MODAL SPACE - IN OUR OWN LITTLE WORLD

by Pete Avitabile

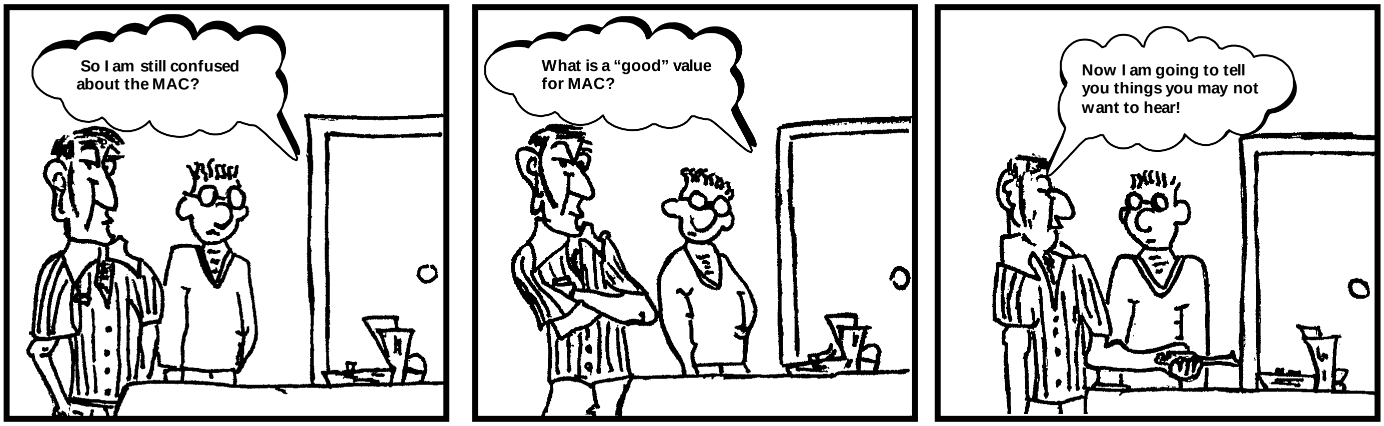

Illustration by Mike Avitabile

So I am still confused about the MAC? What is a “good” value for MAC?
Now I am going to tell you things you may not want to hear!

This is a subject that I find many people have a hard time understanding and accepting. Many would like to act like an ostrich and “stick your head in the ground” and hope that the problem will go away. Well…I am about to show some things here that many people have a hard time accepting. But first let me just start with a few words concerning correlation and orthogonality because there is a big difference between the two and many would like to think that the Modal Assurance Criteria (MAC) is the same as the orthogonality check – but in fact they are dramatically different.

As a formality, the two equations describing the MAC and Orthogonality, respectively, are:

$$MAC_{ij}=\frac{[\{a_i\}^T\{b_j\}]^2}{[\{a_i\}^T\{a_i\}][\{b_j\}^T\{b_j\}]}$$

$$ORT_{ij}=\{u_i\}^T[M]\{j_j\}^T$$

And the first thing to point out is that the MAC is really nothing more than a vector dot product that is scaled such that the values will range between 0 and 1. And if the value of the MAC is close to 0.0 then we say that there is little correlation between the two vectors. And if the value of MAC approaches 1.0 then the two vectors are very similar. But you notice that the word *orthogonal* was never used. Only the word *similar* was used.

Now the orthogonality is a mathematical property that results from the eigensolution of the mass and stiffness matrices that describe the system. A by product of the eigensolution is that the vectors are *“linearly independent”* and the vectors are *“orthogonal with respect to the mass and stiffness matrices simultaneously”*. So the orthogonality is a property that is guaranteed as a result of the eigensolution. The MAC has no such guarantees with that calculation.

The orthogonality check is a much more rigorous check that is performed and often times it is mandated as part of the certification process in the aerospace and military applications. The analytical/finite element model mode shapes are often compared to the measured experimental vectors from test. The governing bodies have mandated that the mass orthogonality of similar vectors must be greater that 90% or 95% and that different vectors must have values no higher than 5% to 10%. That is to say that the diagonal terms of the mass orthogonality matrix must be greater than 90% and all the off-diagonal terms must be lower than 10%. Now in these industries, the MAC is not typically used for the validation of the model because the orthogonality is a better correlation identifier.

Now what about other industries. Well there really isn’t a mandate or governing body so many times companies or industries have “good practices” that are generally followed. The MAC really resulted from a test environment where test engineers wanted to identify if the measured shapes from one test to the next were similar, or one prototype was similar to a production configurations or … (on and on with many different ways that we can use the MAC).

So why didn’t they use the orthogonality check. Well, remember the MAC started from the testing guys and back 30-40 years ago, the test guys didn’t have access to a mass matrix; only a few of the analytical engineers had access to very primitive finite element modeling tools. Plus the MAC was an easy calculation to perform. Let’s face it, they didn’t have the mass matrix and didn’t want to bother with the much more intensive mass orthogonality check. The test guys were just trying to get some simple comparisons.

But then everything “grew up” and all of a sudden we had people using the MAC to correlate the finite element mode shapes with the measured test data. And all of a sudden we had people "correlating" models using MAC and then they started to use some of the same general criteria that were developed with orthogonality checks.

But often times I will hear people saying that they will accept MAC values for correlated vectors with values that are lower than 90% and sometimes accept correlation values as low as 80% because "we are working with real structures and not simple academic examples". Well I am not so sure that I agree with that mentality because there is no mass matrix involved in the MAC calculation.

I want to show two examples to show some MAC values that result from modes that are clearly not similar at all. The first case is to compare the rocking rigid body mode of a free free beam with the first cantilever mode. Figure 1 shows that there is about 60% similarity indicated by MAC.

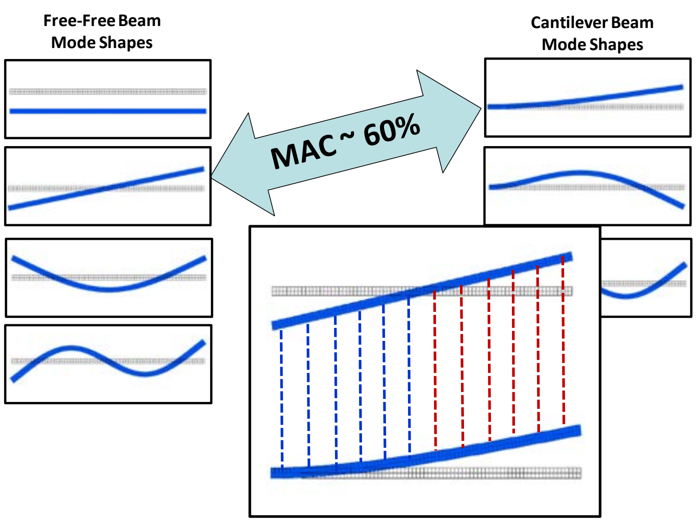

Figure 1: Mode Shapes for Free-Free Beam (left) and Cantilever Beam (right)

Good Golly Miss Molly! These two vectors have nothing to do with each other but MAC shows 60% correlation. How could that possibly happen. Well if you look lower right inset in Figure 1, you will see that the values of the cantilever mode shape from the base to midspan of the beam are very small and their contribution in the MAC calculation are very small. And if you look at the mode shape from the midspan to the tip of the beam, the values are much larger and with the naked eye you can see that the rocking rigid body mode looks quite similar. And that is what the MAC is indicating. But we know these two modes are not similar at all. Now you may argue that I have no right to compare these modes, but I did just to illustrate what a 60% MAC indicates. How bizarre is that !!!!!

Now let's proceed on with two cantilever beams – one with a uniform mass distribution and one with an additional lumped mass at the center of the beam (20% of the weight of beam). Now if I looked at the MAC between Mode 1 and Mode 2 with the uniform mass distribution, the MAC will show little correlation as expected. But when the MAC is performed on the beam with the additional lumped mass, the MAC between Mode 1 and Mode 2 is almost 80%. *Missy Molly cannot believe this at all.* But if you look at the uniform beam modes in the upper right portion of Figure 2 and compare them to the modes in the lower right portion in Figure 2, you start to see the same type of issue as discussed in the first case. The mode shape values close to the built in end have very little contribution to the MAC. And the values of the shape towards the end of the beam for Mode 1 and Mode 2 look very similar. And that is what MAC is indicating again. But we know that these two modes are orthogonal to each other but the MAC completely breaks down here. However, the mass orthogonality clearly identifies the vector status because the proper mass distribution was included in the orthogonality check. The MAC has no way to account for the uneven mass distribution for this case.

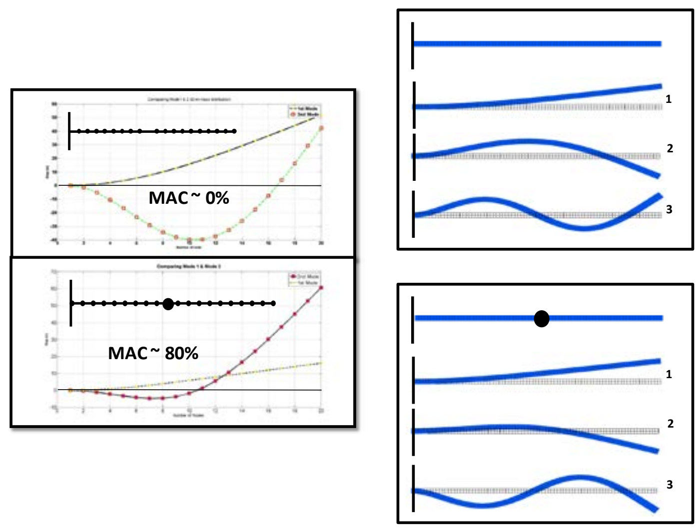

Figure 2: Cantilever Beam Mode 1 and Mode 2
with Uniform Mass Distribution and Center Lumped Mass

So I hope you can see that the MAC values can be deceiving. The MAC has no mass matrix. That is its biggest strength ***and*** its biggest downfall. Now don't get me wrong…I use MAC all the time to help with sorting out model and test results. But I am very leary when the supposedly "correlated modes" have MAC values that do not have very strong indicators.

I hope that this helps to explain the questions you had. We will likely talk more about MAC in a future article. If you have any other questions about modal analysis, just ask me.