Windows and Leakage Brief Overview

When converting a signal from the time domain to the frequency domain, the Fast Fourier Transform (FFT) is used.

The Fourier Transform is defined by the Equation:

$$X(f) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

which requires a signal sample from –∞ to ∞. The Fast Fourier Transform (a version of the Discrete Fourier Transform) however only requires a finite number of samples (which must be a value of 2^n where n is an integer. i.e. 2, 4, 8, 16 … 512, 1024). The FFT is defined as:

$$X_k = \sum_{i=0}^{n-1} x_i e^{-j2\pi ik/n} \qquad \text{for } k = 0,1,2,\ldots, n-1$$

The Fast Fourier Transform is commonly used because it requires much less processing power than the Fourier Transform. Like all shortcuts, there are some compromises involved in the FFT. The signal must be periodic in the sample window or leakage will occur. The signal must start and end at the same point in its cycle. Leakage is the smearing of energy from the true frequency of the signal into adjacent frequencies. Leakage also causes the amplitude representation of the signal to be less than the true amplitude of the signal. An example of a non-periodic signal can be seen in Figure 1.

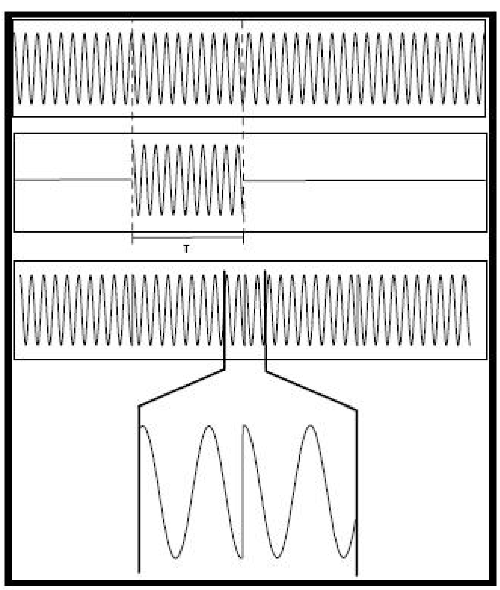

Figure 1: Example of Non-Periodic Signal.

The FFT of a Periodic and non-periodic signal can be seen in Figures 2 and 3.

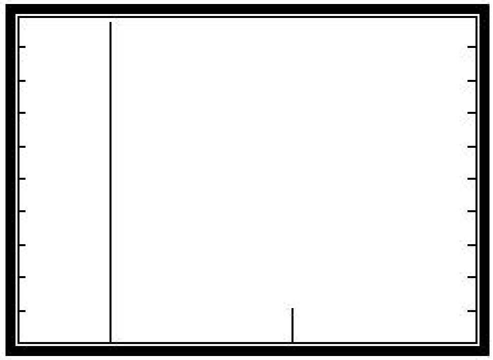

Figure 2: FFT of periodic Signal

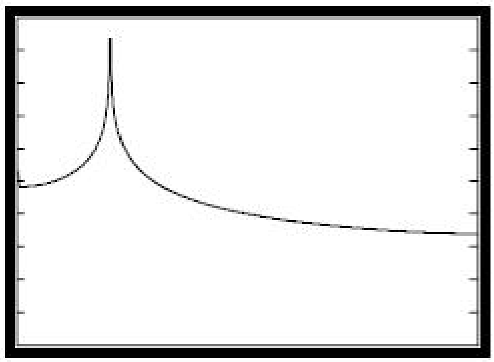

Figure 3: FFT of Non-Periodic Signal.

By comparing Figures 2 and 3 it can be seen that the frequency content of the signal is smeared into adjacent frequencies when the signal is not periodic. In addition to smearing, the amplitude representation of the signal is less than the true value.

To help reduce this smearing and preserve the amplitude of a signal, windows are used. Windows work by weighting the start and end of a sample to zero while at the same time increasing the amplitude of the signal at the center as to maintain the average amplitude of the signal. The affect of a Hanning window on a non-periodic signal in the Frequency Domain can be seen in Figure 4.

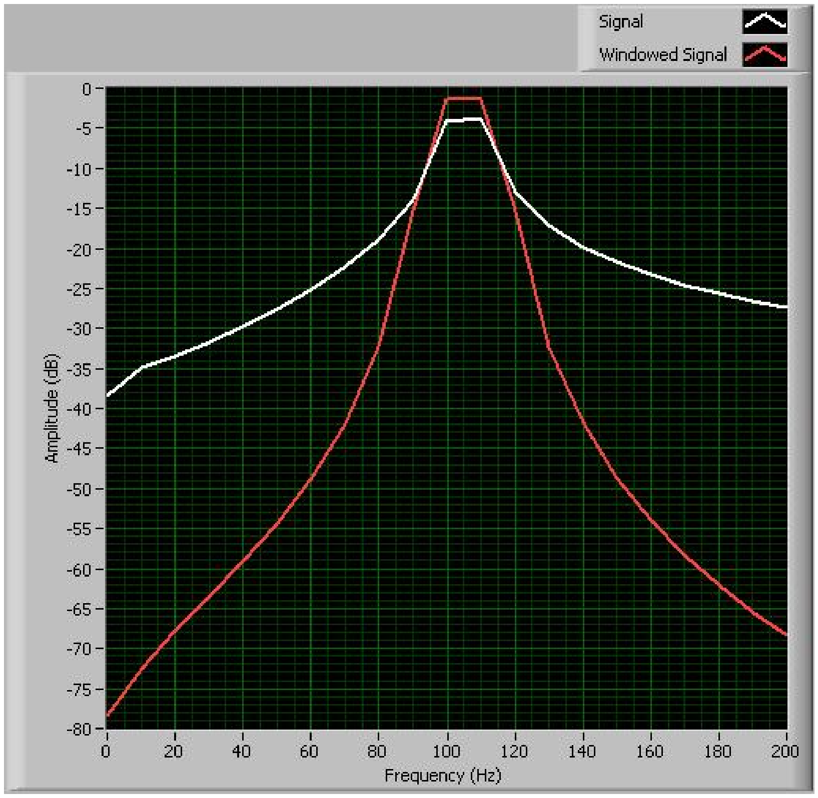

Figure 4: Comparison of Windowed and Un-windowed Signal in the Frequency Domain.

Figure 4 shows that the window reduces smearing and better preserves the amplitude of the signal. The effect of the same Hanning Window on the time domain signal can be seen in Figure 5.

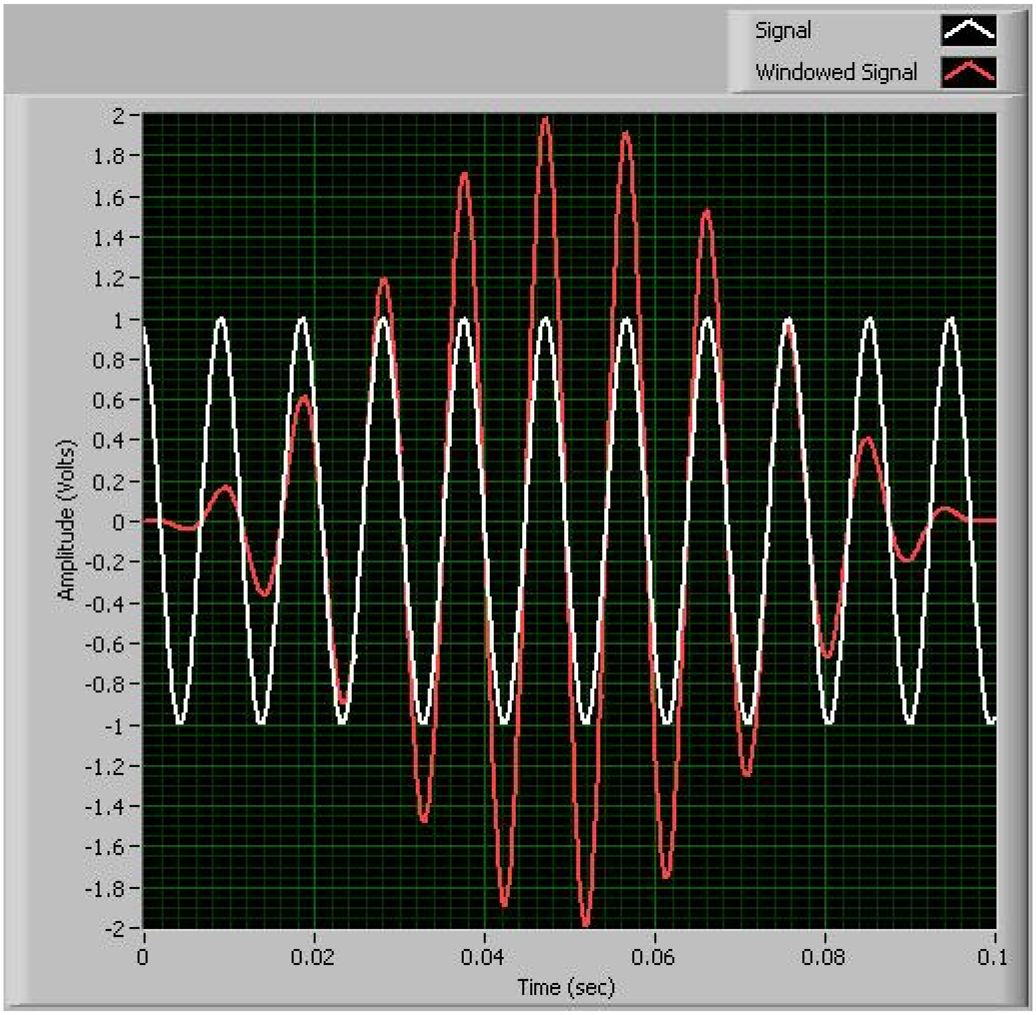

Figure 5: Effect of Hanning Window on Time Domain Signal

Figure 5 shows how the Hanning window weights the beginning and end of the sample to zero so that it is more periodic during the FFT process.

There are several very commonly used windows in signal processing. The first type of window is called the "rectangular" window; it does not weight the signal in any way and is equivalent to saying that no window was used. This is used whenever frequency resolution is of high importance. This window can have up to 36% amplitude error if the signal is not periodic in the sample interval. It is good for signals that inherently satisfy the periodicity requirement of the FFT process.

The Flat Top window is used whenever signal amplitude is of very high importance. The flat top window preserves the amplitude of a signal very well; however it has poor frequency resolution so that the exact frequency content may be hard to determine, this is particularly an issue if several different frequency signals exist in close proximity to each other. The flat top window will have at most 0.1% amplitude error.

The Hanning window is a compromise between the Flat Top and Rectangular windows. It helps to maintain the amplitude of a signal while at the same time maintaining frequency resolution. This window can have up to a 16% amplitude error if the signal is not periodic.

The last common type of windows is used in impact testing. It is called the Force-Exponential window. The Force-Exponential window is actually two separate windows. In impact testing a force transducer mounted on the impact hammer to measure input force. A force window reduces noise in the force signal so that the FFT of the input spectrum will be more accurate. An example of the force window can be seen in Figure 6.

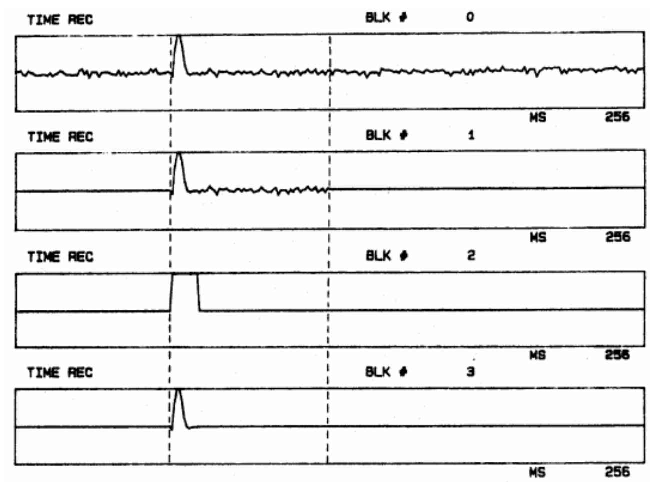

Figure 6: Force window.

The exponential window is used to make a measurement from a vibrating structure more accurate. It is used when the "ringing" of a structure does not attenuate adequately during the sample interval. An example of the Exponential window can be seen in Figure 7.

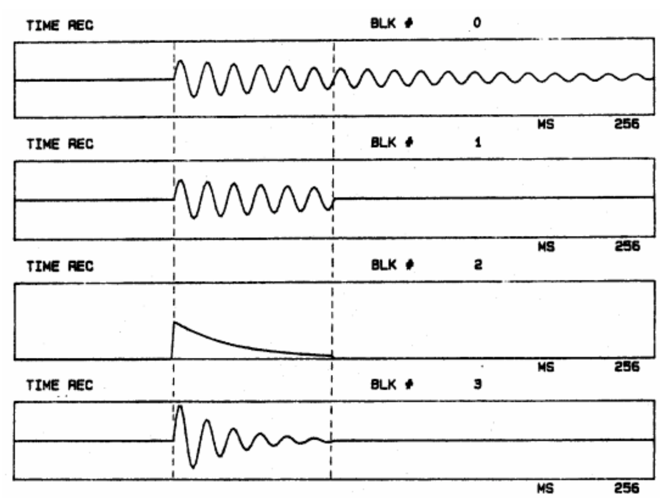

Figure 7: Exponential Window.