MODAL SPACE - IN OUR OWN LITTLE WORLD

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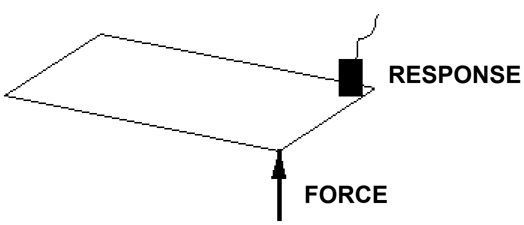
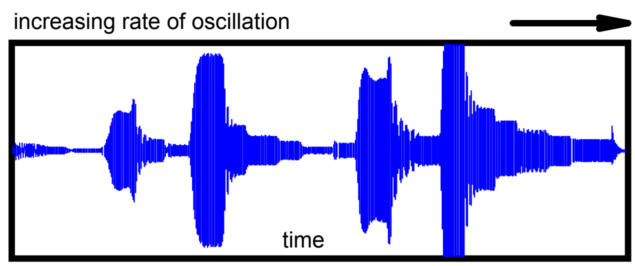
Could you explain modal analysis for me?
Well...it will take a little bit but here's one that anyone can understand.

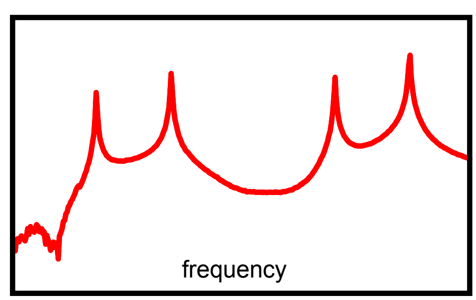
You're not the first one to ask me to explain modal analysis is simple terms so anyone can understand it. In a nutshell, we could say that modal analysis is a process whereby we describe a structure in terms of it's natural characteristics which are the frequency, damping and mode shapes - it's dynamic properties. Well that's a mouthful so let's explain what that means. Without getting too technical, I often explain modal analysis in terms of the modes of vibration of a simple plate. This explanation is usually useful for engineers who are new to vibrations and modal analysis.



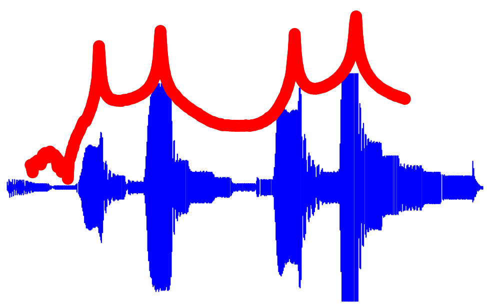
Let's consider a freely supported flat plate. Let's apply a constant force to one corner of the plate. We usually think of a force in a static sense which would cause some static deformation in the plate. But here what I would like to do is to apply a force that varies in a sinusoidal fashion. Let's consider a fixed frequency of oscillation of the constant force. We will change the rate of oscillation of the frequency but the peak force will always be the same value - only the rate of oscillation of the force will change. We will also measure the response of the plate due to the excitation with an accelerometer attached to one corner of the plate.



Now if we measure the response on the plate we will notice that the amplitude changes as we change the rate of oscillation of the input force. There will be increases as well as decreases in amplitude at different points as we sweep up in time. ***This seems very odd*** since we are applying a constant force to the system yet the amplitude varies depending on the rate of oscillation of the input force. But this is exactly what happens - the response amplifies as we apply a force with a rate of oscillation that gets closer and closer to the natural frequency (or resonant frequency) of the system and reaches a maximum when the rate of oscillation is at the resonant frequency of the system. When you think about it, that's pretty amazing since I am applying the same peak force all the time - only the rate of oscillation is changing!

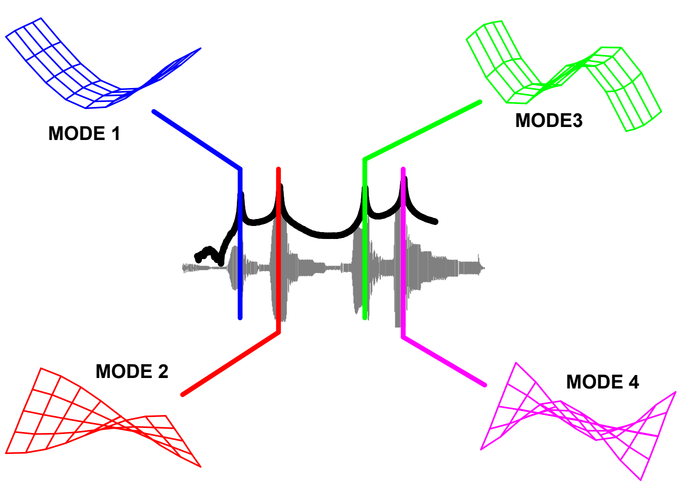


This time data provides very useful information. But if we take the time data and transform it to the frequency domain using the Fast Fourier Transform then we can compute something called the frequency response function. Now there are some very interesting items to note. We see that there are peaks in this function which occur at the resonant frequencies of the system. Now we notice that these peaks occur at frequencies where the time response was observed to have maximum response corresponding to the rate of oscillation of the input excitation.



Now if we overlay the time trace with the frequency trace what we will notice is that the frequency of oscillation at the time at which the time trace reaches it’s maximum value corresponds to the frequency where peaks in the frequency response function reach a maximum. So you can see that we can use either the time trace to determine the frequency at which maximum amplitude increases occur or the frequency response function to determine where these natural frequencies occur. Clearly the frequency response function is easier to evaluate.

You thought it was pretty amazing how the structure has these natural characteristics. Well, the deformation patterns at these natural frequencies also take on a variety of different shapes depending on which frequency is used for the excitation force.



Now let's see what happens to the deformation pattern on the structure at each one of these natural frequencies. Let's place 45 evenly distributed accelerometers on the plate and measure the amplitude of the response of the plate with different excitation frequencies. If we were to dwell at each one of the frequencies - each one of the natural frequencies - we would see a deformation pattern that exists in the structure. The figure shows the deformation patterns that will result when the excitation coincides with one of the natural frequencies of the system. We see that when we dwell at the first natural frequency, there is a first bending deformation pattern in the plate shown in blue. When we dwell at the second natural frequency, there is a first twisting deformation pattern in the plate shown in red. When we dwell at the third and fourth natural frequencies, the second bending and second twisting deformation patterns are seen in green and magenta, respectively. These deformation patterns are referred to as the mode shapes of the structure. (That's not actually perfectly correct from a pure mathematical standpoint but for the simple discussion here, these deformation patterns are very close to the mode shapes, for all practical purposes.)

Now these natural frequencies and mode shapes occur in all structures that we design. Basically, there are characteristics that depend on the weight and stiffness of my structure which determine where these natural frequencies and mode shapes will exist. As a design engineer, I need to identify these frequencies and know how they might affect the response of my structure when a force excites the structure. Understanding the mode shape and how the structure will vibrate when excited helps the design engineer to design better structures. Now there is much more to it all but this is just a very simple explanation of modal analysis.

Now we can better understand what modal analysis is all about - it is the study of the natural characteristics of structures. Both the natural frequency and mode shape (which depends on the mass and stiffness distributions in my structure) are used to help design my structural system for noise and vibration applications. We use modal analysis to help design all types of structures including automotive structures, aircraft structures, spacecraft, computers, tennis rackets, golf clubs, ... the list just goes on and on.

I hope this very brief introduction helps to explain what modal analysis is all about. I know I explained modal analysis to my Mom using the example above and I think for the first time she actually understood what I actually do. Since then, she has been heard explaining it to her friends using a variety of words closely resembling *modal analysis*, of which the best one was the time she referred to it as *noodle analysis* ... but that's another story!