MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

Illustration by Mike Avitabile

I made a stiffness modification to a free-free system. The flexible modes shifted down! What's up?
Now this needs to be discussed.

Alright – that's a pretty bold statement that will turn almost anyone's head. I think we need to first describe what actually happened in this particular case. But rather than using the specific data originally presented, a simple beam can be used to describe what happened in this case.

The way this problem was described was that a free-free system was tested and then the system was constrained to fix or constrain the corners of the system. When the actual modification was performed to constrain the free-free system, the modes obtained were lower than the flexible modes of the unconstrained system.

Of course, the first thing that everyone stated was that if you increase the stiffness of any system, then the modes must shift upwards because

$$\omega_{initial}=\sqrt{\frac{k}{m}}$$

and if the stiffness is increased then

$$\omega_{modified}=\sqrt{\frac{k+k_{added}}{m}}$$

So it stands to reason that the frequencies must shift upwards – and the fact that the frequencies were lower just doesn't make sense.

So let's start with a simple beam that was analyzed and tested in a free-free condition. The first several free-free modes were 164 Hz, 452 Hz and 888 Hz. The unconstrained modes of the planar beam are shown in Figure 1 for reference.

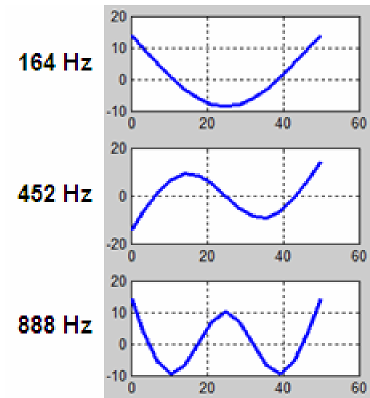

Figure 1 – Flexible Modes of Free-Free Beam

Then the simple beam was tested in a pinned (or constrained) condition. The first several free-free modes were 72 Hz, 288 Hz and 647 Hz. The constrained modes of the planar beam are shown in Figure 2 for reference. Clearly, the modes did not shift up as expected.

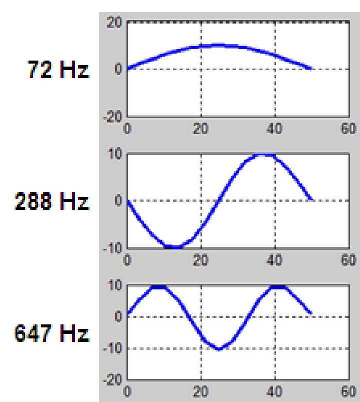

Figure 2 – Pinned Modes of Free-Free Beam

So exactly what happened here???

This is a very simple problem. But I have heard this many times over many years and inevitably the same problem exists.

Typically, people are concerned about the flexible modes of the system because those are the modes that generally cause all the noise and vibration problems that we encounter. But those are not the only modes that are needed to describe the entire system.

The basic problem here is that everyone forgot that an unconstrained system has flexible modes AND the rigid body modes. Now most times people don't measure the rigid body modes in test and they don't include them as part of the flexible modes measured during a modal test. And from an analytical standpoint, many times the eigensolution is performed but either a shifted eigenproblem is solved or only the flexible modes are obtained.

While the rigid body modes exist, many times people ignore them – mainly because they are not the source of the noise and vibration problems that are of concern. So Figure 3 shows the set of modes for the beam to more correctly include the rigid body modes as well as the flexible modes. So now once we realize that the first modes are actually at 0 Hz from the analytical model or a very low frequency from a test, then the intuition that adding stiffness shifts the modes upwards makes proper sense. Table 1 shows the frequencies of the free-free beam with rigid body modes along with the pinned modes

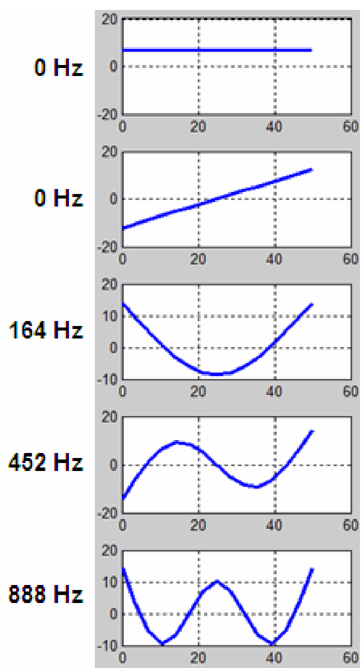

Figure 3 – Flexible Modes of Free-Free Beam with Rigid Body Modes

Table 1: Free-Free and Constrained Modes for Beam

Mode	Free	Constrained
1	0.	72.
2	0.	288.
3	164.	647.
4	452.	
5	888.	

So the basic problem is that the rigid body modes can't be ignored; they are a part of the total description for the beam. Notice now that all the frequencies in Table 1 do shift upwards as the stiffness is increased.

One way to easily prove this to yourself is to make a simple free-free beam model. The next model to develop is the beam with two very soft springs. Then make subsequent beam models where the spring stiffness is increased until ultimately the spring is so stiff that it is an approximation of a pinned end condition.

Along the way, it would be very beneficial to look at the mode shapes. When the springs supporting the beam are very soft, then the mode shape for the beam looks very much like a rigid body.

But as the stiffness of the support beam increases, the frequencies will increase and the mode shapes will start to migrate from rigid body modes to modes that have some rigid body mode component but also start to develop some more flexible attributes.

When the support spring stiffness gets larger and larger, the rigid body mode characteristic will diminish as the flexible characteristic becomes more pronounced. Ultimately, the rigid body characteristic will disappear and the flexible characteristic will completely dominate the mode shape.

This little exercise will then clearly show that the rigid body modes are critically needed to describe the modes of the system.

I hope this simple example clears up any misconceptions that you may have had. If you have any more questions on modal analysis, just ask me.