MODAL SPACE - IN OUR OWN LITTLE WORLD

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Illustration by Mike Avitabile

Are you sure you can get mode shapes from one row or column of the H matrix?
Sure! Let's walk through an example.

Let's use the beam that we have discussed before as an example. For this beam, we considered three measurement points. There are a total of nine possible input-output FRFs that can be measured. Remember we discussed that these measurements can be obtained from either shaker or impact testing. So that we have some numbers to discuss, the beam mode shape values are shown in Figure 1. (The values will be kept simple for discussion purposes.)

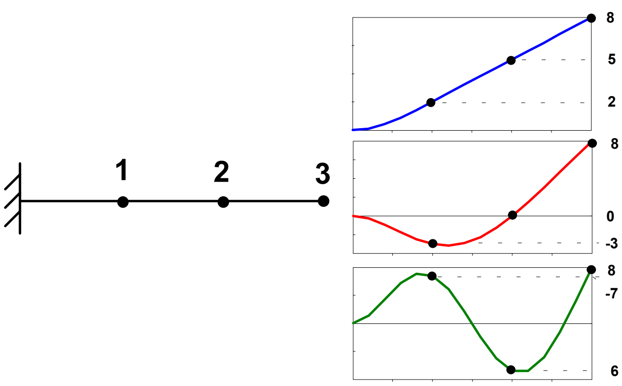

Figure 1

Awhile ago we described how the peak amplitude of the imaginary part of the FRF is directly related to the residue (which is directly related to the mode shape). In fact, we said that the residue was approximated by

$$a_1 = \sigma\, h(j\omega)\,\big|_{\omega\to\omega_n}$$

and that the individual values of the mode shape can be obtained from

$$a_{ijk} = q_k u_{ik} u_{jk}$$

Now let's plot the FRF matrix for this beam with three measurement points. I could show any one of the different parts of the FRF, but it turns out that the imaginary part of the FRF is most informative for this discussion since it shows both magnitude and direction; all the plots have the same -10 to +10 scale. This is shown in Figure 2.

Now let's use the third row of measurements to determine the mode shape for mode 1; this implies that point three is the reference location. Now if I were to pick the peak of the FRF for mode 1, the amplitudes are proportional to the shape of the cantilever beam first mode as seen in Figure 3.

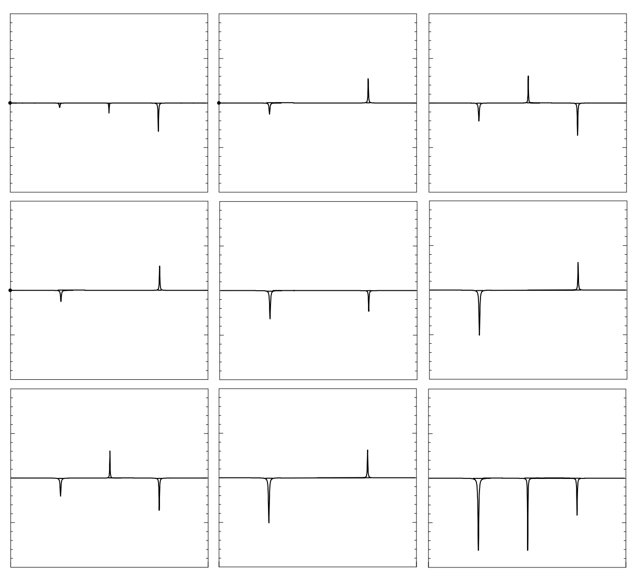

Figure 2

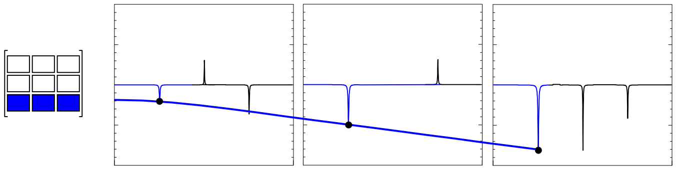

Figure 3

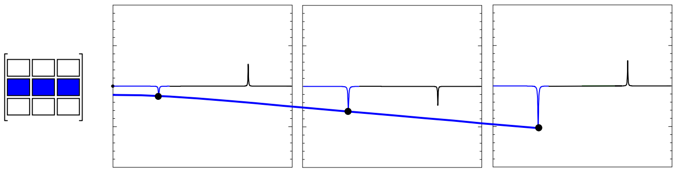

Figure 4

If you look at the values of the amplitude for mode 1 for points 1, 2 and 3, you will see that they are -2, -5, -8, respectively. These values are the values of the mode shape shown in Figure 1. (Notice that I have arbitrarily scaled the values to maintain an easy interpretation of the data. Also notice that the shape could be either plus or minus since the "shape" is the same.)

Now let's use the second row of the FRF matrix. If I pick the peak of the FRF for mode 1, the amplitudes are again proportional to the shape of the cantilever beam first mode as seen in Figure 4.

If you look at the values of the amplitude for mode 1 for points 1, 2 and 3, you will see that they are approximately -1.2, -3.13, -5, respectively. At first glance, these values look different but we can notice that the "ratio" or "shape" is exactly the same as the previous case.

In fact, if I scale the values of the mode shape from the third row by the ratio of the value of the mode shape at reference point 2 (5.0) to the value of the mode shape at reference 3 (8.0), then I will get the mode shape listed above for the 2nd row of the FRF matrix [2 (5/8)=1.2, 5 (5/8)=3.13, 8 (5/8)=5]. This is exactly what I expect to get based on the theory relating mode shapes to residues, so I'm actually not surprised. (We could also look at the first row of the FRF matrix and arrive at the same results.)

So we can see that we can get the mode shape of the beam from any row of the FRF matrix. If we remember that the reciprocity holds true, then we know that the rows and columns contain the same information. So now I can also see that the mode shape in every column of the FRF matrix. So this is why we say that you can use any row or column of the FRF matrix to estimate the mode shape. Of course, I can write out all the equations to show this but the pictorial description is sufficient (and I know how you hate it when I start writing equations!)

Now let's look at mode 2 and use the third row of the FRF matrix. If I pick the peak of the FRF for mode 2, the amplitudes are proportional to the shapes of the cantilever beam second mode as seen in Figure 5.

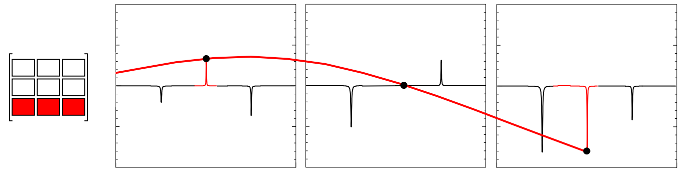

Figure 4

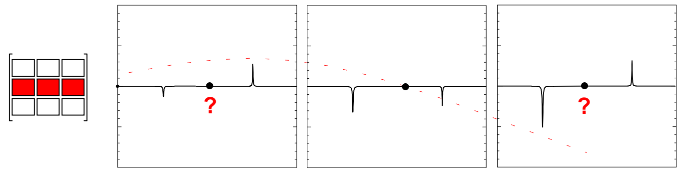

Figure 6

If you look at the values of the amplitude for points 1, 2 and 3, you will see that they are 3, 0, -8, respectively. They are the values of the mode shape shown in Figure 1.

But when I look at the second row of the FRF matrix for mode 2, there is no information pertaining to mode 2. How could this happen? Well, the value of the mode shape for mode 2 at point 2 is zero - its the node of the mode. Anytime we use an input location or response location that is located at a node point (zero shape value) then we will not be able to see the mode from that reference location.

One last picture may help to put it all together for you. Figure 7 shows a waterfall plot of the imaginary part of 15 measurements taken on the beam; the three measurements corresponding to the ones in Figure 2 are shown in color. In this plot, the information pertaining to mode 1 is shown in blue, mode 2 in red and mode 3 in green. We see that the mode shapes can be obtained from the peak of the imaginary part of the FRF. From these plots we can see the first, second and third bending shapes for the cantilever beam.

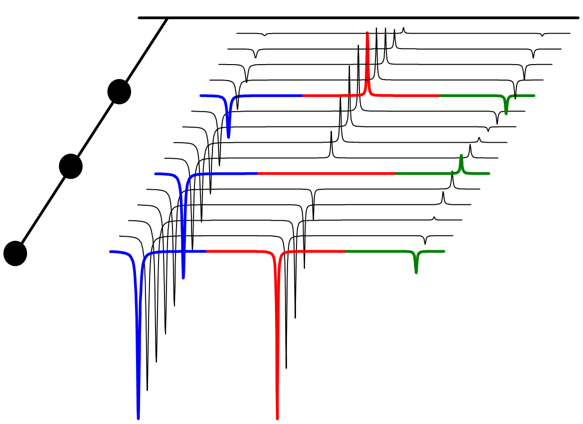

Figure 7

So, in conclusion, we can say that you can use any row or any column of the FRF matrix to estimate any mode of the system, provided that the reference is not located at the node of a mode. I hope this answers your question. If you have any other questions about modal analysis, just ask me.