

## MODAL SPACE - IN OUR OWN LITTLE WORLD

by Pete Avitabile

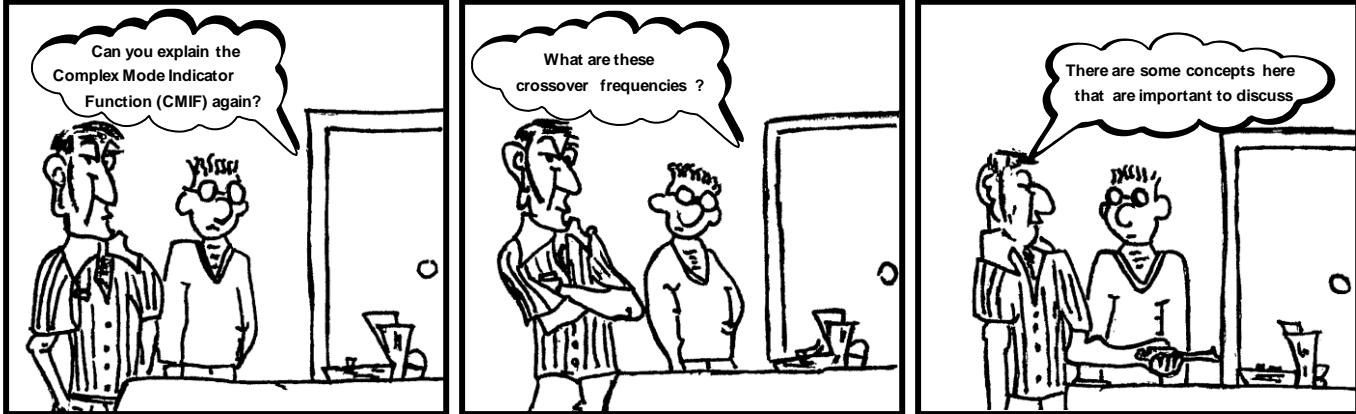


Illustration by Mike Avitabile

Can you explain the Complex Mode Indicator Function (CMIF) again? What are these crossover frequencies? There are some concepts here that are important to discuss.

Now I know I have heard many people comment on this as a point of confusion. But it really is not that hard to explain and I have examples to put it in perspective.

First let's get some of the messy math out of the way but that is needed to start the conversation. Basically, the Complex Mode Indicator Function (CMIF) uses the frequency response matrix and performs a singular value decomposition (SVD) to identify how many "significant" eigenvalues exist in each individual spectral line of the frequency response function (FRF) matrix.

WOW – now that is surely a mouthful to say the least. Let's present the equation and then try to pull it apart and make some sense of it all. If we collected a set of FRF data with multiple references, then I would have a matrix of FRFs. And I could write that equation using SVD as

$$[H] = [U][S][V]^T$$

And I could also write it down in expanded form to see some of the important pieces as

$$[H] = \begin{bmatrix} \{u_1\} & \{u_2\} & \{u_3\} & \dots \end{bmatrix} \begin{bmatrix} s_1 & & & \\ & s_2 & & \\ & & s_3 & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} \{v_1\}^T \\ \{v_2\}^T \\ \{v_3\}^T \\ \vdots \end{bmatrix}$$

Now we can see that there is a matrix [U] and matrix [V] which contain eigenvectors (left and right hand vectors to be specific) and a diagonal matrix of scalar values [S] called singular values; an earlier article about ten years ago describes this in much more detail. But of course the most important part of the SVD is that this can be written as the sum of each of the individual pieces that make up this FRF matrix.

Now when we perform an SVD and plot the singular values, we will get as many curves as there are references for all of the spectral lines in the band considered. So if there are three references then there will be three singular values, namely,  $S_1$ ,  $S_2$ ,  $S_3$ , which will exist for all the spectral lines in the band considered. Now if we plot these, then there will be three separate curves over that frequency band; the singular values are shown in the diagonal [S] matrix in the second equation. It is these three lines that are of interest for CMIF (but the eigenvectors will also play a small role as will be explained shortly). A set of CMIF curves are shown in Figure 1. The upper set of curves shows one form of the CMIF and the lower set of curves shows another form of the CMIF.

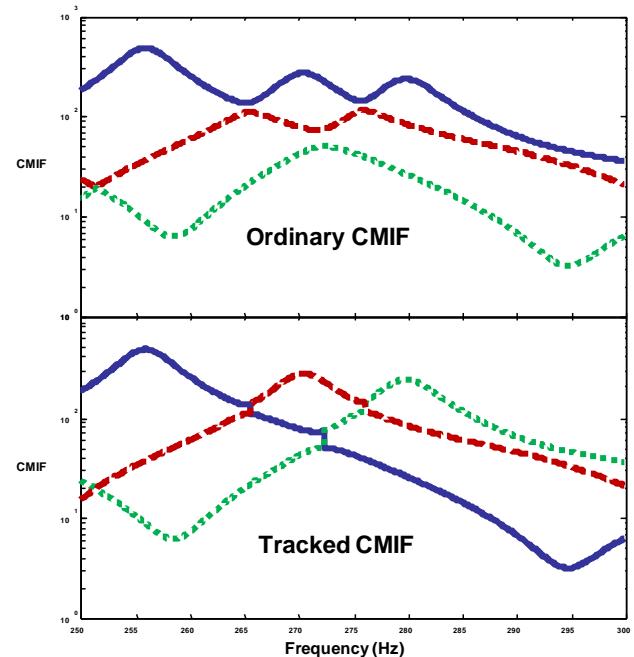


Figure 1 – CMIF (upper) and Tracked CMIF (lower)

Now in the CMIF in Figure 1, notice that the blue curve is larger than the red and green curves in the upper plot. To interpret CMIF, wherever there is a peak in the CMIF, there will be an indication of a mode at that frequency; in the upper plot there are three peaks in the blue curve so there are three modes identified by the CMIF in this case. If there is a second peak **at the same frequency as the first peak** then there is an indication that there are two modes at that frequency – but the second curve **must** peak at the **same** frequency as the first peak, otherwise it is not another mode.

So what are the peaks in the red and green curve and why aren't they indicators of modes? Well these are a result of these crossover frequencies you asked about. It all comes down to what you want to track from the singular values. Do you want to track the biggest singular value or do you want to track the vector related to the singular value. The lower plot in Figure 1 tracks the vector associated with the singular value rather than the largest singular value. So you can see that when we track the blue vector, it peaks at a lower frequency and then steadily declines. We also see that the red vector starts off small and then peaks in the middle of the frequency band and then steadily declines. And then you can see that the green line starts off very small and then eventually peaks as the blue and red decline. So it makes a difference whether I track the largest singular value (upper plot) or the vector related to the singular value (lower plot) – it just depends on what you want to look at in the CMIF.

If this still isn't clear, I have a good example. If you happen to like the horse races, then I can relate the horse race to the SVD for the CMIF. Just before the horse race starts, all of the horses are lined up at the starting gate and they are all at the same point. But as soon as the race starts, different horses will end up in different positions and that will change as the race progresses. But do you want to track who is ahead or do you want to track your horse – it is just a matter of preference (and depends on whether you are betting to win, place or show, etc). Figure 2 shows a schematic of the race at different points during the race. Clearly, at different points during the race, a different horse is in the lead. In terms of who wins, you want to track who is ahead and that changes during the race. But you might also want to track your favorite horse but he may not be in the lead.

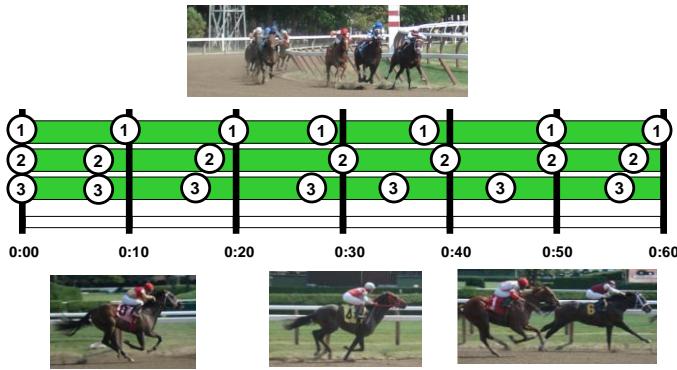


Figure 2 – CMIF/SVD and Horse Racing

In the first example, there was only one prominent mode at three different frequencies in Figure 1. But what would happen if there were multiple modes at the same frequency. Well then the CMIF would show one or more of the singular values peak at the same frequency in the CMIF plot. Figure 3 shows exactly this case where there are actually three modes at the same frequency at the first peak in the expanded view of the CMIF plot followed by three separate peaks higher in frequency – so in that expanded band there are actually six separate modes indicated by the CMIF plot. (That would be analogous to three horses nose to nose and if they were at the finish line, there would have to be a photo finish to determine the winner.)

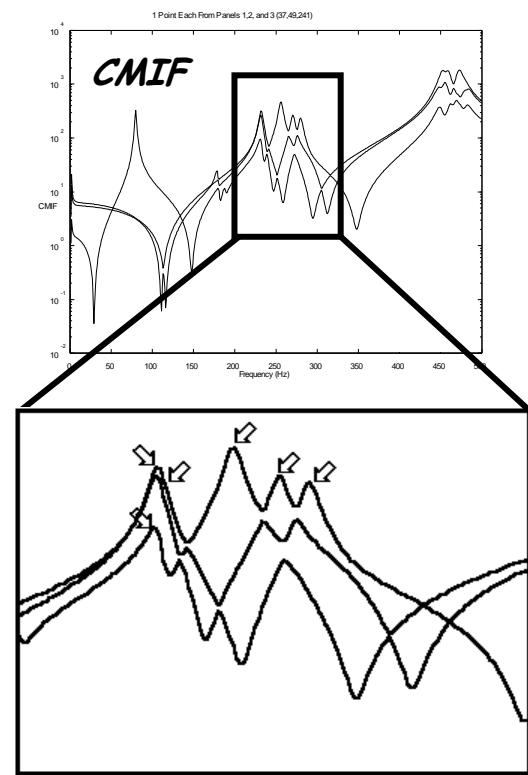


Figure 3 – CMIF with Multiple Modes

So I hope this explanation helps to clear up any confusion related to the CMIF and the interpretation of the curves as well as why the crossover frequencies exist.

And for some reason, it always seems that I never pick the right horse no matter which way I track them. If you have any more questions on modal analysis, just ask me.