MODAL SPACE - IN OUR OWN LITTLE WORLD

by Pete Avitabile

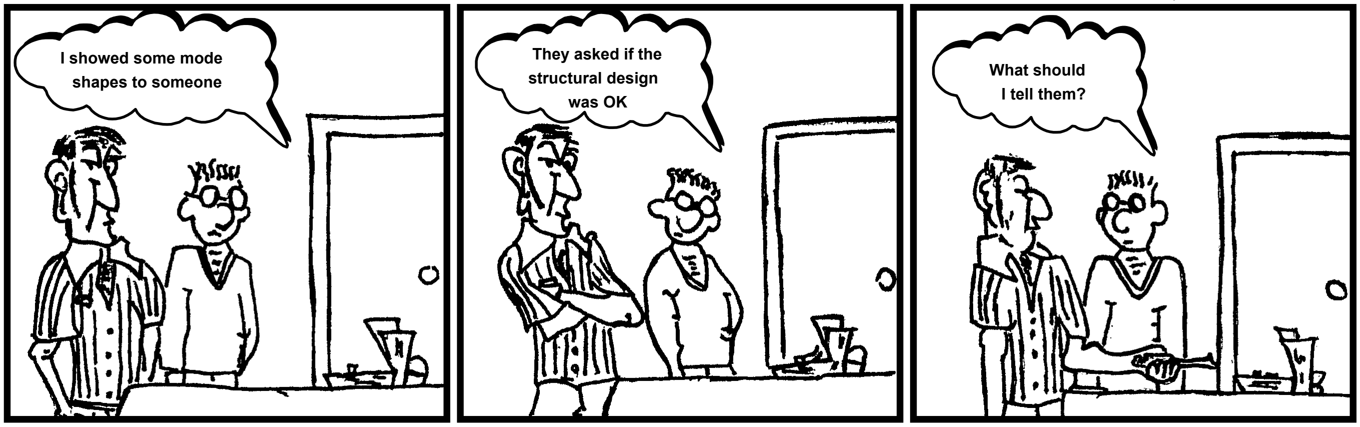

Illustration by Mike Avitabile

I showed some mode shapes to someone
They asked me if the structural design was ok
What should I tell them ?

If I could have a dollar for every time I have heard that question, I'd be rich! The basic answer is you just don't have enough information to answer that question. People who ask that question have no idea what they are asking about. You have to be very diplomatic in telling them that the question is a silly one to ask.

One of the reasons why they are apt to ask the question is because you probably showed them an animation and their impression is that the structure is deforming (since they see the deflections on the computer screen). Of course, you know that this is only a characteristic shape that the structure will undergo when subjected to a force that excites that mode. Sometimes I've been known to say "well let's increase the amplitude of the animation and see if we get the structure to break on the screen". (Of course, this is ridiculous!!! This can't happen.) I use this statement to start to explain what shapes are all about. Animation is only a mechanism to understand how the structure may deform if that mode is excited by the forcing function.

One of the key points here is that we need to know the applied force. For some reason, people forget that we need a force applied to the system to get a response. The physical equation of motion is

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\}$$

and the equivalent modal space representation is

$$\begin{bmatrix}\ddots & & \\ & \overline{M} & \\ & & \ddots\end{bmatrix}\{\ddot{p}\} + \begin{bmatrix}\ddots & & \\ & \overline{C} & \\ & & \ddots\end{bmatrix}\{\dot{p}\} + \begin{bmatrix}\ddots & & \\ & \overline{K} & \\ & & \ddots\end{bmatrix}\{p\} = [U]^T\{F\}$$

Notice that there is a force on the right hand side of this equation. When we solve for the characteristic equation of the system, we assume that there is no force on the right hand side. This is how we obtain the dynamic characteristics of the system. One way to look at it is that the modes of the system are nothing more than a very elaborate set of filters which have the ability to amplify and attenuate an input signal on a frequency basis. If we just look at the filters themselves, can we make any assessment whether the filters are good or bad for a particular application? Of course not! All we can say is that the filters have some characteristics which relate to a center frequency, rolloff and some gain settings as seen in Figure 1.

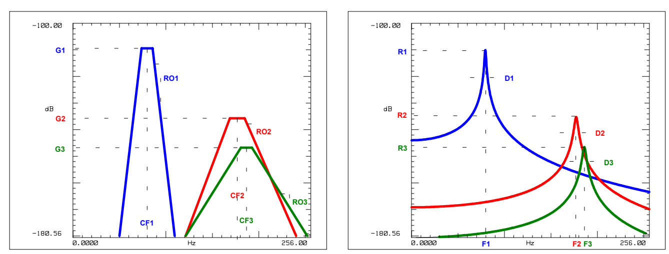

Figure 1

Well ... the dynamic characteristics of a structural system are quite the same. We can identify each mode (each filter) as having a natural frequency (center frequency), damping (rolloff) and residue/mode shape (gain). We need to very clearly understand that the mode shapes are only characteristics and we cannot determine the goodness or badness of a mode unless we know the forcing function - that is, the right hand side of the equation.

As another example, let's say we wanted to determine the stiffness of a cantilever beam. Well, we could go out in the lab and apply a force to the tip of the cantilever beam and measure the resulting displacement. We know that we could determine the stiffness as K = F / X. Now this stiffness is an important parameter or characteristic of the beam. But once I determine the stiffness, do I know if the beam will fail or not? Of course not! I would need to know the actual force that was applied to the beam - wouldn't I? You see, in the test lab we applied an arbitrary force and measured the displacement due to that force in order to determine the character of the beam. Someone needs to identify the actual real world force before I can compute the actual displacement. And then I need to have some specification defined as to how to assess the acceptability of the structure due to the design or real world forces - which brings me to another important point.

One thing that people often forget is that once the mode shapes are obtained and a dynamic design force is specified, the response can be computed, but someone needs to identify a specification defining what is acceptable and unacceptable for the response. This, at times, can be one of the most frustrating parts of the structural dynamic response modeling process. The responses can be computed but no one has defined what the level of acceptance is. Many times this very important detail is overlooked in the process of extracting pretty animated mode shapes. Then everyone asks... how much deflection is acceptable, how long will the component life be, does it "feel" good, is the response too noisy, etc.

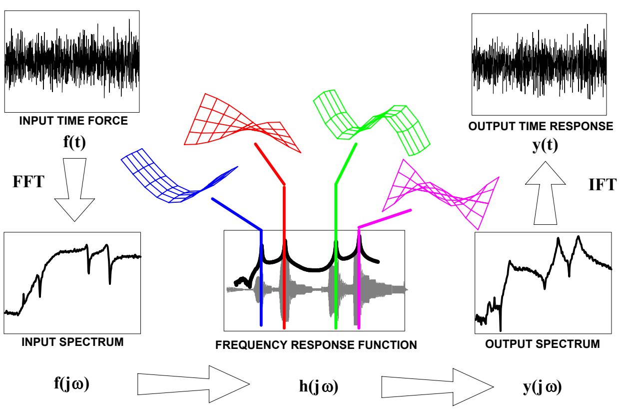

Figure 2

So now that we have discussed a few of these things, let's go back to our plate example that we have discussed before concerning different aspects of modal analysis. Figure 2 shows a schematic of a typical forced vibration problem. There is some force which is applied in the time domain. Well, this time signal is very confusing so it helps to identify some important characteristics of this force if it is transformed to the frequency domain using the FFT process. Now I know that this force is multiplied times the frequency response function in order to get the output of the system. That output could then be transformed back to the time domain if desired. Well, the important point to make here is that the FRF is multiplied by the force spectrum.

That means that the input force spectrum is amplified and attenuated by this multiplication. The FRF controls how this force is amplified and attenuated on a frequency basis. In the figure above, the FRF appears to have contribution for all four modes shown. That assumes that the applied force and response location exists at a point where there is participation of each of the four modes of the system.

But what if the force was applied at a location of a node of a mode. Let's say that the force was applied along the symmetry line along the length of the plate. Then, the applied force would not excite any of the torsional modes from that location; then we say that those modes don't participate in the response of the plate due to that force. The same is true for the response location. So we can see that both the input and output locations will have an effect on the response of the system. (In fact, the mode shape amplitudes have a strong influence on how much a particular mode contributes to the overall response.)

While we could say that certain modes may not participate in the response of the system, that does not imply that those modes don't exist - they just are not needed to compute the response of the system. But the modes still exist - they define the dynamic characteristics of the system. Depending on the location of the applied force and the point where response needs to be measured (as well as the frequency content of the signal), will determine how the structure responds. Some modes may be more dominant in the level of response and others may be less dominant in the response - again depending on the particular input-output location selected. But all the modes exist - they just may not all be activated on a uniform basis.

So what we need to remember is that a modal test only defines the character of the system. We apply an arbitrary force which is measured along with the response of the system due to the applied force. This enables us to determine the dynamic characteristics of the system - the frequency, damping and mode shapes. These are only characteristics of the system. We display the mode shape (animate them) to better understand how the structure may deform if a force is applied to the system that excites one or more modes of the system. Remember, modal analysis doesn't use the force on the right hand side of the equation - the mode shapes are independent of the force.

Now I hope you understand why you can't answer the question that you asked. If you have any other questions about modal analysis, just ask me.