MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

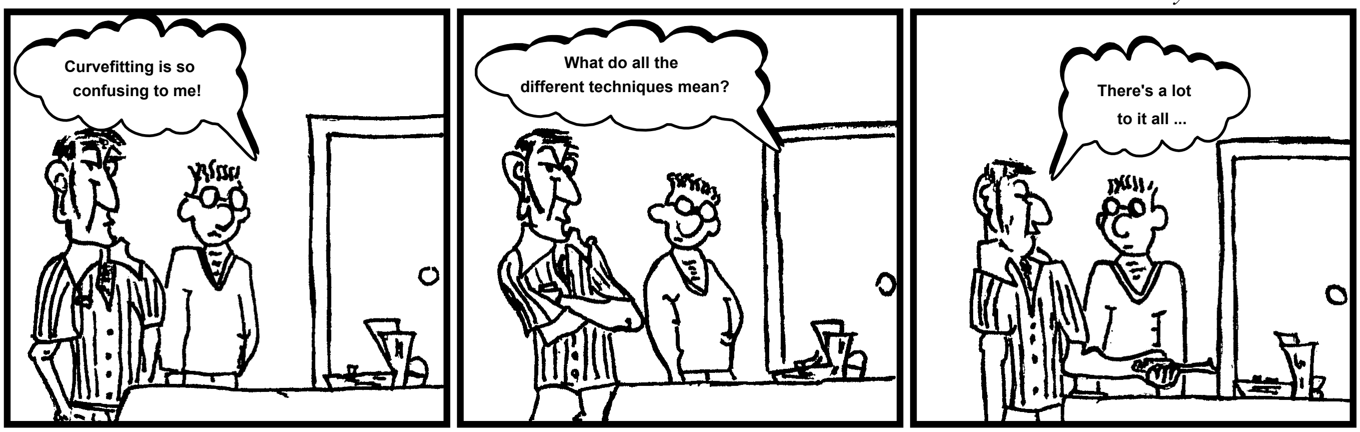


Illustration by Mike Avitabile

Curvefitting is so confusing to me!
What do all the different techniques mean?
There's a lot to it all ...

Curvefitting is probably the most difficult part of the whole experimental modal analysis process for most people. Actually, its better to refer to it as *modal parameter estimation*. But that's a mouthful - so we usually just call it *curvefitting*. But we are actually trying to extract modal parameters (frequency, damping and mode shapes) from measured data. Let's discuss a few general items first.

Basically, we need to describe the system in terms of it's modes of vibration. For example, the three mode system shown in Fig 1 can be described by the following frequency domain representation of the system as

$$h_{ij}(j\omega) = \sum_{k=1}^{3} \frac{a_{ijk}}{(j\omega - p_k)} + \frac{a^*_{ijk}}{(j\omega - p^*_k)})$$

or broken down into the contribution of each of the modes as

$$h_{ij}(j\omega) = \frac{a_{ij1}}{(j\omega - p_1)} + \frac{a^*_{ij1}}{(j\omega - p^*_1)} + \frac{a_{ij2}}{(j\omega - p_2)} + \frac{a^*_{ij2}}{(j\omega - p^*_2)} + \frac{a_{ij3}}{(j\omega - p_3)} + \frac{a^*_{ij3}}{(j\omega - p^*_3)}$$

Now as you start to look at this measurement, some quick thoughts come to mind. How many data points should I use? What should the order of the model be? Are there any effects from modes outside the band of the curvefitter? Does the same technique need to be applied to all the modes? When do I use a SDOF vs. a MDOF technique? Should I use a time or frequency domain curvefitter? (And the most important thing that should come to mind is - Oh how I wished I listened in modal class that day instead of going out to party!)

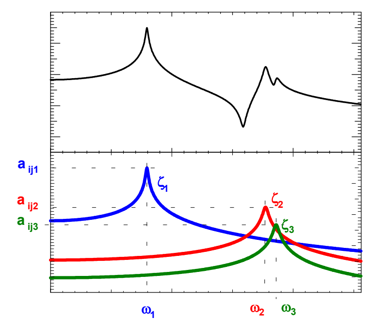
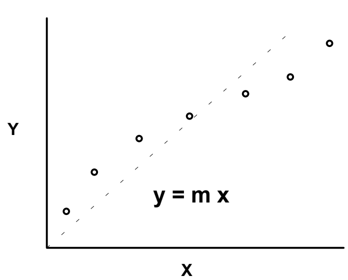
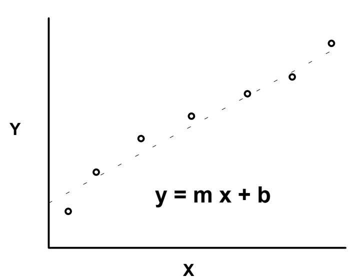


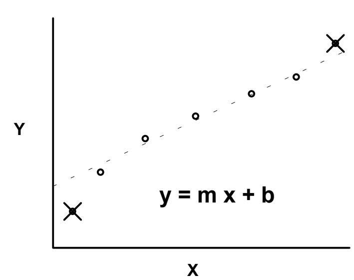
Figure 1 - Three mode system



Force data to pass thru zero
Figure 2a



Allow for compensation
Figure 2b



Use only part of the data
Figure 3b

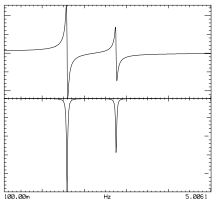


Figure 4a

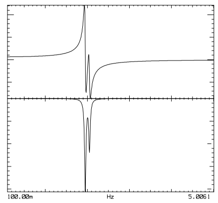


Figure 4b

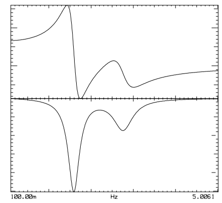


Figure 4c

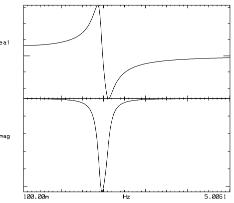


Figure 4d

First let's consider the some simple force gage calibration data in Fig 2. Now if the force gage should read zero at zero load, then Fig 2a represents the best straight line fit of the data - but that fit doesn't look very good. But what if the force gage had a preload. Then it may be necessary to allow for some compensation as shown in Fig 2b. And what if some of the measured data was outside the useful range of the force transducer. Possibly only a portion of the data should be used as shown in Fig 2c. And who said that the force gage was linear with a first order approximation of y=mx+b? I could possibly envision a cubic function that would better describe the measured data. For some reason everyone understands this force gage example but have a hard time realizing that my measured FRF has the same characteristics. Basically the analyst must decide on the order of the model, the amount of data to use and the need for residual compensation as shown in Fig 3. The basic equation to address this measurement is

$$[H(s)] = \text{lower residuals} + \sum_{k=i}^{j} \frac{[A_k]}{(s - s_k)} + \frac{[A_k^*]}{(s - s_k^*)} \quad + \text{upper residuals}$$

Basically, I select a band of modes to fit, specify the order of the model and decide on inclusion of residual terms.

Now I need to know when to use a SDOF or MDOF technique. What I need to know is how much modal overlap exists from one mode to the next. Fig 4 shows a variety of different situations for a two DOF system. Fig 2a shows modes that are well separate with very light damping. These types of modes can be approximated with a SDOF fit. Fig 2b shows modes that are closely spaced with very light damping.

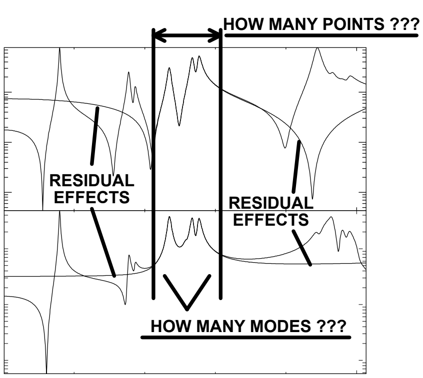


Figure 3 - Schematic of Analyst Curvefitting Decisions

There is some overlap from one mode to the next which may not be correctly compensated with a SDOF fit. It is likely that a MDOF fit may need to be employed for these two modes. Fig 4c shows well separated modes but damping causes some overlap which may also require a MDOF fit. But for both of these last two cases, you may try a SDOF fit for comparison with the MDOF fit. Fig 4d shows modes that are closely spaced with heavy damping. A MDOF fit would be needed for this case.

The last thing to consider is whether to use a time or frequency domain technique. The mathematical relationship is basically the same - it just looks different. Many times we write a relationship in a given form because there is some mathematical *gimmick* that makes the equation easier to solve or more efficient from a computational standpoint. But, in essence, both domains are equivalent. However, many times we tend to use the time domain techniques for lightly damped systems and the frequency domain techniques for heavily damped systems.

If I now look at Fig 5, what would I think would be appropriate for estimating parameters for this measurement. Well it is probably allright to use a SDOF for that first peak. But modes 2 and 3 are too closely spaced to use a SDOF, so most likely a MDOF technique would be used for these modes. Another thing to realize is that the cursors don't need to overlap or cover the whole frequency band. Remember that we are trying to extract parameters that identify the frequency, damping and residue for the system for each of the modes.

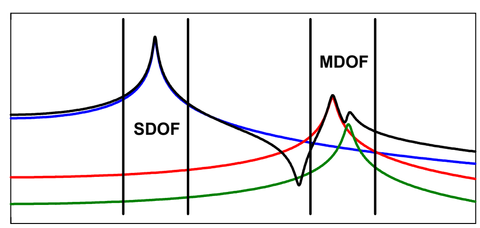


Figure 5 - Possible Curvefit Bands

We really need to spend a lot more time discussing all the details of each of the techniques but there isn't enough time right now to cover everything. But this quick overview should give you an idea of some of the concepts involved. Think about what we have discussed and maybe another time we can discuss each of the techniques in more detail. If you have any more questions on modal analysis, just ask me.