a_{33k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = q_k \begin{bmatrix} u_{1k}u_{1k} & u_{1k}u_{2k} & u_{1k}u_{3k} & \cdots \\ u_{2k}u_{1k} & u_{2k}u_{2k} & u_{2k}u_{3k} & \cdots \\ u_{3k}u_{1k} & u_{3k}u_{2k} & u_{3k}u_{3k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Notice that there is redundancy in this matrix. Each column contains information that is related to the kth mode shape of the system times the reference DOF. (Also note that due to symmetry, the rows contain the same information.) This very important fact is the reason why many modal parameter estimation algorithms utilize multiple reference data from a modal test. Each of the references contains the same basic information that is only scaled by the reference DOF for a particular mode. Therefore, this redundant information can be extracted and used in the curvefitting process.

More importantly, if there is one reference that does not excite a particular mode very well (ie, the reference is located close to the node of a mode for that mode), then there are other references that may be much better reference locations for the determination of that mode. So using multiple references minimizes the need to be absolutely certain that all of the modes of the system can be reasonably well excited from only one reference location. The modal parameter estimation process uses weighting terms, called modal participation factors, in order to utilize all of the referenced data to extract valid modal parameters. So the use of multiple referenced data is a tremendous help in determining modal parameters. The use of redundant data allows for the selection of several references, each of which may be very good for several modes, but not all the modes, of the system. However, using multiple references allows the adequate description of all the modes from the combination of references. This way, many references gives the best possible chance to adequately determine all of the modes of the system. This may not be totally possible using only one reference - even though theoretically, it is possible!

I hope this explanation helps you to understand why multiple references are useful even though they are not theoretically necessary. If you have any other questions about modal analysis, just ask me.