MODAL SPACE - IN OUR OWN LITTLE WORLD

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How do you interpret the stability diagram? And how do data points affect the fit? There are some concepts here that are important to discuss.

The parameter estimation process is a very important part of the extraction of a model (poles and residues). Usually this is broken down into two parts – the extraction of the poles in the first step and then the estimation of the residues in the second step. The stability diagram is a tool that is used in the development of the extraction of the pole from the data. Let's discuss the estimation of poles and the use of the stability diagram. A few simple examples are included here to drive home the point of critical issues in the estimation process.

Let's assume that we have a set of data as shown in Figure 1. As a starting point, a third order fit will be assumed to describe the phenomena well. In general, the fit is reasonable as evidenced by the R^2 coefficient which is large. But when the variance tolerance is included (dotted lines), there is a fair amount of variation possible. One point is clearly seen as an outlier to the fit of the data. If this outlier point is removed from the data set as seen as in Figure 2, then the R^2 coefficient increases. So from the set of data shown here, it becomes very clear that the data quality is very important to the extraction of a valid set of parameters. It is of paramount importance to have good quality data for the estimation process.

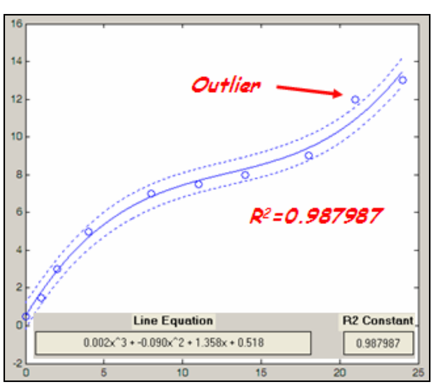

Figure 1 – Fit of Data with Obvious Outlier

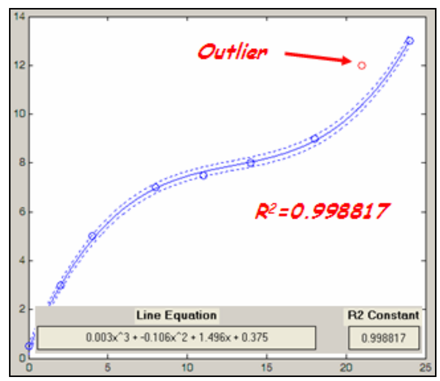

Figure 2 – Fit of Data with Outlier Removed

From this simple example, it is clear that good data is important. Now consider the data set shown in Figure 3. This is a very simple set of data that appears to have a very simple first order characteristic. Let's study the estimated parameters as the order of the model is increased.

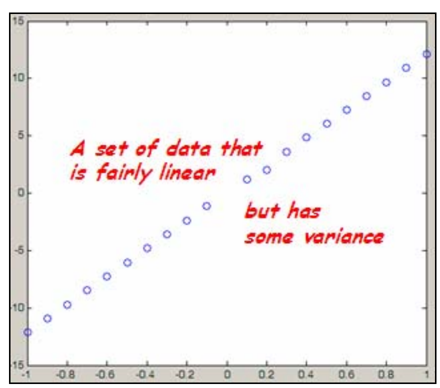

Figure 3 – Set of Fairly Linear Data

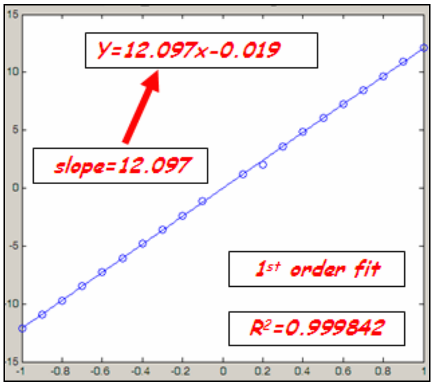

Figure 4a – First Order Fit of Data

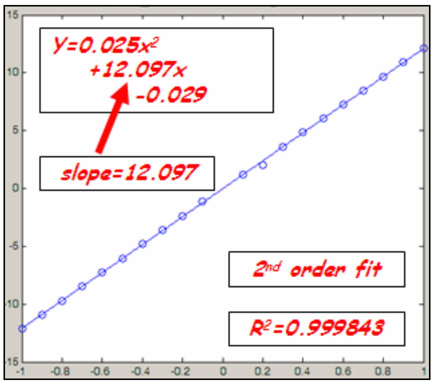

Figure 4b – Second Order Fit of Data

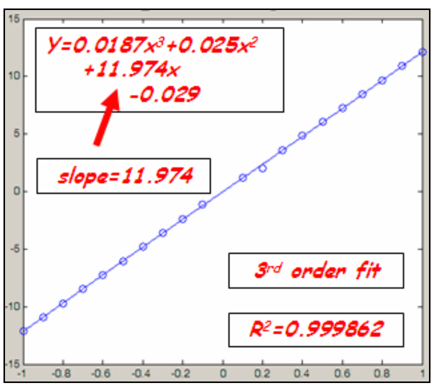

Figure 4c – Third Order Fit of Data

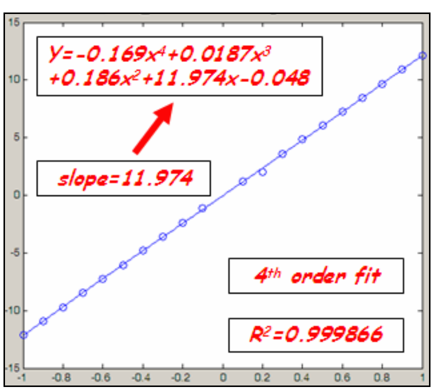

Figure 4d – Fourth Order Fit of Data

The plots in Figure 4 show the progression of the estimation of the slope as the order of the model is increased from first order to fourth order. In Figure 4a, the first order fit produces a slope of 12.097 with a very good R^2 value. Now as the order of the model is increased to second order, the slope is still 12.097 with a good R^2 value. So increasing the order of the model to second order has not produced a change in the estimation of the slope. Of course, the higher order terms are basically making adjustments to account for the variance on the measured data.

As the order of the model is increased to third order, the slope is 11.974 which is very close to the slope previously computed from the first order and second order models. In fact, the slope is only 1% different. So we could argue that the slope is basically the same and has not changed significantly from the previous estimates. And as the order model is further increased to a fourth order model, the slope is again estimated to be 11.974 which is unchanged.

So after this process is complete, the general consensus would be that the parameter of the slope of the data is approximately 12.0 and that very little change occurs as the order of the model is increased. Also note that it doesn't matter which order model I use because to within the tolerance of 1%, all orders produce essentially the same slope!

This simple example really provides an understanding of exactly what goes on behind the scenes in the development of the stability diagram. As the order of the model is increased, there will be estimates of poles. If the pole estimated only changes very slightly from one order model to the next, then the software will provide a flag (or indicator) to help understand if the pole has reached some "stable value" within some specified tolerance. (These tolerances might be set set to 1% on frequency and 5% on damping to identify pole stabilization.) There are usually some indicators that will be provided superimposed on a SUM, MMIF or CMIF plot. A typical stability plot is shown in Figure 5 for reference. The stability diagram helps to identify which poles are "consistent" or stable as the order of the model is increased.

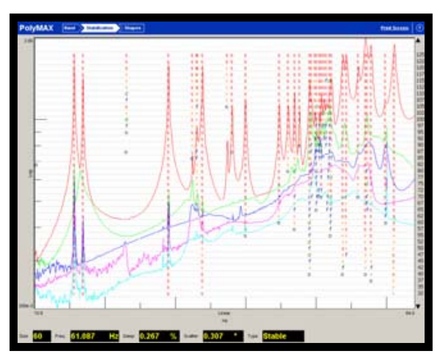

Figure 5 – Typical Stabilization Diagram.

If you have any more questions on modal analysis, just ask me.