

Step Response of First-Order Systems

INTRODUCTION

This tutorial discusses the response of a first-order system to a unit step function input. In particular, it addresses the time constant and how that affects the speed of the system's response. An RC circuit, as shown in Fig. 1, is used as an example of a first-order system.

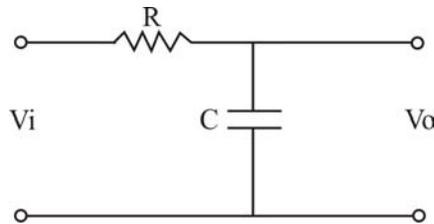


Fig. 1. An RC circuit.

A Simulink model of this circuit is used to illustrate its response; a basic understanding of Simulink is therefore useful but not necessary.

Other documents are available which contain more detailed information on RC circuits and first-order systems in general.

SYSTEM MODEL

The first-order differential equation describing the RC circuit is

$$\tau \dot{x} + x = f(t), \quad (1)$$

where

x = output voltage,

\dot{x} = time rate of change of output voltage,

τ = time constant = RC , and

$f(t)$ = the input, a step function.

The response of the circuit can also be represented using a Simulink model, such as that shown in Fig. 2.

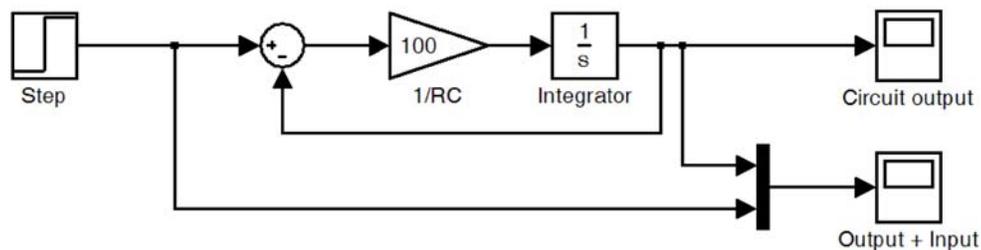


Fig. 2. RC circuit model with unit step input.

GENERAL FORM OF RESPONSE

For a first order system subjected to a unit step input, where the system is originally at zero displacement, the solution is

$$x(t) = 1 - e^{-t/\tau} \quad (2)$$

Note that $e^{-t/\tau}$ decays to zero with increasing t , so $x \rightarrow 1$ as $t \rightarrow \infty$. The general shape of the response can be seen in Fig. 3, which shows the unit step response of a first-order system with $\tau = 0.01$ seconds.

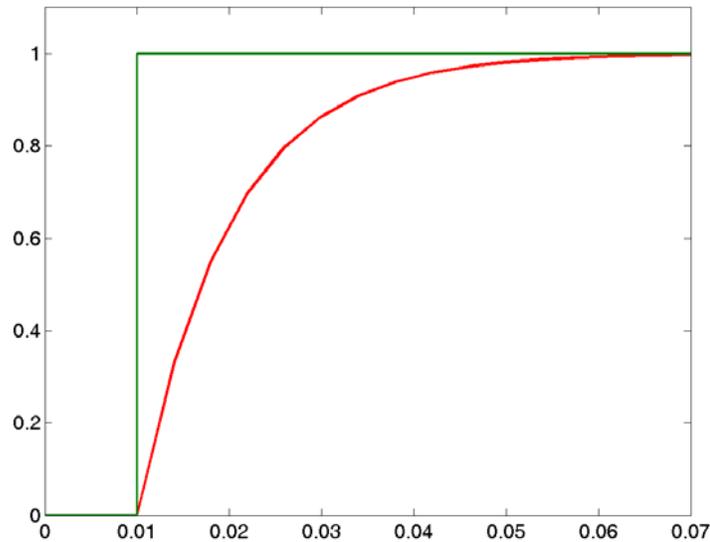


Fig. 3. General shape of step response

EFFECT OF TIME CONSTANT

The speed of the system's response depends on the time constant of the system, τ . To illustrate this, two systems with different time constants will be compared. Each system is subjected to a unit step input. The input is shown on the left-hand side of Fig. 4, and the two responses are shown on the right.

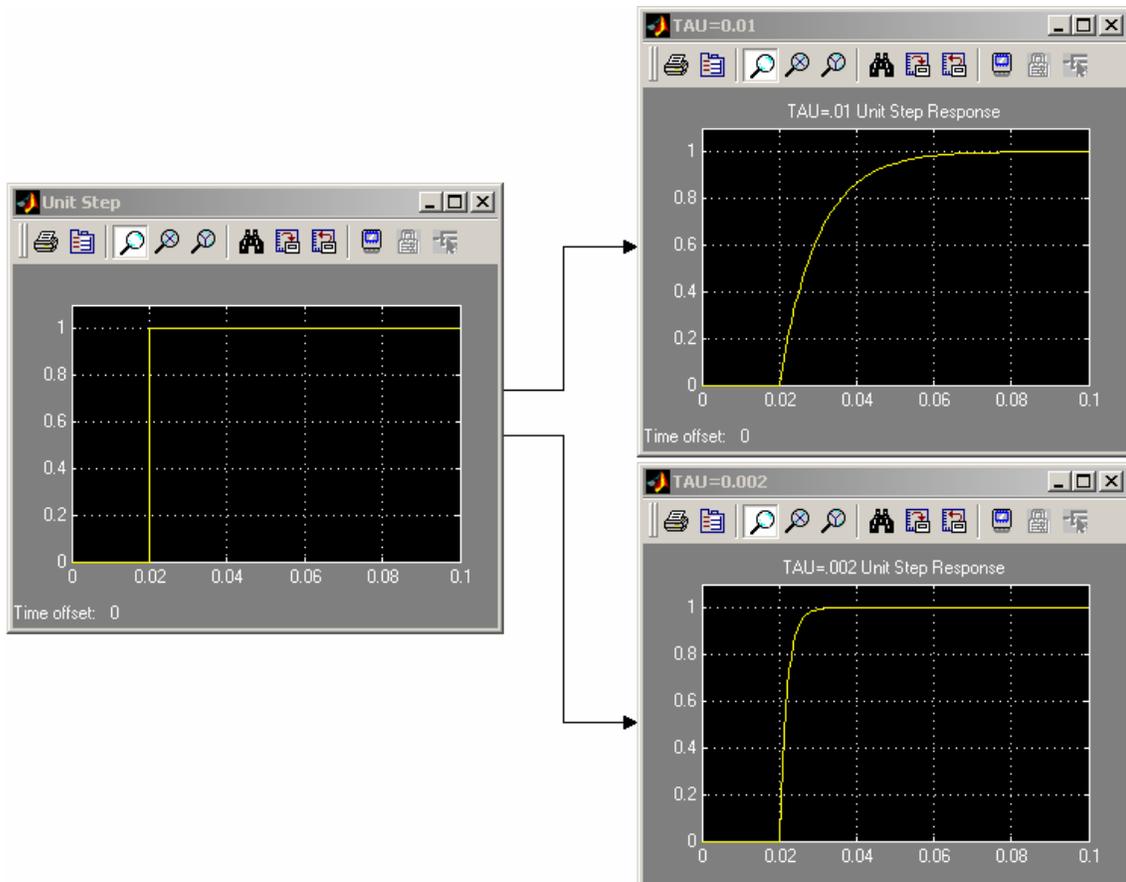


Fig. 4. Response to unit step input, for two systems with different time constants.

The top right-hand plot in Fig. 4 shows the response of a system with $\tau = 0.01$ seconds. The bottom right plot is the response of a system with $\tau = 0.002$ seconds. Clearly, the circuit with the smaller time constant responds more quickly—its response more closely tracks the input. In the limit, as $\tau \rightarrow 0$, the response will exactly follow the input.

Fig. 5 illustrates another way of understanding how the time constant affects the speed of the response.

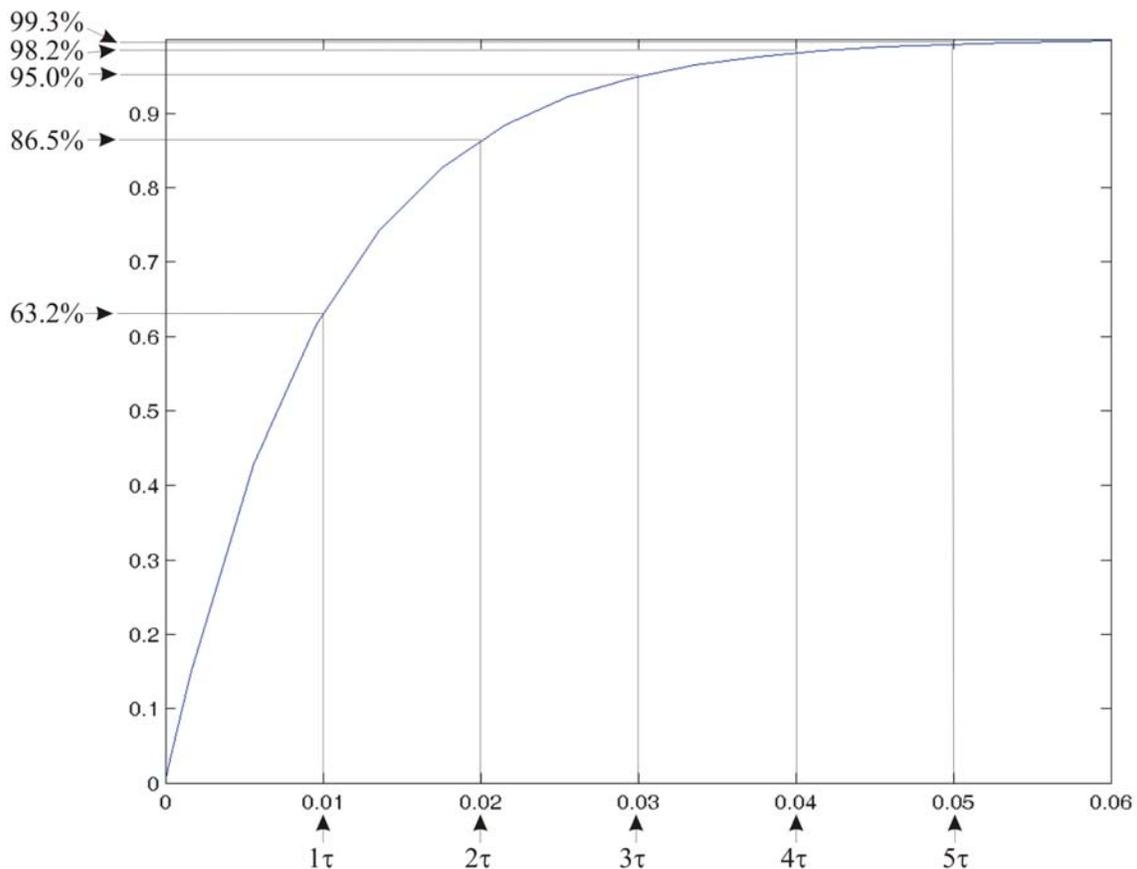


Fig. 5. Effect of time constant on system response.

This plot shows the unit step response of a system with $\tau = 0.01$ seconds. As the plot shows, after one time constant a first-order system will have reached 63.2% of the final steady-state value. There is no point identifying when the system has reached steady state, but often 3τ , 4τ , or 5τ is chosen, depending on the desired accuracy.

FINDING THE STEP RESPONSE IN MATLAB

The step response of a first-order system can be found using a Simulink model like that shown in Fig. 2, but it can also be found in MATLAB. The code shown below produces the plot in Fig. 6.

```
% Define the time constant of the system
tau = 0.01;

% identify the numerator and the denominator of the system transfer
% function
num = [1];
den = [tau 1];

% plot the step response
step(num,den)
```

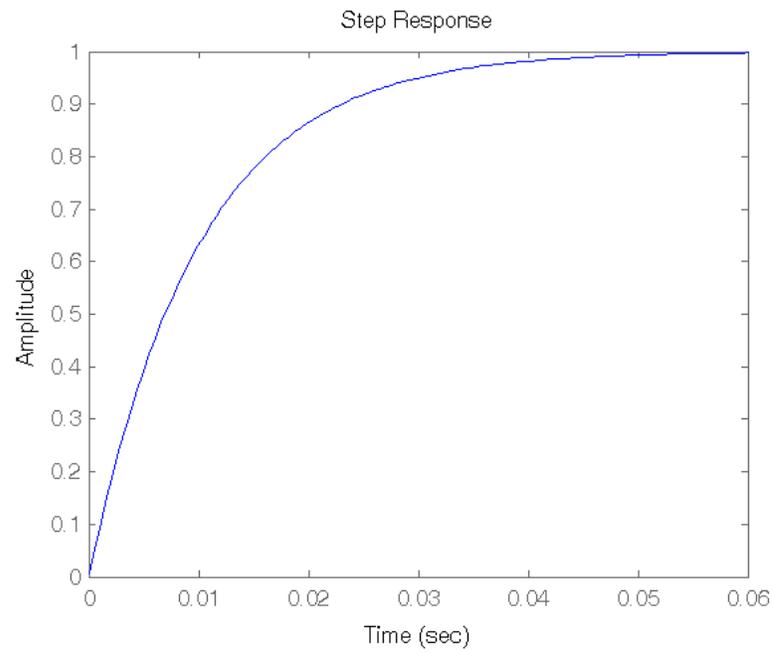


Fig. 6. Step response of a first-order system, found in Matlab.