MODAL SPACE - IN OUR OWN LITTLE WORLD

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What is the difference between all the mode indicator functions? What do they all do ?
Let's discuss this.

This is a good question. The indicator functions are very useful. There are several different mode indicator functions that are routinely used in experimental modal analysis when data is reduced. Let's talk about each of the most common tools to show their strengths and weaknesses and how to interpret data from each.

Of course, the measured frequency response function (FRF) can be viewed also but with only one FRF it may very difficult to identify how many modes exist. This is a problem because all of the modes may not be active in the particular FRF measured. The modes may be directional and from one measurement all the modes may not be easily observed. This might also be especially true of the drive point measurement where all the peaks will have the same phase; two very closely spaced modes may be very difficult to observe. So to assist in the process of pole selection, many different tools have been developed over the years. The main tools used are:

- SUM – Summation Function
- MIF – Mode Indicator Function
- MMIF – Multivariate MIF
- CMIF – Complex Mode Indicator Function
- Stability Diagram

So let's discuss each of these. For an example structure, a simple plate will be used as shown in Figure 1. But this plate has some closely spaced modes which will tax all of the mode indicator tools. The plate is subjected to MIMO testing with 2 shaker reference points and 15 accelerometer locations.

The first tool discussed is the Summation Function, SUM. This is a very simple formulation. Basically, it is the sum of all of the FRFs measured (or sometimes only a subset of all the FRFs is used). The SUM will reach a peak in the region of a mode of the system. The idea is that if all the FRFs are considered then all of the modes will be seen in the majority of the measurements. As more and more FRFs are included, there is a greater chance that all of the modes will be seen in the collection of FRFs summed together. This is obviously better than one particular measurement where all the modes may not be present.

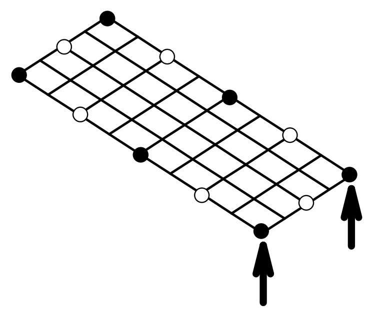

Figure 1 - Plate Test Setup with 2 References

A SUM function for all the measured response functions is shown in Figure 2. The SUM function will identify modes reasonably well especially if the modes are well separated. In the figure, there are five peaks observed which indicates that there are at least five modes in the frequency band shown. Another important feature of the SUM function is that each of the peaks is generally fairly wide and if closely spaced modes exist, then this may not show all of the modes well.

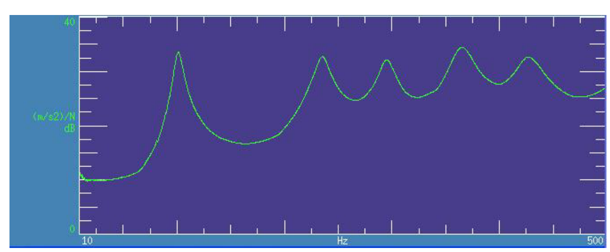

Figure 2 - SUM for 2 References and 15 Accelerometers

While the SUM function is useful, it is not always very clear when modes are closely spaced. The original Mode Indicator Function (MIF) was formulated to provide a better tool for identifying closely spaced modes. Basically the mathematical formulation of the MIF is that the real part of the FRF is divided by the magnitude of the FRF. Because the real part rapidly passes through zero at resonance, the MIF generally tends to have a much more abrupt change across a mode. The real part of the FRF will be zero at resonance and therefore the MIF will drop to a minimum in the region of a mode. An extension of the MIF is the Multivariate MIF (MMIF) which is an extended formulation of MIF for multiple referenced FRF data. The MMIF follows the same basic description of a single MIF. The big advantage is that multiple referenced data will have multiple MIFs (one for each reference) and can detect repeated roots. This is shown in Figure 3.

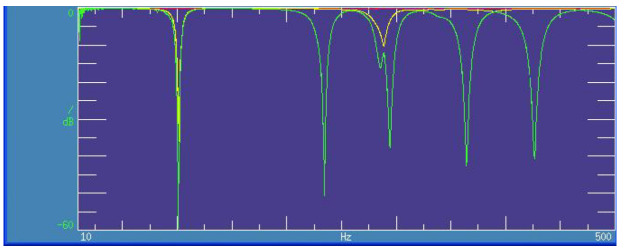

Figure 3 - MMIF for 2 References and 15 Accelerometers

If the first MIF drops, then there is an indication that there is a pole of the system. Every one of the drops in Figure 3 for the MIF1 (shown in green) indicates a mode of the system. Notice that there are six dips in the function – one more than was observed in the SUM function. Clearly, there is one mode that is closely spaced around 300 Hz which was not clearly identified in the SUM function.

Now if the second MIF also drops at the same frequency as the first MIF, then there is an indication that there is a repeated (or pseudo-repeated root). Clearly, the second MIF in Figure 3 (shown in yellow) indicates that there is a repeated root at the first dip in the MIF close to 100 Hz. (Note that the SUM only indicated one mode in this range.) However, the other small dip in the second MIF close to 300 Hz is not an indication of a mode because the second MIF does not dip at the same frequency as the first MIF. In order to have an indication of two roots both MIFs must dip at the same frequency.

The MMIF is a much more accurate tool for indication of modes. However, the assumption is that real part of the FRF is zero at resonance. If the measurements have some distortion or if there is some phasal information in the measurements (associated with non-real normal or complex modes) then the MMIF may not be able to accurately depict the modes accurately.

The Complex Mode Indicator Function (CMIF) is a better tool if this is the case. The CMIF is based on a singular valued decomposition of the FRF matrix to determine all the principal modes that are observed in the set of measurements. The plot of the singular values also helps to identify poles of the system. The CMIF will peak where maximum values exist indicating poles of the system. There will be one CMIF curve for each reference. Figure 4 shows the CMIF.

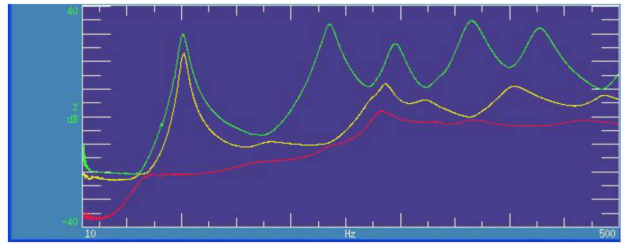

Figure 4 - CMIF for 3 References and 15 Accelerometers

Clearly, the two CMIF curves peak close to 100 Hz indicating that there are two peaks at that frequency. In the 300 Hz frequency range, there is an indication that there are two (or possibly three) modes in that range. The CMIF function provides some additional insight into the number of poles in the frequency band of interest.

All of the tools assist in the selection of poles during the extraction process. The last tool is the Stability Diagram, SD. The basic philosophy is that poles that are extracted from increasing order mathematical model will repeat as the order is increased if the pole is a global characteristic of the system. Other indications of roots will not maintain consistent indication as the order of the model is increased. A plot of these characteristics when a pole migrates to a stable configuration provides yet additional insight into the poles of the system. Figure 5 shows a stability diagram over a narrower frequency range than previously shown. Notice that there is an indication of a repeated root near 100 Hz and another pair of roots close to 300 Hz. (Discussion on details of the stability diagram will be discussed in a future article.) So this confirms the findings from the MMIF and CMIF.

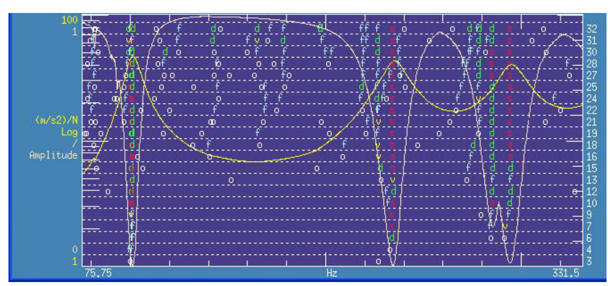

Figure 5 – Stability Diagram for FRF Data

There is a lot more that can be discussed but the majority of the mode indicator tools are explained in this article. If you have any more questions on modal analysis, just ask me.