MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

Illustration by Mike Avitabile

Sometimes the mode shapes appear to be rotated from what is expected. Are the modes wrong? What's up? Now this is something that needs to be discussed.

Now this is a topic that comes up often. So it is going to need some discussion. I have seen people often get confused about the mode shapes for a system. Often times people have a preconceived notion as to what the results "should be". When the mode shapes appear different than that expected, then you might think that the modes are wrong.

Most often when modes of a structure are very closely spaced the mode shapes that satisfy the system can be linear combinations of each other. Therefore, the shapes might be rotated from what you might have expected. The only real requirement is that the modes of the system are orthogonal with respect to the system mass and stiffness matrices. Each of the modes of the system are unique from each other.

In order to help describe this, a simple geometry example will be used along with a simple beam and plate to describe what can be happening here.

A simple x-y coordinate system to describe a rectangular shaped area is shown on the left in Figure 1. Now we selected the coordinate system arbitrarily and aligned the x and y to the side walls in the lower left hand corner. That just happens to be convenient. And then all of our dimensions are easy to understand.

But what if I have the irregular shaped area shown in the right in Figure 1. Now the selection of the reference coordinate system can be selected in several locations and no one location appears to be better than another. With the reference selected at the upper most corner, then the description of the original reference location in the lower left corner is described differently because of the coordinate system selected.

This just means that the description of any point in the area will be described differently. But the point in the area will not change.

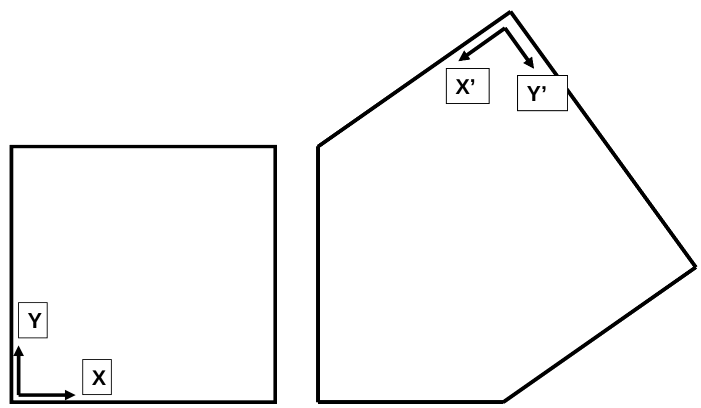


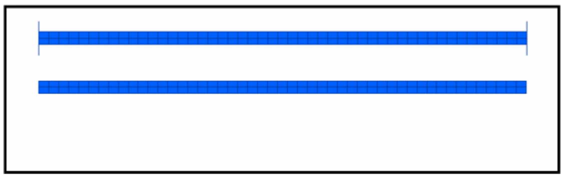
Figure 1 – Rectangular and Irregular Shaped Area

So using this simple geometry example will help set the stage for the discussion of a general mode shape that may be described in different ways depending on how the coordinate system is selected. Figure 2 and 3 show a description of the rigid body modes for a simple planar beam structure.

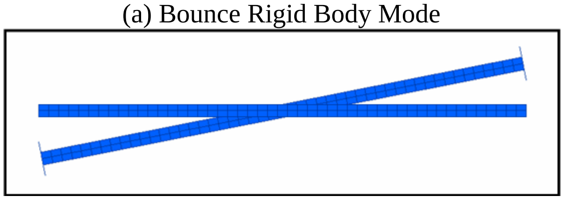
In the first case shown in Figure 2, the first two rigid body modes of the beam consist of a classic bounce mode and a rocking mode that occurs about the geometric center. This is exactly what everyone would expect those two modes to be. And if this were to occur, not one would question this at all.

But for the second case shown in Figure 3, the first two rigid body modes have a slightly different appearance. At first glance, most people would say that those rigid body modes were not correct. And that statement would only be made because it wasn't what you were expecting. You will notice that one mode is mainly bounce but has a little bit of rocking and that the other mode is mainly rocking but not about the geometric center.

While they may not look like what you would expect (or like) to see, these modes are perfectly correct. Because they are essentially at the same frequency, any linear combinations of these modes form a linearly independent set of vectors that are orthogonal with respect to the system mass and stiffness matrices.

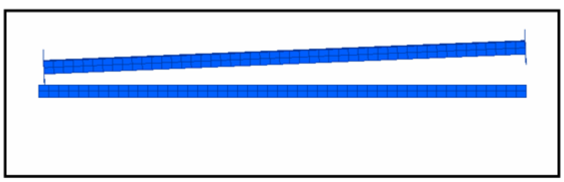


(a) Bounce Rigid Body Mode

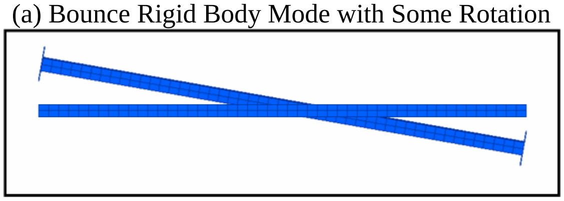


(b) Rocking Rigid Body Mode About Center

Figure 2 – Rigid Body Modes Geometric Center



(a) Bounce Rigid Body Mode with Some Rotation



(b) Rocking Mode Offset From Center

Figure 3 – Rigid Body Modes Not About Geometric Center

This can also happen with the flexible modes of the system when the frequencies are repeated or pseudo-repeated. Figure 4 shows a set of modes that are pseudo-repeated – they occur at essentially the same frequency. These modes are seen as first bending and first torsion as expected. But these same modes are also seen in Figure 5 but they do not appear as simple bending and simple torsion. But these modes just have a different coordinate system to describe them. As long as the modes represent an orthogonal set of vectors then they are mathematically correct. They just may not be what you would expect to see.

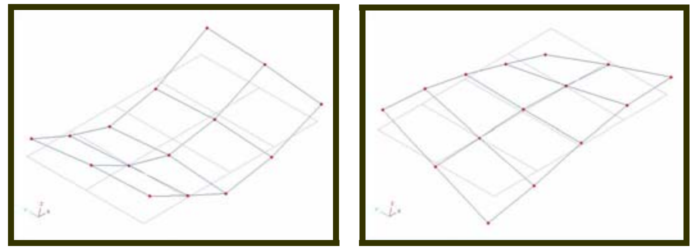


Figure 4 – Pure Bending and Pure Torsion Modes

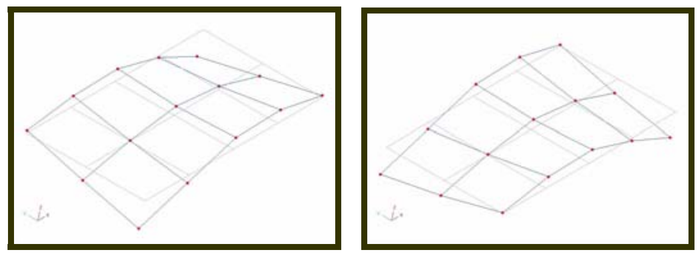


Figure 5 – Modes with Mixing of Bending and Torsion

This issue occurs with structures that have double symmetry and when either repeated roots or pseudo repeated roots occur. Another time it can happen is when using different numerical solution algorithms. Because the solution will typically iterate to a set of solution vectors, there is no reason why the vectors should converge towards a particular reference coordinate system. Actually the beam solutions shown in Figure 2 and 3 were obtained using two different finite element eigensolution approaches – one solution just happened to converge to the modes the way we would have expected them to occur whereas the other solution scheme did not. The modes in Figure 4 and 5 were obtained from actual test data on a structure that is known to have pseudo-repeated roots.

I hope this clarifies your confusion about modes shapes and their possible orientations. If you have any more questions on modal analysis, just ask me.