

## The Effect of Friction in Pipe Flow Problems -- An Example

A good example of how friction affects the flow of incompressible fluids can be illustrated nicely by examining the drain time of a tank with various lengths of exit pipe. In addition, varying the orientation (vertical or horizontal) and the diameter of the pipe can add significant insight into this classical internal flow problem. Since one of the goals of the Gravity Driven Flows (GDF) graphical interface was to be particularly useful for studying viscous pipe flow problems, we should be able to study these effects using this easy-to-use interactive tool.

As a specific example, let's consider an open vertical cylindrical tank that has a diameter of 1.5 ft and a height of 3 ft. The initial height of water in the tank is 2 ft and we want to observe how long it takes to drain the tank down to the 1 ft level. Copper pipe ( $\epsilon = 5 \times 10^{-6}$  ft) is available in lengths of 1, 2, 3, 4, 5, 7.5, and 10 ft and the pipe can be connected to the tank to give either a horizontal exit pipe arrangement or a vertical drain configuration. Note that, if no pipe is attached, then the horizontal and vertical heights are both zero ( $L_h$  and  $L_v$ , respectively). In addition, the copper tube is available with two different internal diameters of approximately  $\frac{1}{4}$  and  $\frac{1}{2}$  inches. Finally, to complete the specification of the piping system, we will assume a total "minor loss" coefficient of  $K = 0.5$  for all configurations (to approximate the losses in the contraction from the tank to the exit connection and from the connection to the exit pipe).

Now, the idea is to use the GDF application to study this system by varying the above parameters and observing the time it takes to drain the tank. Let's start with the  $\frac{1}{2}$  copper pipe attached in a horizontal configuration. You should run the GDF program, initialize the plot, and then select the Vertical Cylindrical geometry option. Then, in order, you should set the final fluid height, initial fluid height, tank height, and tank diameter as indicated above to complete the tank geometry specification. Since the default fluid is water and the default top pressure is fixed at 14.7 psia, these options do not need to be changed for this example.

Concerning the Outflow Pipe Parameters, for now, you should leave the exit pipe diameter at 0.5 inches and the vertical and horizontal pipe lengths at zero, but change the pipe roughness and minor loss coefficient to the values indicated above. With all these values set, you should see a nearly linear decrease in the fluid height with time, from an initial height of 2 ft to a final height of 1 ft. The drain time for this particular situation should be about 164 seconds (see the lower right hand corner of the GUI for the precise simulation time needed to reach the desired height).

Now, systematically change the horizontal length of the exit pipe to 1 ft, 2 ft, etc. as indicated above (leaving all the other parameters unchanged), and record the drain time for each configuration. Upon doing this, you will see that the drain time increases with the value of  $L_h$  as shown in Fig. 1. This behavior is exactly as you should have expected -- since you would expect the friction loss term,  $h_L$ , to increase with  $L_h$  as implied in the head loss equation,

$$h_L = f \frac{(L_h + L_p)}{D_h} \frac{v^2}{2g} + \left( \sum_i K_i \right) \frac{v^2}{2g} \quad (1)$$

Also, since the pressure head and the elevation head do not change with  $L_h$ , we can see from the general energy equation that the exit velocity,  $v = v_2$ , will decrease with an increase in  $h_L$ , or

$$\alpha_2 \frac{v_2^2}{2g} = \left( \frac{P_1}{\gamma} - \frac{P_2}{\gamma} \right) + (z_1 - z_2) - h_L + \alpha_1 \frac{v_1^2}{2g} \quad (2)$$

It should be noted however, that since both  $h_L$  and  $v_1$  are dependent on  $v_2$ , the relationship in eqn. (2) between  $v_2$  and  $h_L$  is clearly nonlinear. Nevertheless, increasing the horizontal length of the exit pipe increases the friction loss, which decreases the flow velocity, which subsequently increases the tank drain time -- as clearly observed in Fig. 1. Thus, the basic behavior of the drain time vs  $L_h$  was relatively easy to predict, and the GDF program simply allowed us to easily quantify this relationship.



**Fig. 1 Dependence of the tank drain time versus  $L_h$  for  $D_p = \frac{1}{2}$  inch.**

Now, to continue this analysis, you should reset  $L_h = 0$ , and allow the vertical length,  $L_v$ , to vary over the same range as before. Again record the drain time for each case and make a simple plot of drain time vs.  $L_v$  (similar to Fig.1). What has happened here for the case with  $D_p = \frac{1}{2}$  inch? In this case you should see the drain time decrease as the vertical pipe length increases. The head loss term,  $h_L$ , is still proportional to the pipe length, so what has caused the direction change? Can you explain this behavior?

Now do one more parametric study by varying  $L_v$  again, but this time, let  $D_p = \frac{1}{4}$  inch. Again record and plot the drain time vs  $L_v$ . What happens to drain time vs  $L_v$  for this case -- does the drain time increase or decrease with  $L_v$ ? Does this behavior make sense? Is this what you expected before you ran the GDF simulations? Explain...

In summary, you should try to explain the observed behavior for the three parametric studies performed here and show that everything is indeed self consistent as explained via the general energy equation.