

Block Diagram Modeling of First-Order Systems

INTRODUCTION

Block diagrams are a method of describing the behavior of a dynamic system. In a block diagram, each discrete component, or block, represents part of the system. These blocks are connected together, representing how the “signal” flows between components. This can aid in understanding the interaction between various components, particularly for complex systems. Simulink allows block-diagram modeling of systems, and will be used to produce the examples in this tutorial. The concepts described here, however, are applicable to block diagrams in general.

First-order differential equations are commonly studied in Dynamic Systems courses, as they occur frequently in practice. In this document, the basics of modeling a first-order equation with a block diagram will be discussed. The equation which will be used is that describing a first-order RC circuit. There are other tutorials available which offer more general information on first-order systems.

CONSTRUCTING THE MODEL

The differential equation describing the RC circuit is

$$\dot{x} + \frac{1}{RC}x = \frac{1}{RC}f(t), \quad (1)$$

where

x = the output voltage,

\dot{x} = the time rate of change of the output voltage,

R = resistance (constant),

C = capacitance (constant), and

$f(t)$ = the forcing function (input voltage).

For the purposes of this tutorial, (1) will be rearranged to give

$$\dot{x} = \frac{1}{RC}[f(t) - x]. \quad (2)$$

This equation will now be analyzed piece by piece. First, notice that the quantity in the brackets consists of the x term subtracted from the $f(t)$ term. If each of these terms is thought of as outputting a signal, this relationship can be modeled as shown in Fig. 1.

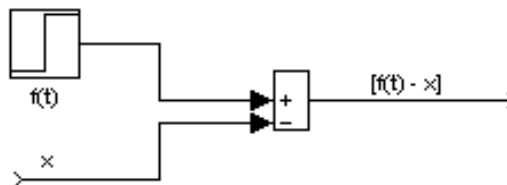


Fig. 1. Summing the input signals.

The block in the center of Fig. 2 is called a summing junction, and is a common component of block diagrams.

Examining the original equation, it can be seen that the bracketed term $[f(t) - x]$ is multiplied by a constant $1/RC$. In the diagram, this is represented as passing the signal through a gain block as shown in Fig. 2.

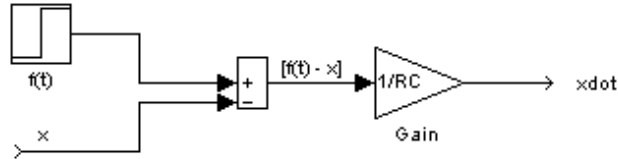


Fig. 2. Applying a gain to the output.

Because all of the terms on the right side of (2) are now accounted for, the output signal must therefore be equal to the left side of the equation, \dot{x} . Note that in the Simulink model labels, “xdot” refers to \dot{x} .

However, in this case the quantity of interest is x , the output voltage, rather than \dot{x} . Therefore, \dot{x} must be integrated once with respect to time. This is done using an integrator block as shown in Fig. 3.

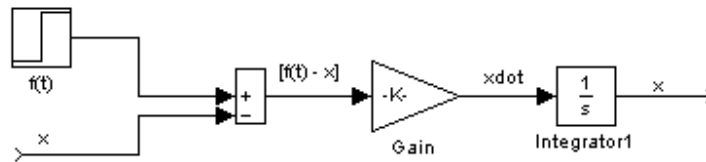


Fig. 3. Integrating the output signal.

The desired output, x , has now been obtained. However, x is also an input to the system. In our model, as shown in Fig. 3, the x input to the summing block is a “dead” branch. In order to make x both an output from and an input to the system, a feedback loop is created by tapping off the output x signal and feeding it back into the system at the input point. A block diagram which has a feedback loop is referred to as a closed-loop diagram.

After some manipulation of the lines, and the addition of a Scope block (“Output”) so that the output can be viewed, the model should look like Fig. 4. The block diagram is now complete.

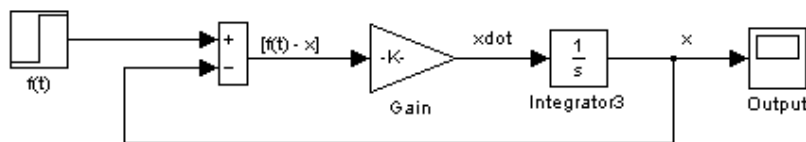


Fig. 4. The finished Simulink model.

EXAMPLE

The Simulink model shown in Fig. 5 below will now be used to find the step response of the first-order system.

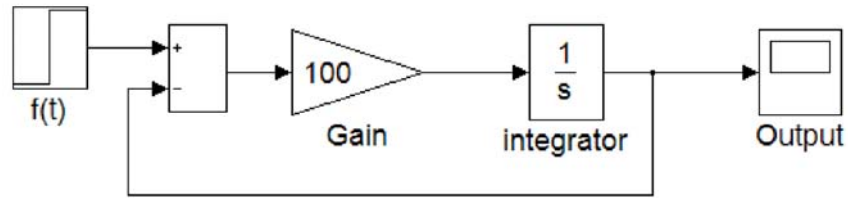


Fig. 5. Simulink model of first-order system.

The values used in the model are listed in Table 1.

Table 1. Values used in Simulink model.

Step block	Step time	0
	Initial value	0
	Final value	1
Gain block	Gain	100
Integrator	Initial condition	0

When the model is run and the scope opened, the response will appear as shown in Fig. 6.

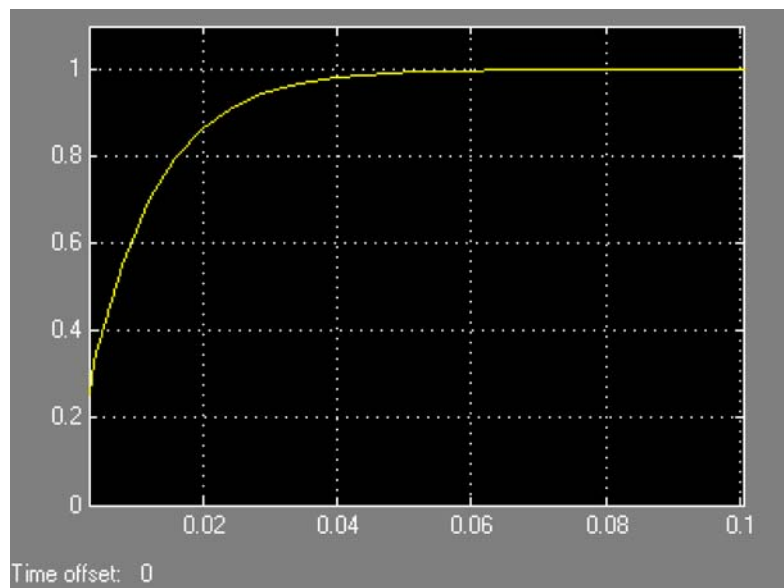


Fig. 6. Step response found using Simulink model.