MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

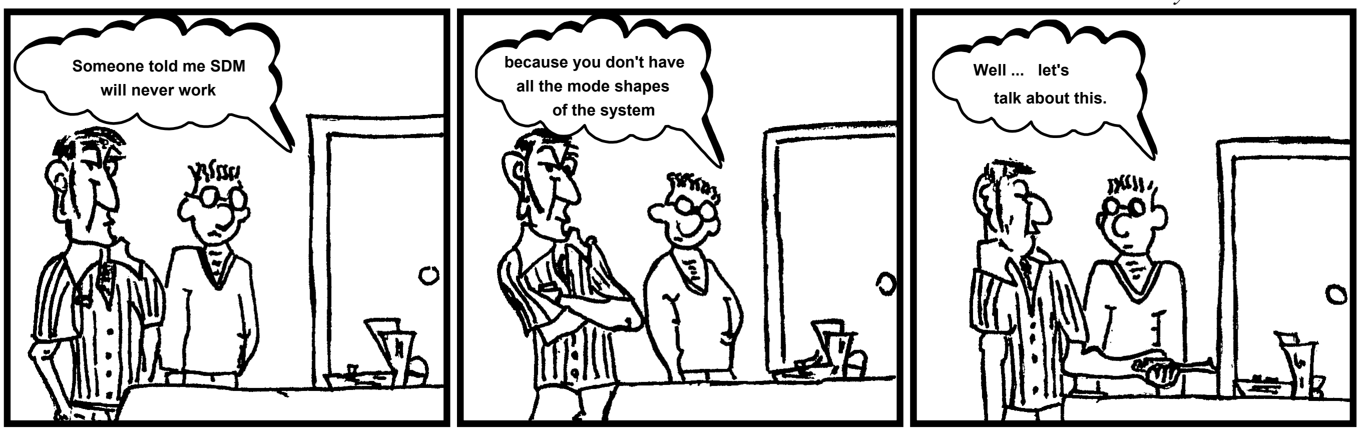

Illustration by Mike Avitabile

Someone told me SDM will never work
Because you don't have all the mode shapes of the system
Well, let's talk about this.

Structural Dynamic Modification (SDM) became a popular tool in the early 80's. Due to some misunderstandings of the technique, some erroneous results could be obtained. But given the right circumstances, SDM is a very powerful tool to help the design engineer make very good design decisions. First, let's briefly recall the technique and show how the technique can be sensitive to its biggest problem - modal truncation.

Basically, SDM is an analytical tool that uses modal data (either analytical or experimental) to estimate how the system dynamic characteristics will change when basic changes in the mass damping, stiffness of the system are investigated. Note that only modal data (frequency, damping and mode shapes) are used for the prediction - the original FEM or test data need not be modified to explore these changes. However, once a set of desired changes are obtained, then it is strongly recommended to re-run the modified FEM or re-test the modified test article.

The physical system equations can be developed and the eigensolution obtained. The modal representation can be obtained from either an analytical model or from test data. The modal representation of a physical system in modal space is given by

$$\begin{bmatrix} \diagdown & & \\ & \overline{M} & \\ & & \diagdown \end{bmatrix}\{\ddot{p}\}+\begin{bmatrix} \diagdown & & \\ & \overline{C} & \\ & & \diagdown \end{bmatrix}\{\dot{p}\}+\begin{bmatrix} \diagdown & & \\ & \overline{K} & \\ & & \diagdown \end{bmatrix}\{p\}=[U]^T\{F\}$$

Now changes to the physical system mass, ΔM, damping, ΔC, and stiffness, ΔK, can be represented in modal space (through the modal transformation equation) as

$$[\Delta\overline{M}]=[U]^T[\Delta M][U];[\Delta\overline{C}]=[U]^T[\Delta C][U];[\Delta\overline{K}]=[U]^T[\Delta K][U]$$

Assuming a proportionally damped system, an eigensolution can be obtained for the modified system. One important part of this solution is the computation of the final physical modes of the system from

$$[U_2]=[U_{12}][U_1]$$

which implies that the final modified modes of the modified system are made up from linear combinations of the unmodified modes of the original system. It is this important equation that we will use to show the effects of truncation of the predicted results.

Let's consider a simple example of a free free beam which we will use to make two simple structural changes - a simple support and a cantilever beam. We will modify the structure using two springs to ground and perform the SDM equations to obtain the modified frequencies and mode shapes. The original unmodified frequencies and resulting modified frequencies are shown in the Table 1 (Note: The frequencies identified in italics are an approximation of the constraint modes of the system and are beyond the scope of this discussion).

Notice that the simple support produces very accurate modified modes using only the first 5 modes of the original unmodified system whereas the cantilever beam does not. Why does the simple support do so well and the cantilever does not? The answer lies in how the mode shapes are formed from the original system modes.

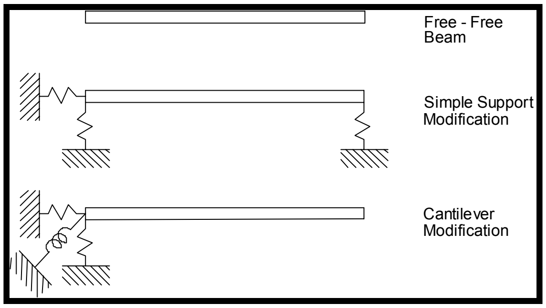

Fig 1 - Models Evaluated

		Simple Support		Cantilever	
#	Free	Ref.	SDM	Ref.	SDM
1	0.	71.9	72.0	21.6	24.8
2	0.	285.7	288.4	139.3	162.8
3	128.	636.5	646.0	396.1	476.0
4	367.	1114.9	*9108.3*	781.8	*1274.5*
5	738.	1706.3	*9593.6*	1292.0	*9437.8*

Table 1 - Frequencies of Different Systems

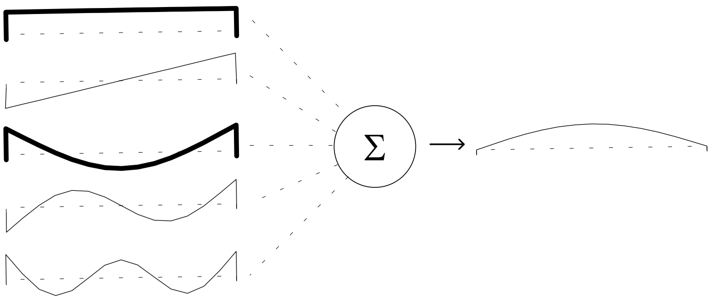

Fig 2 - First Simple Support Mode

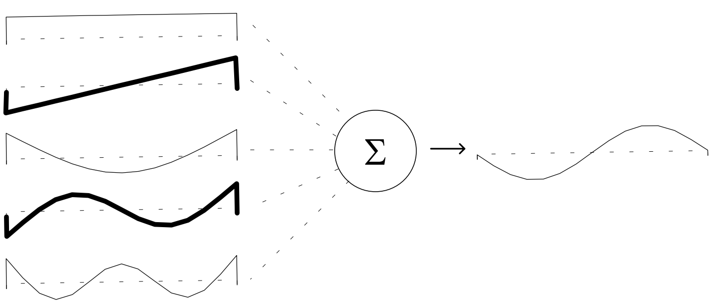

Fig 3 - First Simple Support Mode

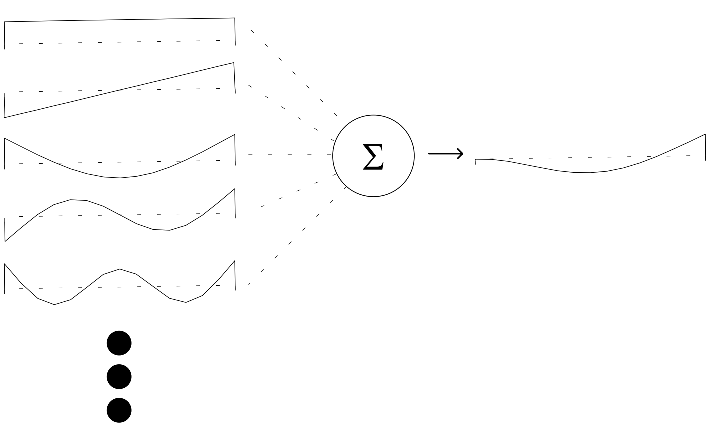

Fig 4 - Second Cantilever Support Mode

The simple support modified modes are easily made up from linear combinations of the unmodified modes of the original system. When we look at Figure 2, we notice that mode 1 and 3 are the most significant contributors to the first final modified mode for the simple support beam. And when we look at Figure 3, we notice that modes 2 and 4 are the most significant contributors to the second final modified mode for the simple support beam.

But when I consider the modes of the cantilever beam modification, there is a significant contribution from all 5 modes of the unmodified system. In fact, many more modes are needed to improve the accuracy of this cantilever predicted modes. (Note: Mode 2 is shown in Figure 4 for the cantilever)

It turns out that the simple support can be easily made from the available linear combinations of the 5 free-free modes of the original system whereas the cantilever can not! So that fact that all the modes are not available (modal truncation) is not always a problem. The real problem is that the final modified modes must be able to be formulated from the original unmodified modes.

Another important item to note is that the rigid body modes of the free-free beam are very important to the accurate prediction of the modified modes. If the rigid body modes are not available, then the predicted modes will be in error. This is an important consideration for the development of the experimental modal database since, often times, rigid body modes are not acquired as part of the test. However, it can be easily seen that the rigid body modes are very important to the success of the modification, even for the case of the simple support modification.

The bottom line is that in order to compute an accurate modified model using SDM, the final modes must always be made up of linear combinations of the unmodified modes. If this is possible, then good results can be obtained. If not, then errors will result due to modal truncation.

Without getting into all the detailed equations, some simple graphics were used to illustrate how SDM uses the unmodified modes of the system to obtain estimates of the modified modes of the system. I hope that this helps you to understand how SDM could be affected by modal truncation. If you have any other questions about modal analysis, just ask me.