# MODAL SPACE - IN OUR OWN LITTLE WORLD

*by Pete Avitabile*

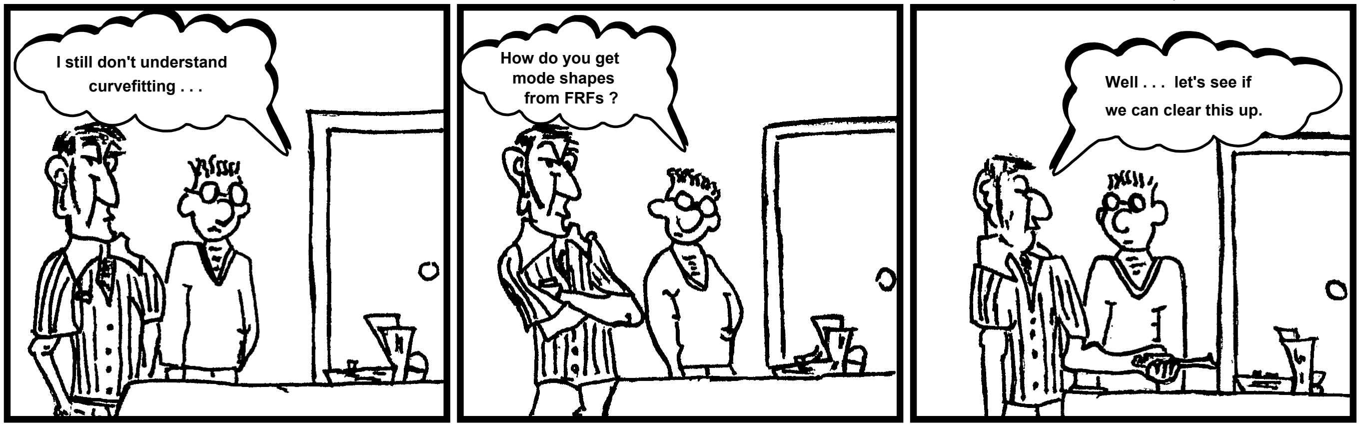



*Illustration by Mike Avitabile*

I still don't understand curvefitting ...
How do you get mode shapes from FRFs?
Well . . . let's see if we can clear this up.

Modal parameter estimation (commonly referred to as curvefitting) is probably, by far, the hardest part of experimental modal analysis for most people to understand. I know I can write out all the equations to explain this. But I will probably bore you to death. Not only do I have to write out all the equations relating to the modal parameter estimation process, I also have to show the equations relating the residue to the mode shape. And, of course, the concept of a residue is another abstract concept. (Oh, how I wished they had called it a mode shape rather than a residue since this only confuses everyone.)

Last time (Feb 1999), we talked about the curvefitting model and the basic equation we use for estimating parameters, of which one form is

$$\left[H(s)\right] = \text{lower residuals} + \sum_{k=i}^{j} \frac{\left[A_k\right]}{\left(s - s_k\right)} + \frac{\left[A_k^*\right]}{\left(s - s_k^*\right)} + \text{upper residuals}$$

Now those terms in the matrix, [A], are the residues which are obtained from the curvefitting process; we also get the poles, or frequency and damping, from the denominator of the equation. Now these residues can be shown to be related to the mode shapes. Without going through all the steps, the resulting relationship is shown below (with some terms expanded)

$$\left[A(s)\right]_k = q_k \left\{u_k\right\} \left\{u_k\right\}^T$$

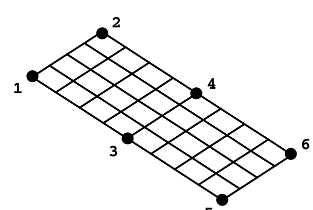
$$\begin{bmatrix} a_{11k} & a_{12k} & a_{13k} & \cdots \\ a_{21k} & a_{22k} & a_{23k} & \cdots \\ a_{31k} & a_{32k} & a_{33k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = q_k \begin{bmatrix} u_{1k}u_{1k} & u_{1k}u_{2k} & u_{1k}u_{3k} & \cdots \\ u_{2k}u_{1k} & u_{2k}u_{2k} & u_{2k}u_{3k} & \cdots \\ u_{3k}u_{1k} & u_{3k}u_{2k} & u_{3k}u_{3k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

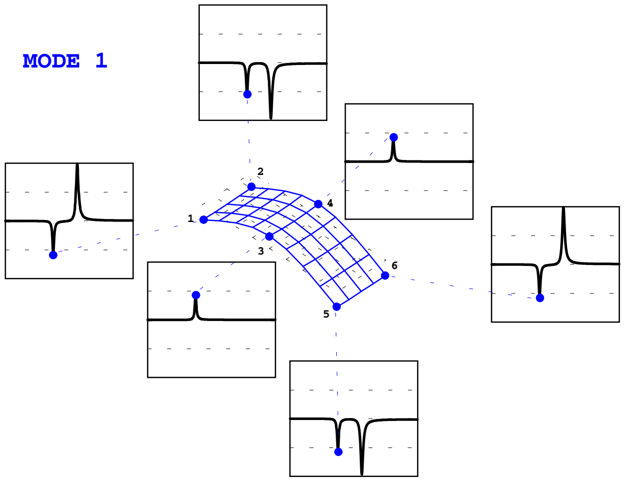
And if we were to look at each of the columns we would see the mode shape is contained in the column with some scalar multipliers; we would also see that due to reciprocity, the rows also contain the mode shapes. If we were to look at one column, such as the first column, then we would see

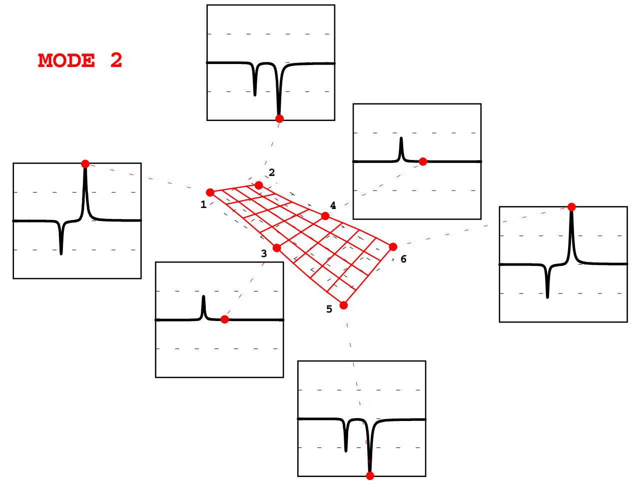
$$\begin{Bmatrix} a_{11k} \\ a_{21k} \\ a_{31k} \\ \vdots \end{Bmatrix} = q_k u_{1k} \begin{Bmatrix} u_{1k} \\ u_{2k} \\ u_{3k} \\ \vdots \end{Bmatrix}$$

The residues are, therefore, nothing more than the mode shape multiplied by a scalar which is the value of the mode shape at the reference location, u, and the scaling constant, q. (The q scale constant allows for mode shapes to be represented with different scale constants (unit modal mass, unit length, etc.)

Great, so here are some equations that you may or may not fully understand or appreciate. Maybe a better way to explain the concept is through some simple pictures. Let's go back to that simple plate that we discussed some time ago (Feb 1998) and explain very simply how we can get mode shapes from measurements (then maybe you'll appreciate what the math is doing for us).






Now let's take some measurements on the plate so that we get a total of 6 FRFs - at the 4 corners and at the 2 mid-points. We want to be able to determine what the first two mode shapes look like from these measurements. Now we could look at the log magnitude of the FRFs but this is not very useful since all the peaks would be positive in this plot.

A more informative plot is the imaginary part of the FRF. This shows both amplitude and, most importantly, the direction of the response. Without getting into all of the technical math, we know that the peak amplitude of the imaginary part of the FRF is directly related to the residue (and the residue is related to the mode shape). This approximate equation is shown below

$$h(j\omega)\Big|_{\omega\to\omega_n} = \frac{a_1}{(j\omega_n+\sigma-j\omega_d)} + \frac{a_1^*}{(j\omega_n+\sigma+j\omega_d)}$$

$$a_1 = \sigma\, h(j\omega)\Big|_{\omega\to\omega_n}$$

This very simplistic approach to determining mode shapes is commonly referred to as peak picking since we are picking the peak of the FRF. Now let's look at some of the peaks for each of the measurements at each of the points. (In all of the plots shown, the amplitude of the scale ranges from minus one to plus one and the dashed line is one-half. In addition, the frequency axis has been removed.) Now let's just concentrate on mode 1 first and then go on to mode 2.

Look at the FRF for mode 1 for point 1. Notice that this amplitude is 0.5 and it is negative. If we look at point 2, then we see that the amplitude is also 0.5 and it is also negative. This means that point 1 and 2 are moving with the same amplitude and in the same direction for mode 1. If we look at point 5 and 6, we see the same thing as point 1 and 2. So we can see that points 1, 2, 5, and 6 are all moving with the same amplitude in the same direction.

A important point to make here is that if I only measure these four points, then it would appear to me that the mode shape of the plate would be a rigid body mode (all four points moving together with equal amplitude). This is a common problem encountered when too few points are used to describe the mode shape of a system.

Now look at point 3 for mode 1. Notice that since amplitude is 0.5 but that it is positive. Then same can be said for point 4. So we see that point 3 and 4 have the same amplitude and move in the same direction together. But we also notice that points 3 and 4 are moving in the opposite direction from the rest of the points. Now, while we haven't measured more than 6 points, we start to see that the plate is deflecting into a pattern that is plate bending in characteristic. If we measured more points, then we would see a much better defined mode shape.

Now if we look at mode 2 we can step through and look at all the points and what we will see is that point 1 and 2 have the same amplitude but now they are moving in opposite directions. The same is true for points 5 and 6. But we notice that points 1 and 5 are also moving in opposite directions; the same is true for points 2 and 6. So we see that there is some type of twisting or torsional type deformation pattern for mode 2. If we look at points 3 and 4, we notice that these points have zero value. This is because points 3 and 4 are node points for the torsional mode of the plate. Again, adding more points better defines the shape.

So now we can see that the peaks of the imaginary part of the FRF are directly related to the mode shape of the plate for each of the modal peaks. Without going through all the math, the residues are terms that are extracted from the curvefitting process and these residues are directly related to the modes shapes of the plate. This was shown pictorially to keep things simple.

I hope that this helps to clear up the mystery as to how we get mode shapes from FRFs. Think about it and if you have any more questions about modal analysis, just ask me.