

Modulated submillimeter laser interferometer system for plasma density measurements

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A high resolution submillimeter interferometer system for measurement of electron densities in the 10^{14} - $\text{cm}^{-3} \leq n_e \leq 2 \times 10^{15}$ - cm^{-3} range has been developed for use in high density tokamaks. Phase modulation at ~ 1 MHz is accomplished by difference frequency mixing of two cavity tuned laser oscillators. The optically pumped CH_3OH lasers, which operate on the $118.8\text{-}\mu\text{m}$ line, feature a novel output coupling design that permits good mode quality and low beam divergence. The beat signals are detected using a newly developed Ge:Li photoconductor, and a direct measurement of the phase shift is obtained from the time lag between probe and reference signals. The sensitivity of the resulting phase measurement is independent of the instantaneous phase and unaffected by fluctuations in the amplitude or in the frequency of the modulation.

I. Introduction

The electron density of plasmas confined in tokamak devices has recently been increased to higher levels ($n_e \gtrsim 5 \times 10^{14} \text{ cm}^{-3}$), resulting in improved confinement properties.¹ Investigation of this promising new regime of operation requires the development of new diagnostic techniques. The plasma density is generally measured by detecting the phase shift of electromagnetic radiation that passes through the plasma; this phase shift is proportional to the integral of the electron density along the optical path. The frequency of the radiation must be considerably larger than the plasma frequency ω_{pe} if reflection and refraction effects are to be avoided, that is, $\omega > \omega_{pe} = (n_e e^2 / \epsilon_0 m)^{1/2}$, where e and m are the electronic charge and mass. While millimeter waves were appropriate for this purpose in the lower densities ($n_e \lesssim 10^{14} \text{ cm}^{-3}$) formerly achieved in tokamak devices, the higher frequencies of submillimeter wave radiation are now required. In this paper we describe a new dual laser modulated interferometer system, based on optically pumped submillimeter lasers, suitable for performing density measurements on the very high density ($n_e > 5 \times 10^{14}$ - cm^{-3}) plasmas characteristic of the Alcator¹ tokamak, and also on the large radius, moderate density ($n_e \sim 10^{14}$ - cm^{-3}) plasmas of such devices as the Toroidal Fusion Test Reactor (TFTR).²

Laser action at submillimeter wavelengths, from $40 \mu\text{m}$ to $> 1\text{mm}$, has been demonstrated in a number of molecular gases when these are pumped by CO_2 laser radiation; over 500 lines in this range have been observed.³ These optically pumped lasers offer important

advantages over gas discharge types; they provide exceptional frequency stability⁴ and immunity from stray magnetic fields, both of which are important for measurements on large scale plasma experiments. We have chosen the $118.8\text{-}\mu\text{m}$ line of methyl alcohol to operate on the Alcator device, which achieves plasma densities in the 10^{14} - $\text{cm}^{-3} < n_e < 2 \times 10^{15}$ - cm^{-3} range; at this wavelength, we can expect phase shifts of $0.70 \leq \phi/2\pi \leq 15$ for a plasma radius of 10 cm and an approximately parabolic density profile.

II. Phase Modulated Interferometry

A simple interferometer, such as the common Michelson and Mach-Zender configurations, is limited in resolution by amplitude fluctuations. Furthermore, since the output $I \propto (1 - \cos\phi)$, the resolution $dI/d\phi \propto \sin\phi$ is a function of the value of the fringe shift, and for $\phi = m\pi$ the resolution is zero. The situation may be improved considerably by introducing a time dependent phase modulation into one arm of the interferometer,^{5,6} such that the output is of the form $I \propto \cos(\Delta\omega t + \phi)$, where $\Delta\omega$ is the modulation frequency, which can be compared to a reference signal proportional to $\cos(\Delta\omega t)$. The phase shift is then obtained directly from the time lag between the two signals, since $\phi = \Delta\omega\Delta t$, where Δt is the time lag between the two. The technique is insensitive to amplitude variation and gives a resolution that is independent of phase.

The modulation scheme used in the present system is shown in Fig. 1. Two cw optically pumped CH_3OH lasers are located on a single granite base and pumped by a single CO_2 laser. The cavities are tuned such that the two oscillate at frequencies differing by Δf of the order of 1 MHz. A reference signal x at the difference frequency is then generated by mixing a portion of the

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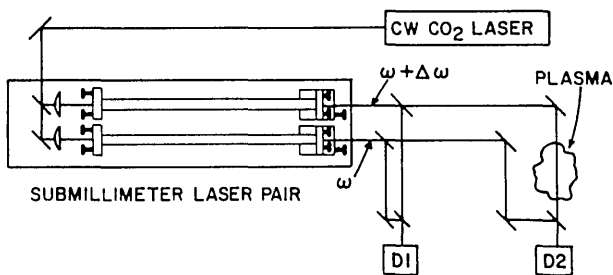


Fig. 1. Dual beam modulated interferometer system. Two optically pumped lasers are operated at frequencies differing by $\Delta\omega$. Mixing the two outputs in detector $D1$ provides a modulated signal $x \sim \cos(\Delta\omega t)$, while detector $D2$ sees a similar signal $y \sim \cos(\Delta\omega t + \phi)$ with an additional phase shift due to the plasma. The magnitude of this phase shift is then obtained directly from comparison of the two signals.

two beams in a detector $D1$; similarly a phase shifted signal y is obtained in the second detector $D2$, where one of the beams has passed through the plasma before entering the detector. The phase lag between the two signals, x and y , is then obtained electronically.

The time resolution of the system is in principle given by the period of the beat signal or by the bandwidth of any filters used in the detection system. Here the value of 1 MHz was chosen to be fast enough to follow the evolution of the density during fast instabilities and also to resolve density fluctuations due to MHD phenomena, while remaining low enough to permit convenient analysis and to be small compared to the approximate 10-MHz gain bandwidth of the laser oscillator.

III. CH₃OH Laser System

The CH₃OH lasers are of the waveguide type,⁷ consisting of a quartz guide with i.d. of 12 mm and cavity length of ~ 1 m. The two lasers are pumped with ~ 6 W each at the 9.69- μm $P(36)$ line by a commercial CO₂ laser, the pump radiation entering the guide through a 1-mm hole in the rear cavity mirror. The output coupler assembly is located on a translation stage driven by a differential screw, permitting accurate tuning of the cavity length and hence of the laser frequency.

The output coupler of an optically pumped laser must serve the dual role of acting as a partially transmitting reflector for the submillimeter laser radiation and as a good mirror to trap the ir pump radiation inside the guide so that it may be efficiently absorbed. A hole coupled mirror has commonly been employed for this purpose, but has the drawback of producing a highly divergent and nonuniform output mode. Since mode quality is of great importance for low divergence beam properties and for efficient mixing, we have rejected hole-coupling and developed instead a new device, the capacitive coupler.⁸

The properties of metal mesh and its complementary structure, the capacitive grid, as optical elements in the far ir have been extensively studied by Ulrich.⁹ It is found that for a grid that satisfies the condition $g/\lambda < 1$, where g is the grid constant and λ the wavelength of the radiation of interest, only the zero order reflected and transmitted waves propagate, and diffraction does

not occur. A grid may be completely described, neglecting absorption losses, by a complex reflection amplitude coefficient Γ ; furthermore, as a consequence of Babinet's principle, the transmission and reflection properties of metal mesh (inductive grid) and its complementary "capacitive structure" are complementary:

$$R_c = |\Gamma_c|^2 = 1 - |\Gamma_i|^2 = T_i.$$

This leads to the nonintuitive result that, for a capacitive grid, as $g/\lambda \rightarrow 0$ the transmissivity $T \rightarrow 1$.

In particular, for a capacitive grid with $g/\lambda \lesssim 0.7$ it is found that the reflectivity is given by⁹

$$R = 1 - T = |\Gamma|^2 = 1/(1 + Z_0^2 \Omega^2), \quad (1)$$

where $\Omega = g/\lambda - \lambda/g$,

$$Z_0 = [2 \ln \csc(\pi a/g)]^{-1},$$

and $2a$ is the width of the strips in the grid (or the wire in the inductive mesh). In the above, absorption loss is neglected, and a grid thickness $t \ll a$ is assumed.

For an output coupler we require a device with a reflectivity of the order of 90% that corresponds to a grid constant of the order of $\lambda/2$ to $3\lambda/4$. We note that for the ir pump radiation $\lambda_p \approx 10 \mu\text{m}$ the structure is in the diffraction region $g/\lambda \gg 1$, and the reflectivity can no longer be described by the simple expression Eq. (1). Experimentally it is found that 65% of the incident pump radiation is specularly reflected. Thus, the capacitive grid structure can act both as an output coupler for the submillimeter radiation and a reasonable reflector for the ir pump.

In practice, the capacitive coupler is made by depositing a 2500–3000- \AA thick aluminum coating through an appropriate mesh onto a z -cut crystal quartz substrate; the resulting pattern closely corresponds to the complement of the mask. The transmission properties, however, are somewhat modified by the presence of the substrate due to two effects. First, the presence of the dielectric modifies the transmissivity of the grid structure; second, the interaction of the wave with the second surface of the substrate must be treated as in the case of an asymmetric Fabry-Perot etalon, unless the substrate is intentionally wedged to prevent this effect. The transmissivity can be calculated in terms of equivalent circuits using standard transmission line analysis techniques. In this case we find⁸ that the transmission is given by

$$T = \frac{T_1 T_2}{[1 - (R_1 R_2)^{1/2}] + 4(R_1 R_2)^{1/2} \sin^2(\phi + \delta)}, \quad (2)$$

where T_1 and R_1 are the transmissivity of the grid including the effect of the substrate¹⁰; T_2 and R_2 are the transmissivity and reflectivity of the second surface; ϕ is the usual phase ($2\pi n l/\lambda$), where l is the thickness of the substrate and δ is an additional phase shift due to the reflection properties of the grid. Then

$$T_1 = \frac{nT}{R + [(1+n)/2]^2 T}, \quad R_1 = 1 - T_1,$$

$$T_2 = \frac{4n}{(1+n)^2}, \quad R_2 = \left(\frac{1-n}{1+n}\right)^2,$$

$$\delta = \frac{1}{2} \cos^{-1} \left(\frac{1 + R_1 - nT_1}{2(R_1)^{1/2}} \right),$$

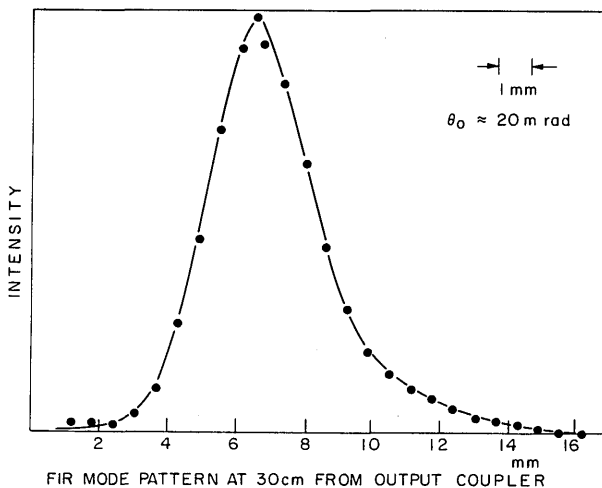


Fig. 2. Laser mode shape. The mode is linearly polarized, and the shape is consistent with the EH_{11} dielectric wave guide mode. This mode is well suited for efficient mixing.

where R and T are the values for the “free-standing” grid obtained from Eq. (1). Absorption loss has again been neglected in the above calculation.

Optimum performance was obtained with couplers for which $g = 76 \mu\text{m}$, $a = 10.4 \mu\text{m}$, and measured reflectivity of $\sim 85\%$. Single laser outputs of 5.5 mW with 12 W of pump power are obtained in a single transverse mode with divergence of the order of 20 mrad. The mode, shown in Fig. 2, is found to be linearly polarized, and the shape is consistent with the EH_{11} dielectric waveguide mode, which is expected to be the low loss mode for such a guide. Threshold pump power is ~ 2 W, and with the 12-W pump beam split equally between the two submillimeter lasers outputs of ~ 1.5 mW/laser are obtained. For this configuration the optimum operating pressure is 200 mTorr, and the gain bandwidth over which lasing is obtained is ~ 8 MHz.

IV. Detectors

Mixing of the two lasers is accomplished by combining the beams on a mesh beam splitter and observing the beat signal on a detector, which must have a sufficiently fast response to follow the beat frequency, and as low an NEP as possible, since the wideband noise from the detector will impose the ultimate limitation on the resolution of the interferometer. The detector used in the majority of the work presented in this paper was a He cooled, Li-doped germanium photoconductor,¹¹ which has a strong response in the region from 75 cm^{-1} to 125 cm^{-1} and is sensitive, with proper biasing, up to a few MHz.

The detector element used in the present work has a doping concentration of $5 \times 10^{11} \text{ cm}^{-3}$ and a dc resistance at 4.2 K of $1.6 \times 10^5 \Omega$. The element is in the shape of a rectangular polyhedron with a 10-mm \times 15-mm rectangular surface and a thickness of 5 mm. The detector is biased in the constant current configuration with optimum bias for high frequency response of approximately 40- μA dc. The radiation is directed onto the detector, which is immersed in liquid He, by a 1-cm diam metal light guide 1 m long. The respon-

sivity of the system is approximately 10^3 V/W . The SNR at 1 MHz is approximately 1 order of magnitude higher than that observed using an extrinsic GaAs photoconductor¹² in the same configuration, and preliminary measurements indicate the NEP at low frequency to be less than $10^{-10} \text{ W/Hz}^{1/2}$. Work on optimization of a doping concentration and contact techniques is currently underway.

V. Mixing Characteristics—Experimental

The mixing signal is displayed on the spectrum analyzer trace shown in Fig. 3. This modulation frequency may be adjusted by manually tuning the cavity length of either or both lasers over a range of several MHz. The tuning equation is

$$\frac{\Delta\nu}{\Delta l} = \frac{\nu}{l} = \frac{2.5 \times 10^{12}}{1\text{m}} = 2.5 \text{ MHz}/\mu\text{m}.$$

Since the resolution of the differential screw used to tune the cavity is $0.025 \mu\text{m}$, the difference frequency may be accurately adjusted without difficulty. The short term stability of the beat frequency is currently limited to ~ 25 kHz by mechanical vibrations that are communicated to the system through the supports. We note that the amplitude of vibrations required to cause such instability is of the order of 100 \AA . This degree of stability was obtained by supporting the granite base on two foam rubber strips [uncompressed thickness of 7.6 cm (3 in.)] and represents an improvement by a factor of 6 over the situation in which the base rested directly on the laboratory table, with no isolation from the table or the floor. We feel, based on experience with other systems, that it should be possible to improve the short term stability by as much as an order of magnitude by employing air-suspension type isolation mounts to eliminate these vibrations; however, as will be shown below, such small variations in the modulation frequency need not contribute any significant uncertainty

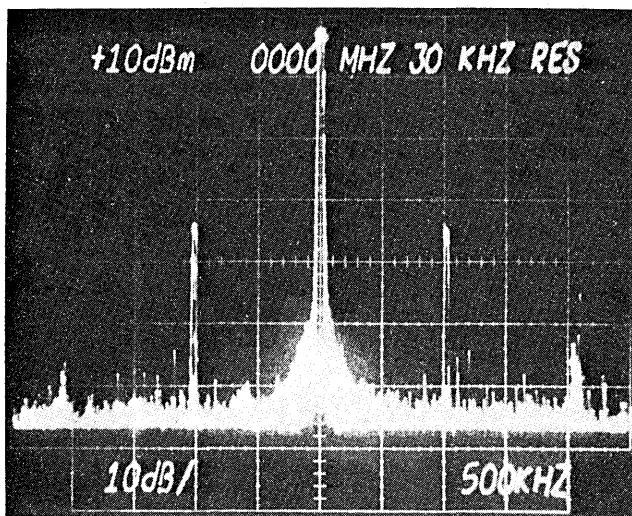


Fig. 3. The 1.0-MHz modulation signal. The spectrum analyzer trace shows a single beat signal at 1 MHz with a width of less than 30 kHz. The small peaks at the second harmonic represent a slight nonlinearity in the detector response. Single transverse mode operation is verified by the presence of only one beat signal.

to the phase measurements, and the present system already exhibits sufficient stability. The long term drift of the modulation frequency is limited by thermal expansion of the granite base and is observed to be of the order of 100–200 kHz/h. Since readjustment of the frequency is easily accomplished this slow drift does not present any difficulties.

VI. Phase Shift Measurement—Techniques

Two versions of the phase measurement electronics are being investigated. In each case the two modulation signals are first filtered, amplified, and clipped so that the phase detection circuit is presented with two square waves in which only the zero-crossing information is preserved. Thus, only noise at the modulation frequency within the bandwidth of the filters will contribute error in the phase measurement, since only this noise introduces uncertainty in the time of zero crossing. This noise is principally the low level white noise contributed by the amplifier and detector system. Slowly varying amplitude fluctuations due to the laser source, the changing transmission of the plasma, etc. do not affect the phase measurement, and, therefore, sensitivities of the order of 10^{-2} fringe are achievable.

The simpler of the phase measurement circuits consists essentially of a single flip-flop. The zero-crossing transition of the reference detector x sets the output high, while the next transition of the signal detector y resets the state to zero. The output of the flip-flop is therefore a pulse train with the same period τ as the modulation frequency and pulsewidth equal to the time delay Δt between zero crossings of the two signals. This pulse train is then integrated on a low pass filter; the resulting signal is then proportional to the ratio $\Delta t/\tau = \phi/2\pi$, which is the fractional phase shift due to the plasma. With this circuit the output voltage rises to its maximum value as ϕ increases to 2π and then resets to zero as ϕ passes through each fringe. It should be noted that the output is directly proportional to the phase, independent of the frequency of the modulation; thus, even the 2.5% uncertainty in the difference frequency noted above would not introduce any significant uncertainty in the phase measurement. This fact is due to the averaging technique and would not be the case if, for instance, the time lag of the two signals was measured with a fixed frequency clock.

A somewhat more complex phase comparator, which provides an output directly proportional to the total, as opposed to the fractional, phase shift, is that designed by Meddens and Taylor.¹³ In this circuit the clipped square waves are presented to two counters whose outputs are subtracted in a digital adder. The difference between the two counts, which is simply the total phase shift, is put through a digital-to-analog converter, and the analog output is, as in the previous case, time averaged. The signal is again independent of the value of the modulation frequency and in this case is directly proportional to the plasma density.

VII. Conclusion and Comments

We have constructed a modulated interferometer system for plasma density measurements employing a

pair of optically pumped lasers oscillating at slightly different frequencies. The large number of wavelengths available from such lasers provides great versatility, and systems of this type may prove applicable to a wide range of plasma experiments. The modulation scheme based on difference frequency mixing offers excellent time resolution as well as sensitive phase detection.

The system we have described has been designed as a single path interferometer to measure the integrated column density in the present Alcator device. In machines with larger size and greater diagnostic access it will be desirable to employ a multipath system capable of determining, with Abel inversion techniques, the evolution of the density profile in time. For such measurements higher powers than those employed here will be required, and construction of a system that should provide at least an order of magnitude increase¹⁴ in power levels is now in progress.

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