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Dr. Regina M. Panasuk

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Volume XVIII
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Editor
Dr. Regina M. Panasuk

Assistant Editor
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Graduate School of Education—University of Massachusetts Lowell

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The Annual Symposium Journal is published once a year in April.

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2015 Annual Symposium Journal Call for Papers

The Graduate School of Education, University of Massachusetts Lowell

GUIDELINES FOR SUBMISSION

The papers submitted for the Journal must discuss psychological and pedagogical issues and trends related to educational research and practice. Please use the following guidelines:

WHEN SUBMITTING A PAPER, PLEASE USE THE FOLLOWING GUIDELINES:

1 Submit an electronic version of the paper, an abstract, approximately 150 words, and a biographical sketch, about 30 words. All pictures and diagrams must be submitted as a separate document.

2. Use double spacing with one-inch margins.
3. For references, tables, and figures follow the style described in the Publication Manual of the American Psychological Association (APA), Sixth Edition.
5. Paper must be submitted by December 1.
6. Authors will be notified about the status of their papers by January 15.
7. The Symposium is scheduled in April.

A RESEARCH PAPER MUST INCLUDE

a) a rationale and an identification of the research question(s)

b) a conceptual framework or brief statement of relationship to the literature

c) an identification of research methodology

d) a summary of the analytical technique(s)

e) a summary of preliminary findings

The length of the paper length might be up to 30-40 pages, including pictures, tables, figures, and list of references.

A position paper for the **Educational Resources** section can be up to 20 pages. It must present new ideas and developments of major importance to practitioners working in the fields of mathematics and science education, language art and literacy education, and leadership and schooling. It must reflect a variety of research concerns within the fields and deal with didactical, methodological, and pedagogical issues.

An **abstract** for a **poster presentation** can be about 150-250 words; must outline the major ideas of the research study (proposed or completed), or a teacher education program.

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Academic Task Expectations of Students with Autism

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ABSTRACT

Autism spectrum disorders (ASD) is the fastest growing developmental disability in the United States. With this increase in identification comes an increase in the number of services public schools are required to provide for students with autism. This study investigated the types of tasks and duration of tasks that four grade five students with ASD were asked to complete in their substantially separate and inclusion classrooms. A total of 36 observations took place in the sub-separate classrooms, and 41 observations in the inclusion classroom. The majority of classroom time was spent on worksheets, whole group lectures, waiting for classroom routines, and copying notes off the board. These types of tasks did not require interactions to take place.

The number of children identified with autism is increasing (Lynn & Collet-Klingenberg, 2010). According to the Center for Disease Control and Prevention (2012) one out of every 88 individuals is diagnosed on the autism spectrum. This makes autism spectrum disorders (ASD) the fastest growing developmental disability in the United States (Data Accountability Center, 2008).

The Massachusetts Autism Commission Report (2013) suggests that there are approximately 16,000 school-aged individuals with autism in the state. Over one million school age children, ages six to 17, in the United States have autism (Center for Disease Control and Prevention, 2012).

With this increase in identification comes an increase in the number of services public schools are required to provide for students with autism (Stephens, 2005). The increased prevalence in ASD and its presence in public schools is of concern to general education teachers who are trying to provide appropriate inclusion practices but feel they do not have the necessary training and support to teach students with autism effectively (Massachusetts Autism Commission, 2013; Robertson, Chamberlain, & Kasari, 2003; Scott, 2013). Research on how to best educate students with autism in the public school inclusion setting is relatively new and there are still many questions regarding which practices are most effective for this population. This lack of clarity presents challenges to educators who are struggling to provide an effective education for students with ASD (Odom, Collet-Klingenberg, Rogers, & Hatton, 2010).

While there is much to be learned about effective inclusion education, current research suggest that supportive peer relationships are the biggest factor in fostering a successful inclusion program (Overton & Rausch, 2002; Strain & Hoyson, 2000). When typical peers interact with students with ASD they can reinforce social behaviors, facilitate learning, and provide them with a sense that they belong (Overton & Rausch, 2002). Since ASD is most notably identified by pervasive and sustained impairment in socialization and communication one would assume an inclusion classroom would provide opportunities to help develop these areas of weakness (American Psychiatric Association, 2013). A classroom that embeds social instruction into academic tasks has the power to increase social success and independence in students with ASD (Roger, 2000). Indeed, learning is a social phenomena (Vygotsky, 1978) and as such, classrooms are social contexts in which students can be active members in how they acquire and construct knowledge. They learn skills when they interact with other people. The contexts and tasks students are exposed to, therefore, bound what students are able to learn (Bodrova and Leong, 1996). Students learn best when context and task are meaningful, relevant to an individual's life and require both collaboration and cooperative work (Vygotsky, 1978).

Because there is limited research exploring the efficacy of inclusion practices and the social and academic experiences of students with ASD, this study investigated the types of tasks that four grade five students with ASD were asked to complete in their substantially separate and inclusion classrooms. The amount of time spent on social activity was examined to see if social programming was being embedded into academic tasks. The specific research questions are:

1. What academic tasks are asked of the students with ASD?
2. How long do students with ASD spend in each task?
3. Do these tasks require the student with ASD to converse with their peers?

METHODOLOGY

Site

The school chosen for this study was a kindergarten through fifth grade elementary school in an urban district in Massachusetts. This specific school was chosen because it was a convenient sample and had students with ASD involved in the inclusion classroom for some portion of their day.

Table 1
Students' scores on the WISC-IV

WISC-IV	Verbal Comprehension	Perceptual Reasoning	Working Memory	Processing Speed	FSIQ
Victoria	95 (Avg.)	112 (H. Avg.)	102 (Avg.)	85 (L. Avg.)	99 (Avg.)
Craig	87 (L. Avg.)	94 (Low)	74 (Low)	80 (L. Avg.)	N/A
Ben	75 (Borderline)	92 (Avg.)	64 (Ext. Limited)	97 (Avg.)	N/A
David	93 (Avg.)	96 (Avg.)	88 (Low Avg.)	68 (Low)	84 (Low Avg.)

Participants

Four grade five students with ASD were included in this study. Inclusion criteria for this study required the student with autism to be in the general education setting for at least 45 minutes of academic instruction during the school day, according to the individualized educational program (IEP) service delivery page. All of the students needed and had current IEPs. Table 1 is a summary of each student's performance on IQ obtained from their cumulative folder.

Data Analysis

Classroom observations of four individual students with autism were conducted in both the substantially separate and inclusion classrooms. A total of 36 observations took place in the sub-separate classrooms, and 41 observations in the inclusion classroom. Each observation was 30 minutes in length. For each observation the task students were asked to do was recorded. The durations of the tasks were also recorded.

Reliability

Inter-rater agreement was collected on ten percent of the observation sessions. Inter-rater reliability was calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100. Inter-rater agreement on task identification was 87.9%.

RESULTS

The inclusion classroom

Students were asked to execute the following tasks: listening to lectures, copying notes, completing worksheets, engaging in one-on-one question and answer discussions with the teacher and/or paraprofessional, watching videos, reading, participating in small group work, conducting an individual teacher-prescribed experiment, and coloring. The minute breakdown of each of these activities can be seen in Figure 1.

Students with ASD had the opportunity to socialize with their peers in cooperative groups for only 4.4% of the observed time. During this time the students with ASD were on-task, looking at their peers when they spoke and making remarks to contribute to the discussion for 23 of the 54 minutes (42.5% of the time). This meant that instruction that involved social learning was occurring for students with ASD 1.8% of all the observed times.

The substantially separate classroom.

The tasks in the substantially separate classroom that students with ASD were asked to engage in mirrored those seen in the inclusion classroom. The duration of time spent in each task can be seen in Figure 2.

Students were engaged in social instruction or conversation with peers in this classroom for 1.2% of the time. Peer small group work occurred for .8% of the time. In the

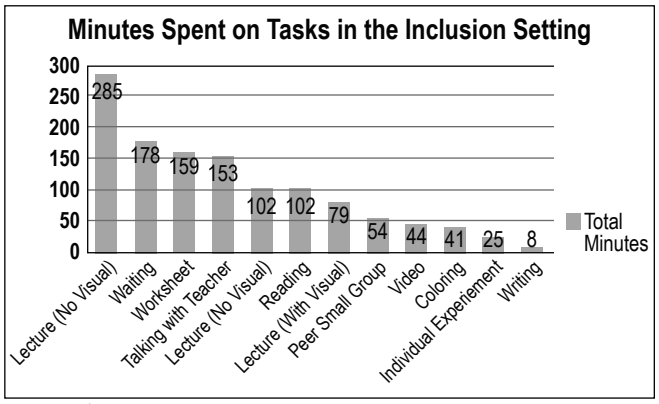

Figure 1.
Minutes spent on tasks in the inclusion setting

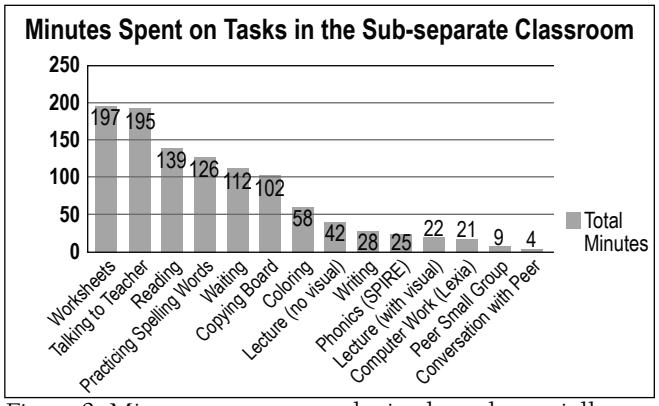

Figure 2. Minutes spent on tasks in the substantially-separate classroom setting.

small group work students were not looking at or interacting with each other; they were merely sitting together. The other .4% of social time was when one student with ASD initiated an interaction with another student with ASD. These short conversations happened on two occasions. One time the students talked about the movie "The Lorax." The other conversation was about summer school.

Discussion

Overall the students with autism spent very little time conversing with each other and with other peers in the academic setting. They were not required to communicate with their peers. The majority of classroom time was spent on worksheets, whole group lectures, waiting for classroom routines, and copying notes off the board. These types of tasks did not require interactions to take place. Students tend to adapt their learning to what is required of them (Doyle, 1983). Since construction of knowledge through meaningful interaction was not required of them, it did not happen. Students were passive learners, sitting and watching the opportunity for an interactive education pass them by. The environment did not support Vygotsky's social learning theory.

The small percentage of time that was spent on group work, a time where students were asked to be social, was not aided to help students with ASD, by diagnosis categorized as socially deficient students. This lack of scaffolding resulted in the student with ASD sitting in proximity to their classmates but not contributing to the group discussion. Inclusion, therefore, was happening through physical presence, not through mental stimulation. The environment needs to be shaped to encourage students to be included beyond simply sitting next to their peers.

Students were frequently asked to execute low level task, such as coloring, copying the board, drill-and-practice, and worksheets. These tasks focused on students strengths in memorization and rules-based concepts by solely asking the students with ASD to either memorize and recite back information (drill-and-practice and worksheets) or spend their education passively doing mundane tasks (copying the board, coloring, and waiting). These tasks direct attention to learning at the surface structure and do not target the conceptual structure of integrating new knowledge into schemas. The conceptual structure is where learning and retrival of information takes place, where as on the surface structure isolated facts are learned (Doyle, 1983). When students with ASD learn information as isolated facts it does not help them target areas they are deficient in. These areas, due to neurological deficits, are associated with theory-of-mind, executive functioning, and weak central coherence, include: organizing information, categorizing information, and integrating and forming schemas for concept recall (Tager-Flusberg, 1985).

The tasks identified in this study were similar to the low level tasks seen in other studies of students with ASD (Klinger & Dawson, 2001; Minshew, Meyer & Goldstein, 2002; Prior, 1979; Sigman, Dissanayake, Arbelle, & Ruskin, 1997). In this study, like in past studies, when the students with ASD were asked to perform these task they did so. They were asked to be passive learners within their academic environment. Asking students to complete tasks that are individual passive assignments does not help them develop compensatory strategies to target areas they are deficient in. When students with ASD are not asked to communicate with their peers they will not learn effective ways to do so. This is detrimental because when these students do not learn these skills in a controlled school environment they will not have these skills when they graduate. By not teaching these students how to be an effective group member during their early years they will most likely have a hard time contributing to society in later years; most jobs require at least some level of interaction with people. By not knowing how to communicate with people opportunities to join the workforce are limited.

Passive learning also limits an individual's potential. Simply asking students to regurgitate memorized information, copy learned facts, and wait around for teachers to develop the necessary skil to effectivelty teach students with ASD is doing a disservice to this student population. They are not going to improve their deficit areas by solely working on their strengths. Therefore, not scaffolding academic tasks towards levels of 'higher-order' thinking and social skills towards reciprocal communication during the school years will most likely lead to students remaining dependent on society post-graduation. Teaching communication independence, instead of solely having one-on-one teacher-to-student question and answer sessions, as was seen in this study, needs to be targeted while students are in school. In schools there are peer groups to help model social behaviors, and these should be used to scaffold social and communication development in students with ASD.

It is important to scaffold academic development and knowledge beyond rote memorization (Doyle, 1983). No Child Left Behind (2001) identifies that these scaffolds to 'higher-order' thinking skills include prerequisite social and communication skills, problem-solving, and emotional-resilience skills. The typical students in this study, observed as congruent models in the inclusion setting, also need 'higher-order' thinking skills asked of them. In the inclusion classroom there was no modification of assignments, instruction, or curriculum for the students with ASD in this environment. This means that the tasks asked of the students with ASD matched the tasks asked of typical peers in

all inclusion observations that took place throughout this study. It also means that all students in the observations conducted seem to be losing out on important higher standards in learning. Students learn what they are asked to learn; they learn what the tasks ask them to learn (Doyle, 1983). By not asking students with ASD to learn how to develop weak areas associated with theory of mind, executive functioning, and weak central coherence these area will continue to remain deficit areas for this population. This means that students with ASD in this environment will not learn how to: take others perspectives, understand others emotions, organize information, tune out unnecessary information, plan the use of information, carry out specific tasks, become flexible in their thinking, use working memory, and think globally (Baron-Cohen & Swettenham, 1997; Frith, 2003; Iland, 2011; Kenworthy, Yerys, Anthony, & Wallace, 2008; Ozonoff, 1997; Quill, 1995; Tager-Flusberg, 1997; Tager-Flusberg, 1985) . Not teaching strategies to compensate for these skill deficits means that these students will be 'left' behind.

The study provided an in-depth task analysis of four fifth grade students with ASD in one school. It is limited because of this narrow focus. Future research can examine how different school and different teachers are providing educational services for students with ASD. Professional development and how that affects change in programming can also serve as a future study.

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Peer-Mediated Gender Socialization: How Peers Influence Young Children's Understanding and Practice of Gender in School

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ABSTRACT

By the time young children arrive at school, they are aware of socially-sanctioned gender norms. Guided by these norms, most children in early-elementary school take up traditional gender performances, and can instinctively determine the extent to which they and their peers align with gender ideals. Just as they monitor themselves, young children monitor each other in an on-going process known as peer-mediated gender socialization. Through peer-mediated gender socialization children judge, influence, and regulate the gender performances of other children. One key underlying goal of peer-mediated gender socialization is to establish and maintain the gendered social order of the classroom. To that end, children employ an array of social rewards and punishments to correct unconventional gender practices and to move peers closer to socially-acceptable gender norms. Young children who conform to gender ideals achieve high standing and social power in the classroom. Those who are unable to inhabit socially-sanctioned gender roles endure social deprivation. In addition to normalizing gender performances and upholding barriers between the genders, peer-mediated gender socialization also functions as a tool through which students can co-construct gender norms in relation to each other. Over time, constructions of gender can grow and evolve.

While interactions with teachers and texts influence young students' growing understanding of gender in school, another equally-potent contributing factor grows organically from the students themselves. Peer influence significantly contributes to students' growing knowledge of what it means to be correctly gendered in school. Identifying peers as influential in the formation of young children's ideas about gender roles and identities has its roots in the work of Lamb and Roopnarine (1979) and Lamb, Easterbrooks, and Holden (1980), who first introduced the concept of peer-mediated socialization more than 25 years ago. Considering earlier studies of children at play, Gilligan (1982) seconds this view in her writing; citing the peer group as a key factor in the process through which children come to understand and practice gender in the early elementary years. Other researchers agree that the influence of peers is of prime importance to young children's understanding and enactment of gender in school. Martin's (1990) review of research finds the peer group to be a particularly-profound source for young children as they develop their gender identities. Similarly, Maccoby (1998), in her developmental look at gender and relationships, emphasizes the significance of peers as the foundation through which young children practice and explore gender during the first years of school.

YOUNG STUDENTS UNDERSTAND GENDER NORMS

Any examination of the peer group must first begin with an examination of the individual. Lamb and Roopnarine's (1979) dated but relevant observations of preschool students reveal that from very early ages children retain a keen awareness of gender and of socially-sanctioned gender norms. Kanka, Wagner, Schober, and Spiel (2011) echo this stance in their work; arguing that "by the age of three, children recognize their own sex and which behavior patterns are 'appropriate' for being a boy or a girl" (p. 291). Learning from their parents and family members, and more broadly from the world around them, children come to school with clear ideas about gender norms and ideals. Signorella and Liben (1985), in analyzing their survey of gender-based activities, occupations, and traits, maintain that children in early-elementary school are just as knowledgeable as adults with respect to socially-accepted views of gender. Davies's (2003) work both upholds and complicates this stance. Drawing on her experiences as a participant observer/ethnographer in multiple early-elementary classrooms, Davies (2003) asserts that while young children clearly retain a growing mastery of socially-sanctioned gender roles, it is not something to which they must devote much conscious thought. Rather, gender roles, performances, and relationships stand as a naturally-occurring part of their social lives. Discussing her informal interviews with second-grade students, Boldt (1996) contends that gender is a "normal, commonsensical, [and] even intuitive" aspect of being a young child in school (p. 117). Research holds, therefore, that most young children are aware of socially-acceptable gender roles, and that they incorporate these understandings seamlessly into their daily lives at school.

Children Enact Traditional Gender Identities

Because many of their initial ideas about gender grow from an internalization of socially-sanctioned gender norms, young children often retain markedly-traditional understandings of appropriate gender identities and performances. Accordingly, the views of gender that they identify

with and act upon in school tend to be conventional. In her detailed observations of preschool students, Alloway (1999) notes the extent to which young children embody "socially endorsed patterns of gendered interaction" (pp. 157, 161). Upon entering kindergarten, many children take up what Boldt (1996) refers to as "idealized" gender identities (p. 117). Wohlwend's (2011) findings second those of Boldt (1996). Utilizing critical discourse analysis to better understand her observations of children's free literacy play, Wohlwend (2011) contends that in the classroom and on the school yard young children "do boy" and '"do girl" in customary, socially-endorsed ways. As individuals, young children know with confidence how to inhabit "the correct gendered narrative" (Davies, 2003, p. 75). Their understanding of their own gender identity goes hand-in-hand with their understanding of the gender dynamics of the larger group. Just as children tend toward the traditional when enacting their own gender identities, so too do they tend toward the traditional when participating as members of larger peer groups. Whether playing alone or with their classmates, most children in early-elementary school perform gender in traditional ways.

RULES FOR "DOING GENDER" IN SCHOOL

The most critical rule governing gender performance in early-elementary school is simple; all children know it. Everyone must have a gender. As Davies's (2003) observations of young students make clear, children implicitly understand that an important aspect of their social identity rests on their correct enactment of gender - they must be easily recognizable to teachers and to peers as a boy or a girl. The second rule follows from the first: girls and boys are innately different. Boldt (1996) explains how young children perceive the genders as essentially separate and distinct: "There are certain ways that boys *by nature* feel and behave and certain things they like, and these are often different than the behaviors, feelings, and likes that girls *by nature* have" (p. 117). Blaise's (2005) findings offer a cogent example that builds on and corroborates Boldt's (1996) position. Moreover, Blaise's (2005) conversations with young children make clear that being a girl is a distinct practice from being a boy. For instance, as she notes, "Being a girly girl mean[s] that you w[ear] frilly, ruffly, and cute outfits, with matching shoes, tights, barrettes, and ribbons" (p. 93). Using the dichotomous categories of "boy" and '"girl" as guidelines, children learn which behaviors to engage in and which behaviors to avoid (Fagot, 1977). Guided by these tacitly-understood precepts, students enact gender through their appearance, discourse, deportment, and activities, and so position themselves as boys or girls in school (Davies, 2003).

Not only are boys and girls inherently different, but they are also inherently unequal (Boldt, 1996). Though inarguably a more controversial stance than the previously-explored guidelines, there is research to support the inequitable nature of children's gender roles in school. For example, Reay (2001), argues that school-based gender categories should not be understood merely as groups, but rather should be reconceived as hierarchies. Drawing on her observations and interviews with early-elementary school students during her year-long classroom case study, Reay (2001) attests that being male equates with having more power and status in school. Orellana's work supports this stance. Reflecting on her critical analysis of preschool children's discourse at home and in school, Orellana (1999) claims: "There are also strong forces that establish cultural practices and social identities as binary opposites: You are either a boy or a girl, either weak or strong, either a hero or a damsel in distress" (p. 112). While most children are tacitly aware of the imbalance of power between the genders in school, it is not something they attempt to alter. Blaise's (2005) work with young children again emerges as germane. Her observations shed light on how children understand the gender power dynamics at work within the classroom. During an informal interview, she asked a young female student why the girl liked to pretend to be a boy. The child answered, "Because it's just better. I can be um, be stronger and do more things. You get to do more stuff, be cool, and it's easier" (p. 101). This girl's response hearkens back to Cherland's (1994) observations gathered during her critical ethnography. Therein, the author suggests that while most girls recognize their place in the gender hierarchy, and they do not attempt to alter or subvert the established social order. As will become evident, pretending to be a boy or playing a boy part is one thing, taking on a male identity is something else entirely.

In addition to standing as separate but unequal categories, boys and girls represent entirely exclusive groups. That is to say, one cannot be both a boy and a girl, and neither can one move from one category to the other. Blaise's (2005) work as a participant observer in an urban kindergarten classroom illuminates the aforementioned stratified, bifurcated nature of gender at work within early-elementary school. More specifically, she attests, "There are certain and distinct ways to be either a girl or a boy, with no room for the blurring of these two genders" (p. 97). For most young students, gender is an inflexible set of binary groups, with no room for negotiation or interplay. These distinct categories serve as borders or boundaries. Blaise (2005) further clarifies this point. In early-elementary school, she asserts, gender emerges as a dualistic system which both informs and regulates students' gender performances and relations. Working from the established categories of "boy" and '"girl"

children define themselves in opposition to those of the opposite sex (Davies, 2003). Through their performance of gender in school, children observe and reinforce these symbolic, hierarchal boundaries.

CHILDREN JUDGE THEMSELVES AND THEIR PEERS

Just as most young children retain an innate understanding of the rules for "doing" gender, so too do most young children retain an innate tendency to monitor gender performances in themselves and others. Critique of gender performance often begins with self-awareness and self-judgment. Children critique their own enactments of gender, and understand the extent to which they are able to meet socially-acceptable gender ideals. Such self-regulation seamlessly extends to regulation of the peer group. Young children in school actively monitor and judge the gender performances of their peers. As alluded to earlier, such assessments arise from internalized, idealized understandings of gender; a stance made clear in Kanka et al.'s (2011) study investigating kindergarteners' attitudes regarding gender and gender stereotypes. In other words, children know what girls should do and what boys should do, and they are able to say whether or not their peers are acting typically or atypically for their gender (Signorella & Frieze, 2008). Blaise (2005) lends her voice to this view and offers a cogent example. In her investigation of the ways that gender is created and sustained in kindergarten classrooms, she posits that it would not be considered "normal" for boys to be overly interested in feminine items, such as dresses, skirts, or make-up (p. 94). Children know without thinking whether or not their peers are getting their gender right, and the process of monitoring classmates represents an instinctive and automatic aspect of children's daily life in school. That is to say, just as children do not have to devote conscious thought to the ways they enact gender, neither do they have to devote conscious thought to process of judging classmates' enactments of gender. Thorne (1998), drawing from participant-observation data gathered from her time spent in working-class elementary schools, attests that monitoring the peer group forms a subtle and ubiquitous part of children's work in school. Though it stands as a habitual aspect of daily school life, monitoring peers' gender performances serves an important, and indeed, mandatory function (West & Zimmerman, 1998). In judging their classmates, children take a critical first step toward maintaining the gendered system of the classroom.

Just as they are aware of their ability to judge others, so too are children aware of the ability of others to judge them. Kamler's (1999) work speaks directly to this topic. Observing student's daily practice of morning talk (the Australian equivalent of *show and tell* or *circle share* in a suburban primary school, Kamler (1999) contends that children's awareness of the judgmental perceptions of their classmates shapes their actions in palpable ways. In sharing artifacts and stories with their peers, the children she observed selected topics that "they considered, and that they considered others would consider, appropriate to their gender" (Kamler, 1999, p. 198). For example, the boys in Kamler's study brought in traditionally masculine items, including "cars, a water pistol, a squirt camera, a watch, [and] a boat" (p. 198). The girls, on the other hand, chose more traditionally feminine objects, including "dolls, toy animals, [and] soft toys" (p. 198). Kamler's (1999) observations share much with Blaise's (2005) findings. Like her colleague, Blaise (2005) contends that children use "the public space of show-and-tell to reinforce gender norms, illustrating the powerful ways in which their talk and actions maintain the gendered social order of the classroom" (p. 94). These findings make clear that young children retain a keen awareness of peer judgment throughout the school day. They can sense the eyes of others upon them, and this awareness of peers' on-going judgment affects their performance of gender in school.

Many gender researchers working with early-elementary school students agree that children judge themselves and their peers against a set of internalized gender ideals, and that these judgments can affect classroom interactions. Some researchers push this stance further, emphasizing the judgment/response interplay between young students in school (West & Zimmerman, 1998). Though more than 25 years old, the work of Lamb and Roopnarine (1979) and Lamb et al. (1980) emerges here as highly relevant. Findings from Lamb et al.'s (1980) naturalistic observations suggest that young children not only monitor each other, but also actively endorse or censure the gender performances of their peers; offering support for "sex-appropriate" acts and censure for "sex-inappropriate" acts (Lamb et al., 1980). To use the authors' words: "Young children administer reinforcements and punishments to one another for gender-appropriate and gender-inappropriate behavior" (Lamb & Roopnarine, 1979, p. 1219). In their writing, Lamb and Roopnarine (1979) stress that such reinforcements serve "to *remind* (not inform) children of sex-stereotyping prescriptions of which they [are] already aware" (p. 1222). Blaise's (2005) more-recent investigation of children's propensity to monitor and judge the gender performances of their peers offers up strikingly similar assertions, especially when she states: children offer "rewards for appropriate gendered and heterosexual behaviors [and] punishments for deviations from the conventional or 'normal' ways of being either a girl or boy" (pp. 86-87). In both cases, it seems that when children offer positive and negative

feedback to their peers, they are attempting to prod classmates toward a commonly-agreed-upon set of rules for doing gender in school.

PEER-MEDIATED GENDER SOCIALIZATION

Through the process of monitoring peers' gender performances and responding with positive and negative feedback, young children further cement "the boundaries where femininity meets masculinity" (Reay, 2001, p. 162). Thorne (1998) notes this practice in findings drawn from her periods of participant observation in early-elementary school. She terms the practice of reinforcing gender boundaries " *borderwork* " (p. 671). She argues the purpose of 'borderwork' is to "reaffirm boundaries and asymmetries between girls' and boys' groups" (p. 671). Davies (2003) seconds these views in her ethnographic reflections, proffering a new term - " *category-maintenance work* " (p. 31). Like Thorne (1998), Davies (2003) suggests that children engage in category-maintenance work to remind peers of the right ways to do gender in school (p. 31). Perhaps the most-apt name for this process is also the oldest. A look back to Lamb and Roopnarine (1979) and Lamb et al. (1980) reveals the early use of the term "*peer-mediated socialization*". Building off that language, this paper puts forth a slightly different term—*peer-mediated gender socialization*. As it is used here, peer-mediated gender socialization represents the process by which young children actively endorse and censure their peers' gender performances with the explicit goal of correcting unconventional gender practices and moving peers closer to socially-acceptable gender norms and ideals. Engaged in peer-mediated gender socialization, young children play an active role shaping one another's understandings and enactments of gender in school, and, in so doing, maintain the borders between the genders and uphold the gendered social order of the classroom (Lamb & Roopnarine, 1979).

STRATEGIES OF PEER-MEDIATED GENDER SOCIALIZATION

There are a number of strategies that students can employ when engaged in peer-mediated gender socialization. These various tactics, reactions, and social cues span a vast spectrum - from light-hearted, humorous banter to unflinching social banishment. The first, most common, and least evasive strategy is teasing. Young students in school often tease peers who fail to get their gender right. As a preliminary measure, children use teasing to bring peers back to the correctly-gendered fold. If teasing fails to achieve the desired results, elementary-school students can employ other, increasingly severe social deterrents and punishments which include, social belittlement, exclusion/social isolation, and exile. The purpose of these strategies is to help peers ameliorate their gendered failings, and eventually rejoin the group. The following sections will explore each of these tactics in greater depth.

TEASING

Children use many strategies when engaging in peer-mediated gender socialization. The most common, and inarguably the most effective, is teasing. Maclean (1999), theorizing from past studies, cites teasing as one of the most familiar peer sanctions for inappropriate gender behavior. Boldt (1996) accords; finding that young children who perform gender incorrectly are often teased by peers. Though teasing can take varying tones, it is often a first step, a first reminder for students that they are failing to meet the agreed-upon code for gender performance in some way. Drawing upon her observations of preschool-age children, Fagot (1977) suggests that one of the most grievous infractions young children can commit is identifying as or performing the wrong gender. Thorne's (1998) findings echo this stance, particularly when she argues that children who engage in cross-gender play and cross gender boundaries risk being teased by peers. Moreover, Aapola, Gonick, and Harris's (2005) more recent work bolsters these claims. Therein, they contend that taking up a cross-gender identity, or "doing" the gender of the opposite sex, is a sure way to incite teasing from peers. The concept of gender-crossing becomes more concrete with an on-the-ground example. Boys who take up nurturing, domestic roles and care for dolls are seen as "as being mama's boys and able to be beaten up by girls", whereas girls who don capes and charge enemies with swords are demeaned as "dumb, ugly, and unable to do things" (Keenan, Solsken, & Willett, 1999, p. 44). Though teasing can result in momentary hurt feelings, it serves as a potent tool through which children can confirm their own gender, the gender of their peers, and the gender dynamic of the class as a whole (Cherland, 1994). Teasing serves the larger goals of peer-mediated gender socialization by delineating and maintaining the borders between gender groups. Cherland (1994), in her ethnographic study of slightly-older elementary-school students, identifies how teasing works "to discourage cross-gender [behavior] and so enforce gender boundaries" (p. 48). Davies (2003) seconds this idea and clarifies the role teasing plays in marking and re-marking gender boundaries. In fact, she persuasively argues, the primary purpose of teasing is to maintain gender categories "in the face of individual deviation" (p. 31). Teasing, the most-common and most-effective tool of peer-mediated gender socialization, represents the first line of defense children take up as

they work to uphold the hierarchal gender boundaries of the peer group.

In addition to maintaining the boundaries between the genders, teasing also prompts reform. That is to say, teasing leads young children back into socially-accepted gender performances. In discussing her observations of kindergarten students, Wohlwend (2011) explains that young children who do not get their gender right are viewed by peers "as novices in need of remediation" (p. 18). Teasing spurs errant students to reflect upon their shortcomings and to amend their improper gendered behavior. In their work with young children, West and Zimmerman (1998) found that teasing is often enough to prompt children to appreciate their failings. Once children can see how they have erred, they are quick to reform. Aapola et al. (2005) address this phenomenon in their research. Their interviews with young girls reveal; Girls who expressed views that were deemed 'out-of-line' with the general understanding concerning proper behavior for a girl, were reprimanded by the rest of the group, and they were expected to mend their ways if they wanted to be part of the group. (p. 113)

These findings not only confirm the use of peer-mediated gender socialization as a tool of reform, but also reveal that most children who are teased by peers are eager and anxious to rectify their wayward behavior, comforted by the knowledge that such that compliance will lead to renewed acceptance from their peer group (West & Zimmerman, 1998). Reid (1999) echoes this stance by revealing how quickly teasing leads most young children to alter their gender performance and reclaim their place within the group.

Social Deterrents and Punishments

Most of the time, teasing is enough to bring young children back into the fold of socially-sanctioned gender performances. If teasing fails to prompt the required changes, however, students engaged in peer-mediated gender socialization can branch into a more diverse spectrum of social deterrents and punishments. Like teasing, these strategies are based upon children's shared feelings about the inappropriateness of violating gender norms (Blakemore, 2003). Gender researchers working with children have done much to shed light on this system of deterrents that keeps children from violating socially-accepted gender roles (Orellana, 1999). Aapola et al. (2005) offer some particularly insightful examples of young children's use of social punishments. From their interviews with young girls, they note the passive-aggressive ways children react to peers who fail to conform to gender ideals. The students' compensatory reactions include "excluding a person from social interactions, sulking, talking behind someone's back and seeking other friends as revenge" (p. 119). Such findings make clear that young children who repeatedly deviate from conventional gender ideals will be met with ridicule and social disruption (Blaise, 2005; Cherland, 1994). Children use these social deterrents as the stick, and the promise of renewed social acceptance as the carrot, in their attempts to move their classmates toward a renewed embodiment of appropriate gender performances.

Social Isolation

If such social deterrents are not effective in righting peers' aberrant enactments of gender, young children can embrace more openly-hostile measures. Children who repeatedly fail to align with gender norms find themselves on a slippery slope, where good-natured teasing can quickly devolve into bullying and other more aggressive social sanctions (Aapola et al., 2005). In her work, Thorne (1998) explores some of these more belligerent tactics such as public belittlement and social rejection. Her research reveals the extent to which children use exclusionary tactics to single out and reject those who fail to take up idealized gender performances. A brief section from her field notes illuminates this conjecture: A first-grade boy avidly watched an all-female game of jump rope. When the girls began to shift positions, he recognized a means of access to the play and he offered, "I'll swing it." A girl responded, "No way, you don't know how to do it, to swing it. You gotta be a girl" (p.671)

Young students incapable of "doing gender" in socially-endorsed ways are forced by their peers to the periphery of the group. As Maclean (1999) makes clear, the early-elementary peer group "marginalizes children who do not display gender-appropriate behavior" (p. 73). Repeat gender offenders are relegated to the fringes of the community by their peers. Davies's (2003) work accords with these suppositions. George, a young boy prominently featured in her ethnographic report, shows interest in playing house with female classmates. As a result of his cross-gender interests, his peers reject and isolate him. He is not "recognized by any of the other children as a legitimate kind of person" (p. 89). As with previously-explored measures, these exclusionary tactics are used to encourage children to perform gender in socially-sanctioned ways. Aberrance, once corrected, will result in renewed group membership, and those cast off to the outskirts of the classroom community will be reinstated once their gender performance aligns with those of the dominant group (Wohlwend, 2011).

Exile

If being forced to a marginalized existence *still* does not result in appropriate reform, gender outliers are shunned by their peer group. Though similar, shunning differs from social exclusion. Those students who populate the

fringes of the community still exist, while truly unrepentant gender deviants do not. They are social ciphers. Expelled from the peer group, they are punished with complete and total social isolation (Aapola et al., 2005). In her critical ethnography exploring the gendered school lives of slightly-older, elementary-school girls, Cherland (1994) sheds light on the experiences of one such student, Marcia. Marcia, fails to embrace and enact socially-prescripted gender ideals for girls in her classroom. As Cherland (1994) relates, "She refused to look like they looked. She refused to be a friend. She refused to be 'good'" (p. 65). Faced with such unyielding obstinacy, Marcia's classmates "meted out the only punishment they could: They silenced Marcia, and made her invisible" (65). Resisting the gender expectations of her peers, Marcia became an exile, a social pariah; existing as an example for any student failing to take up an expected gender performance (Boldt, 1996, p. 127). Because she fit nowhere within the gendered social order of the classroom, Marcia was ignored. Her peers acted as if she did not exist: "In classroom project groups, and on the playground, they refused to hear anything she said. . . . Occasionally, she spoke to another girl in her class, but she got no response. She seemed to accept that" (Cherland, 1994, pp. 65-66). Like Marcia, victims of this most hostile incarnation of peer-mediated gender socialization live outside the system, bereft of friends and allies. Though it is difficult to imagine such a harsh fate for such a young child, it is important to remember that even these personae non gratae can be recalled; saved from the unrelenting cruelty of the school yard by expressing a willingness to take up socially-sanctioned gender performances (Fausto-Sterling, 1998).

CONFORMITY WITH GENDER NORMS AND SOCIAL POWER

Conscious of the on-going judgment of peers and keenly aware of the social stigma associated with miscreant gender performances, most children conform to socially-accepted gender norms. In their synthesis of research, the American Association of University Women Educational Foundation (1992) finds that "both girls and boys strive for conformity with gender-stereotyped roles" (p. 17). Blaise (2005) lends her voice to this view, contending that "it is important for the majority of young children to get their gender 'right'" (p. 93). Boldt (1996) explains children's shared desire to inhabit conventional gender roles: "To be 'normal', to have the well-being, privilege, and sanction that go with normalcy, one's physical sex, gender, and sexuality must be enacted in particular ways" (p. 114). For children, getting their gender right conveys a sense of social competency. That is to say, for young children, the ability to "do gender" correctly stands as "an important social accomplishment" (Cherland, 1994, p. 33). Along the same lines, taking up a socially-endorsed gender performance offers children social legitimacy and membership within the larger peer group. For example, Williams (2006) theorizes young students "seek assurance that they are becoming insiders - people who will be accepted by the dominant culture - and not those who will be shut out and shunned" (p. 301). Davies (2003) upholds this line of thinking. She attests that "any person who wants to be recognized as legitimate and competent must be appropriately gendered" (p. 161). By getting their gender right, children prove to their peers that they belong, that they understand the dominant gender narrative, and that they are worthy of inclusion and acceptance (Davies, 2003). Even young children instinctively understand that conforming with socially-sanctified expectations for gender will place them solidly within the majority, a safe and secure place to be (Boldt, 1996).

Besides granting children a sense of acceptance and valuation, and guaranteeing them safe refuge within the confines of the dominant group, gender conformity carries with it the promise of other powers and social rewards. Davies's (2003) observations and interviews with preschool-aged children sheds light on this issue. Her findings make clear that for children, "some forms of 'masculinity' and 'femininity' are 'safe'. If correctly achieved, they are recognized as high-status ways of being. The children who achieve them are popular and other children aspire to be in their group" (Davies, 2003). As Blaise's (2005) work with urban kindergarteners reveals, the popular children are often those who embody socially-sanctioned gender ideals. These boys and girls understand socially-desirable gender performances, and happily provide them. During her time as a participant-observer, Blaise (2005) encountered a handful of examples of such children. Their ability to perform gender masterfully grants them expert status. One student, Alan, inhabits such an authoritarian role. Popular, powerful, and knowledgeable, he acts as a leader among the boys, making important decisions and controlling their play (Blaise, 2005). He retains an implicit mastery of gender roles - what boys are and are not supposed to do. He can say with confidence that "boys neither play with Barbie dolls, nor like the color pink" (p. 97). His expertise extends to girls as well. During an informal interview, Alan reveals that popular "girls w[ear] cool clothes, they [have] to wear make-up and perfume, they [are] not interested in being police because, they like sitting around and being beautiful" (p. 96). As a gender expert, Alan's knowledge in the following passage proves both innate and unshakable:

> Look . . . boys are supposed to do boy things and girls, well, they do all those girly things. That is how it is! Boys play football, girl are cheerleaders . . . And we aren't going to mess with it. That is final! (p. 98)

Alan's ability to perform gender in socially-sanctioned ways grants him power and establishes him as a leader and an expert in the classroom. Cherland (1994) digs into the underlying rationale at work here: children like Alan "have an investment in being accepted, in doing what is expected of them, and in being rewarded for it" (p. 53). Viewed from this perspective, it becomes clear that for some children, on-target gender performances serve as vehicles for accessing social prestige and power within the peer group (Aapola et al., 2005). In a similar vein, these same children set the standard for others. They are the norm that their peers are measured against. Empowered by their ability to take up idealized forms of gender, such children reward those who follow suit. Again, Blaise's (2005) study illuminates this issue. In observing girls, she notes how they mirror and reflect the behaviors and speech patterns of the most popular students, perpetuating traditional gender practices through imitation. Taking cues from a popular girl in class, other girls began "complimenting each other's outfits, especially how particular barrettes and bracelets made them look, 'Oh so beautiful!' (Blaise, 2005, p. 95). Through these practices, a cycle of peer judgment and peer validation continues, recursively highlighting and rewarding those children who best embody socially-endorsed gender roles and performances (Blaise, 2009).

FAILURE TO ALIGN WITH GENDER NORMS AND SOCIAL DEPRIVATION

There are some students, however, who do not or cannot conform to socially-acceptable gender ideals. Before looking more deeply into such cases, it is important note that they are quite rare, particularly in the early-elementary years. Cherland (1994) makes clear that only very occasionally do young children "openly and deliberately break the rules for 'doing boy' or 'doing girl' (p. 64). Where older students can embrace defiant attitudes as part of a conscious rejection of societal norms, very young students are not psychologically mature enough to pursue such stances. As Wohlwend (2011) theorizes, such children do not disrupt peers' gender expectations as part of a teenage rebellion, but rather because gender norms are inherently alien and confining for them. For such students, their whole identity, their whole personhood, rejects traditional ways of doing gender in school. An example of such a student can be found in Reid's (1999) work, where she records and examines narratives of violence in early-elementary school. Therein, she explores the fate of the few very young children who cannot offer socially-acceptable performances of gender. Jodie is such a girl: "Loud, bouncing, [and] dominant, [she] is marked as different from girls like Bianca and Michelle, good quiet girls who don't call out and are not likely to have done violence to boys" (Reid, 1999, p. 184). For Jodie and others like her, inhabiting "their assigned gender category is a straightjacket they have a lot of trouble wearing" (Davies, 2003, p. 132). Conforming to society's ideals and peers' expectations regarding gender is anathema to them. It is untenable. They simply cannot do it; they cannot be other than who they are. Though their inability to take up socially-endorsed gender ideals does not stem from a conscious decision or rejection of societal precepts, it nevertheless sets the stage for a tragic, downward arc of their social status in the classroom.

Those who find themselves unable to take up socially-accepted norms for gender in school suffer a serious social cost, and even the youngest children are aware of the price they will pay for failure. Cherland's (1994) observations make clear that children are conscious of the imminent threat of "social disruption that is the consequence of doing gender incorrectly" (p. 33). In her survey research about children's attitudes regarding peers who violate gender norms Blakemore (2003) echoes this contention. She asserts that children view violation of socially-accepted gender ideals as very undesirable, because they are know all too well the social repercussions associated with such deviancy. Davies (2003) takes a step further, equating correct gender performances with social and emotional survival for young children at school. Children know that failure to "do boy" or "do girl" correctly will be perceived by the peer group as a moral blot on their identity, and will result in a fall from grace (Davies, 2003). Though this system may seem cruel to outsiders, to most children it is equitable and just (Wohlwend, 2011). Those who fall outside assigned gender norms, for whatever reason, must be censured (Davies, 2003, p. 54). Children sanguinely accept and acknowledge the social costs associated with failure to conform to dominant notions of gender (Weedon, 1997).

Those who deviate from conventional or "normal" ways of being either a girl or a boy suffer at the hands of their peers (Blaise, 2009; Reay, 2001). Boldt's (1996) work explores the difficulties young students encounter when the violate socially-held norms for gender. The social deterrents and punishments used by children in an effort to reform their peers' errant gender performances become a permanent way of life. Wohlwend's (2011) writing illustrates what such children are forced to bear; as she explains, gender performances that do not meet peer's expectations have very real and immediate social consequences (p. 8). Children who cannot take up socially-endorsed gender identities are excluded from the peer group. Not as a temporary punishment for a momentary mistake, but as a permanent, unrelenting sentence of exile; a complete social embargo. Davies's (2003) research makes clear that gender outliers suffer continual, unabating banishment. She offers a telling

example: George retains a "clear preference in his play for things that are normally seen as for girls . . . these compromises are not acceptable to other children and so George must often be alone" (p. 137). Unable to meet the expectations of their peers, George and children like him exist in a world devoid of friendship and social support (Davies, 2003). Because they cannot take up socially-sanctioned gender roles, there is no chance of acceptance. They find themselves alone within a crowd; lonely among a group of "friends" (Wohlwend, 2011).

PEER-MEDIATED SOCIALIZATION TO CO-CONSTRUCT GENDER NORMS

Though the fate of gender outliers is lamentable, peer-mediated gender socialization should not be viewed as positive or negative. Rather it should be understood as a tool through which children monitor, judge, influence, and regulate the gender performances of other children in reflexive ways. Through a recursive system of endorsing and censuring their peers' gender performances, young children self-regulate the gender roles and relationships in the classroom (West & Zimmerman, 1998). The constant and critical peer-gaze acts as a tumbler, smoothing out the vagaries and discrepancies of the children's gender performances. In this way, peer-mediated gender socialization stands as a self-regulatory process that produces and upholds socially-shared notions of the right ways to "do boy" and "do girl" (Blaise, 2005). Davies (2003) explains, "Each child must get its gender right, not only for itself to be seen as normal and acceptable within the terms of the culture, but it must get it right for others who will be interpreting themselves in relation to it as other" (p. 21). When they "do gender" in school, children synthesize traditional, internalized views together with feedback from peers, learning from and with each other. Taking this line of thought further, Aapola et al. (2005) persuasively contend that the on-going work of peer-mediated gender socialization should be understood as a communal shaping of the self, in which children process and understand norms in relation to each other.

GENDER EMERGES AS A FLUID AND DYNAMIC CONSTRUCT

Seen in this light, peer-mediated gender socialization reemerges as a system of give and take. Individual children's gender identities are achieved not solely within the mind and practice of each individual child, but are co-constructed socially and relationally; in the space between children (Aapola et al., 2005; Blaise, 2005). Peer-mediated gender socialization becomes the work of a community of practice. Children construct and reconstruct gender in relation to each other through a communal sharing of ideas (Schieffelin & Ochs, 1986). Working with and through social norms, children actively construct gender through "talk, actions, and interactions with each other" (Blaise, 2009, p. 453). That is to say, young children engaged in peer-mediated gender socialization both reflect and affect the gender identities of their peers. Individuals both shape and are shaped by group understandings of gender. In their daily interactions, children work in unspoken symbiosis "to constitute and contest what is means to be a desirable female and male" (Blaise, 2009, p. 458). In a complex dance of give-and-take, alternating seamlessly between active and passive roles, children both embody and reflect what it means to be correctly gendered. As Blaise (2005) explains, young students practice and perform gender in school, they influence each other, and also influence the standard they are working against. Reconceived in this way, it becomes clear that peer-mediated gender socialization retains a further purpose. While inarguably it serves to judge, monitor, influence, and regulate children's gender performances, it can also work to create new gender norms. Blaise (2005) further clarifies, "Young children take an active part in 'doing' gender by socially constructing the meanings about femininities and masculinities from the gender discourses available to them in their everyday worlds" (p. 85). This last piece of the peer-mediated gender socialization puzzle is hugely important because it recognizes that socially-sanctioned gender norms can change, and it is the peer-group who can change them. Working together, young children in school construct and reconstruct an evolving understanding of appropriate gender performances, and gender emerges as a vibrant, living, breathing construct.

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Should I Stay or Should I Go: Helping New Teachers Connect

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ABSTRACT

New teachers are leaving the classroom in staggering numbers. Within the first three to five years, typical nearly half leave the profession. Among the various reasons for this exodus, research often points to the isolating culture of the teaching profession and the sudden immersion from study into practice.

New Teachers' Center's (NTC) Mentoring for Student Success (eMSS) teacher induction program strives to address isolation and new teacher transition to the classroom issues through social media and networked communities connecting knowledge among participants. NTC works with school districts, consortiums, other organizations and non-profits, and institutions of higher education (IHEs) nationwide mentoring new teachers, setting up professional learning development communities, and analyzing school learning and teaching environments.

The purpose of my proposed study is to explore how issues of new teacher isolation and transition might be resolved by interweaving a framework of Connectivism, with a methodology, Netnography, in an eMSS social network.

Should I stay or should I go now?
…This indecision's bugging me…
Exactly whom I'm supposed to be
Don't you know which clothes even fit me?
— The Clash

New teachers are continuing to leave the classroom in staggering numbers. On average, over the past nearly 30 years, nearly half of all novice teachers leave the profession within the first three to five years (Ingersoll, 2002; Inman & Marlow, 2004; Rogers & Babinski, 2002; Voke, 2002). Among the various reasons for this exodus, research often points to the isolating culture of the teaching profession (Rogers & Babinski, 2002; Dodor, Sira, & Hausafus, 2010; Hadar & Brody, 2010) and the sudden immersion from study into practice (Ashton, 1984; Lieberman & Miller, 1984; Veenman, 1984).

The culture of teacher isolation has been well documented in the literature. Beginning with Lortie's (1975) groundbreaking research, which found that teaching practices foster little professional time for interactions, several researchers have concurred. Teachers are separated by a sense of academic autonomy (Elmore, 2000; Little, 1990; Rogers & Babinski, 2002), differences in subject matter (Schlager & Fusco, 2003), and by the physical boundaries of the classroom (Cookson, 2005). Differences from influences both within and outside various local educational communities, in culture, leadership, tools, and cooperation can hinder development and the effectiveness of professional teacher development. Teachers in some schools, according to Schlager & Fusco (2003), are separated by their career stage and discipline, which hinders communications rather than helps support a professional network. Administration often discourages informal leadership and delineates internal from external expertise. Consequently, these factors "promote a culture of privacy and autonomy that can reinforce ineffective professional development strategies and discourage collaboration and sharing of expertise and resources" (p. 19). For new teachers, the results of isolation — for those who choose to stay — are displayed in variable teaching methods "that are inconsistent and even contradictory to their initial pedagogical beliefs, goals, and expectations" (Rogers & Babinski, 2002, p. 3).

The physical structure of the school environment (Cookson, 2005) itself prohibits collaboration among teachers who remain isolated from colleagues. In a culture that promotes academic professional privacy and self-reliance, "opportunities for teachers to engage in genuine professional dialogues are rare in the schools" (Rogers & Babinski, 2002, p. 2). External educational consultants and other external-driven professional development (PD) resources, such as curriculum developers and policy makers, often facilitate opportunities for discussion about teaching practices and associated issues. These sessions take place in workshops and in-service training outside of the classroom, where the facilitator dictates what and how teachers need to learn. Consequently, these opportunities never address the daily issues teachers encounter in their classrooms (McLaughlin & Talbert, 2006, p. 2). Moreover, the workshops focus on institutional-mandated practices, not on the individual practitioner's needs. Paradoxically, professional development in schools stresses teamwork and adherence to institution standards, but neglects to establish a social network of communication among teachers, barring further professional growth and development, which often results in new teachers leaving the profession after only a few years. (Dodor, Sira, & Hausafus, 2010, p. 1).

The "reality shock" of transitioning from student to teacher, as Veenman (1984) noted, is complicated by applying educational theories to practice, especially in areas of student motivation, assessments, and parental interactions. Studying social situations, such as interacting with parents, are often not a part of the teacher-training curricu-

lum; they are more likely encountered through personal and shared experiences. Unfortunately, pre-service training offers little by way of discussions with current first- and second-year teachers, whose "on the job" perspectives would be of considerable importance to students who would be teachers. The transition (Lieberman & Miller, 1984) from student to educator often leaves new teachers with a sense of isolation and loss of community (Ashton, 1984).

According to Wenger (1998), a community of practice in which adults discuss their professional issues, trading experiences, tips, and best practices, requires that participants manifest their identity as participants, not as representatives of an institution. Unfortunately, K-12 teachers lose their most powerful teaching asset when they separate themselves from mature practice by not acting as themselves, but rather as teachers, upholders of curricular demands with a role defined by their institution.

Opportunities to connect with other professionals in communities of practices are increasingly taking place online. Social networking sites are popular across all age groups; although, in recent years, participation in adults between the ages of 30 and 64 has dramatically increased, while frequency of use among younger adults has stabilized (Sistek-Chandler, 2012).

Engaging in online social networking and community building has been available since 1985 when early virtual communities convened in computer-mediated communication (CMC). The growing interest in adult usage of CMC social networks offers an innovative way to "break down the walls" of teacher isolation (Abbott, 2003). The practices of using social media and Web 2.0 technologies today are, according to Redecker, Ala-Murka, Bacigalupo, Ferrari, and Punie (2009), are the educational and training tools of the 21th century, poised for promoting the skills required for future jobs. While Web 1.0, was often more like reading a book than sharing a conversation (Kozinets, 2010), Web 2.0 technologies are engaging. Formats such as videos, podcasts, blogs, wikis and social media, are well-known to a generation of new teachers, who grew up with "considerable proficiencies in using online resources" (Moore & Chae, 2007, p. 223), and invite social networking and online community interaction. Moreover, over the past decade, the proliferation of adult online continuing education sets precedence for how busy working professionals can learn through the social interaction, in collaboration and through reflection, in a convenient, safe, and helpful environment. Despite the advancements in communication technology, the problems associated with teacher isolation, transition persist (Moore & Chae, 2007), and new teachers continue to leave the profession.

This proposal outlines the problems that new teachers encounter with isolation and transition. Moreover, it proposes an exploratory study of how social networking and online communities of practice as part of ongoing professional development may help struggling new teachers survive their beginning years, building support networks, and continue in the profession.

THE RESEARCH PROBLEM

New ideas emerge from learning and collaborating with others. McLaughlin and Talbert (2006) concur that training that happens in isolation from the classroom and away from interactions with other teachers "represent others ideas about needed skills and knowledge but seldom reflect teachers' thoughts about what they need to learn or how to learn it" (p. 2).

The problem of teacher isolation stems from traditional programs of professional development, which "fall short of meeting the pedagogical and personal needs of beginning teachers" (Rogers & Babinski, 2002, p. 5). Instead, they often "deemphasize interaction among participant" (Hadar & Brody, 2010, p. 1641) segmented from the work setting. They found that when teachers work and learn together within their environment, they create a culture in which further learning is stimulated and supported. This finding is supported by research showing that learning in isolation had limited capacities (Brown, 1997).

According to Rogers and Babinski (2002), "new teachers frequently complain that the standard induction model of professional development provides the wrong kind of information at the wrong time and in the wrong way" (p. 5). New teachers, they find, are missing opportunities to reflect on their work in discussion with other teachers in community. "We, too," Rogers and Babinski summarize, "believe that community is important for learning" (p. 12).

An Exploration Into Uses of Social Networking

Online communities and social networks enable people to communicate and interact in ways that are enhanced by the technology. They allow participation at times that are convenient for them, exchanges of ideas, and time and location for the reflection of those ideas. Some educational researchers point to the vast potential to shape the way people learn (Barbour and Plough, 2009; Drexler, Baralt, and Dawson, 2008) online. Hung and Yuen (2010) suggest that educators make instructional use of this new world's "social nature of Web 2.0 in order to create optimal, natural environments for learning to take place" (p. 703). Signer (2008) postulates that the best practices for using technology in instructional professional development requires that the content of staff development focus on pedagogical and not technological skills. Best practices should include reflective work stories shared among teacher participants in dialogue

that is both social and professional. Technology should be a means to this end, enabling sharing and incorporating activities that transform classroom practice.

One online community that integrates CMC and Web 2.0 technologies to enable teachers to share best practice stories and discussion is the New Teacher Center (NTC) based in Santa Cruz, California. Founded in 1998, NTC is a national nonprofit organization focused on opening critical paths for beginning teachers to communicate with mentors, administrators, and other teachers through online networking and traditional face-to-face professional development programs. Specifically, its e-Mentoring for Student Success (eMSS) Program assigns online mentors to work closely with new teachers, or mentees, in varied asynchronous safe and social interactions, from private one-on-one meetings, to mentor communities, to special interest groups, and larger communities of new teachers and mentors nationwide.

THROUGH A FRAMEWORK OF CONNECTIVISM

For new teachers to be successful in breaking the walls of isolation, each must build a personal support network that includes a professional development community. *Connectivism* , a learning theory proposed for the digital era, grounded in theories of social learning and constructivism, provides a framework for analyzing how one builds these personal learning networks, where one must become active online addressing dynamic information needs. George Siemens (2006) defines his "education theory for the 21st Century," connectivism as a framework for learning through modern social networks and Web 2.0 technologies. "Knowing and learning are today defined by connections. Connectivism is the assertion that learning is primarily a network-forming process" (p. 15). Siemens see social tools, like social networks, blogs, and texting, as vehicles for rapid exchanges of knowledge and high levels of dialogue. Web 2.0 technologies offer a variety of media for individual learning styles and collaboration including wikis and online meetings, broadcasts, and shared spaces, also referred to as conduits for information. Knowledge is dynamic. It requires a greater need to stay connected in the new century.

Using Siemens' (2009) checkpoints of connectivism as the theoretical framework of my study, I will structure my analysis of NTC's online interaction data noting how:

- connections are formed
- technology enables these connections
- knowledge is transferred during an interaction between learners.

RESEARCH PROPOSAL QUESTIONS

I propose to conduct a qualitative study that explores how new teachers' interactions in a structured professional community induction program helps lessen isolation and doubts about teaching as they learn from colleagues and establish connections to knowledge that benefit them professionally. Specifically, my study seeks to contribute to the research through non-participatory observations and analysis of the NTC eMSS program's online discussions.

Through an analysis of the online discussions, or *content* , this study seeks to learn how new teachers use technology to build personal networks through the *conduit* of social media and community within the *context* of the learning theory, connectivism. My curiosity about the effectiveness of online social media and community in supporting teacher professional development, therefore, prompts my research inquiry:

1. How do novice teachers use online professional development communities to build a support network?
2. What expressed benefits of participation in this online professional development community are observed?
3. What challenges do participants in an online professional development community encounter?

Study Purpose

Teacher isolation and the difficulties encountered in transitioning from student to teacher are often seen among the key reasons for new teacher attrition. In a technological age, where online knowledge acquisition and social interactions are literally at one's fingertips, young, new teachers can use social networking skills, which have become commonplace among many students (Duggan & Brenner, 2013) to discuss online and foster professional relationships that ease the transition into the profession and the feelings of isolation. Nevertheless, despite access to a large number of people with like experiences and issues through technology, the attrition rate (Ingersoll, 2003) remains high. The challenges of isolation in the classroom are ever-present (Cookson, 2005), even in this new "connected" century. The current literature (Cookson, 2005; Dugan & Brenner, 2013; Moore & Chae, 2007) ponders but seems unable to explain the reluctance of new teachers using social media to address their professional needs.

Technologically infused induction programs, such as the NTC's eMSS program, strive to address the issues of isolation and new teacher transition to the classroom through mentored dialogue and connected knowledge among teacher peers. The organization works with school districts,

consortiums, other organizations and nonprofits, and institutions of higher education (IHEs) nationwide to mentor new teachers, set up professional learning development communities, and analyze school learning and teaching environments. NTC provides both a conduit for online social media and community networks to foster knowledge sharing and growth, and the proper context in which participants can feel safe discussing personal work-related issues.

The purpose of my study is to explore how isolation and the transition to the profession for teacher inductees might be mitigated by interweaving connectivist practice in a social network — a structured online learning community — where the conduit (NTC's eMSS program) provides a structured curriculum and intentional design.

The Framework of Connectivism

Increasingly, teachers turn to the Internet for answers to work problems and network socially to increase their learning collaboratively (Purcell, Heaps, Buchanan, & Friedrich, 2013). According to Siemens (2005), connectivism, "a theory describing how learning happens in a digital age," (p. 30) asserts that learning is primarily a network-forming process, not a product. Learning is a way to open the mind, not simply fill it. We learn through creating and making relevant personal connections between the nodes of knowledge. These nodes can be both external, such as libraries, books, web sites, people, and internal from self-made, personal connections. Over time, these nodes form a personal network of knowledge. Like a network, which remains dynamic, nodes are added, deleted, and updated to provide more efficiency in the connections. Social connections become essential because they allow us to create and share knowledge. "'Knowing where' and "knowing who' are more relevant today than knowing what and how" (p. 32).

Connectivism is seemingly the heir-apparent to the collaborative learning theory throne. Although comparisons in the literature remain sparse, its roots can be traced back to Bandura's (1969, 1989) social learning theory and Vygotsky's (1978) constructivist educational theory.

As Figure 1 shows, connectivism inherits the social aspect of cognitive learning but expands access to new knowledge to entities beyond human interaction. As we move to the right, knowledge expands beyond its collective, product base, to a more individualized, process base, subject to personal discernment and relational connections, and remains available, in its raw form, for retrieval outside one's own mind.

Constructivism seeks knowledge through active participants creatively experimenting with the cognitive knowledge. Downes (2012) contends that connectivism, on the other hand, differs from other social learning theories in that:

> …connectivism denies that knowledge is propositional. That is to say, these other theories are 'cognitivist,' in the sense that they depict knowledge and learning as being grounded in language and logic. Connectivism is, by contrast, 'connectionist'. Knowledge is, on this theory, *literally* the set of connections formed by actions and experiences (p. 85).

Knowledge is not like a prepackaged meal ready to serve all. Instead, knowledge requires a customized recipe of the right ingredients, blended properly, and made palatable to personal taste.

In fact, according to Siemens (2005), multiple definitions of *knowledge* are true. `It can be described as an entity and a process, a sequence of continuums; type, level, and application, implicit, explicit, tacit, procedural, declarative, inductive, deductive, qualitative, and quantitative. "Knowledge rests in an individual; it resides in the collective" (p. 14). While there is a strong social component, connectivism uses social networks to create individual connective nodes. These connective nodes define one's knowledge and learning. Hence, "connectivism is the assertion that *learning* is primarily a networking forming process." (p. 15) Knowledge is not merely constructed but remains an entity outside the learner, like a rough diamond, awaiting personal discovery and processing into something of value that meets personal learning needs.

Social Networks and Community

Connectivism describes three key areas for obtaining knowledge and learning: the conduit, learning space, or community, where the knowledge is stored and discussed; the content, which are postings to the community; and the *context*, or the topics, of discussion. Each area is present in social networking technology. Connectivism, as a new digital educational learning theory has developed because of our increasing reliance on technology. As Siemens states (2006), we are no longer driving technology; instead it has begun to drive us. Evidence of this paradigm shift can be seen elsewhere, in the daily use of smart phones and other mobile devices, for example. We are afforded an immediacy of connection and conversation. This immediacy of interaction has

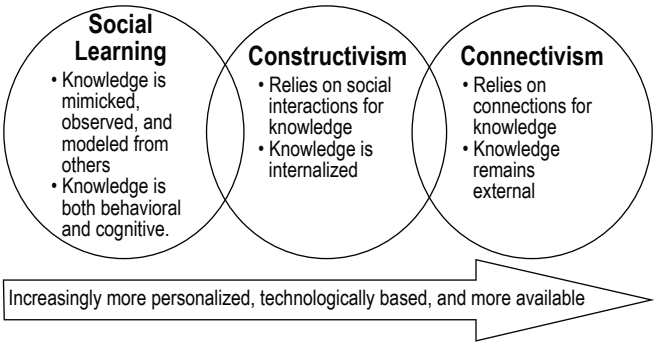

Figure 1. Evolving Learning Theories

changed how we connect and communicate to others. Hence, the technology dictates behavior.

The literature shows the use of social networks as a connection to knowledge and community offers the new teacher the opportunity to exchange knowledge with colleagues throughout their professional careers. Redecker et al, (2009) point to the use of social media and Web 2.0 technologies as being "at the core of education and training, as they promote the competencies needed for future jobs and enable new tools for educational institutions to transform themselves into places that support the competencies needed for participation in 21st century" (p. 14).

Conversely, the literature shows several issues that continue to prohibit teachers from collaborating in a technological age. Unfortunately, education tends to lag behind other professions in technology integration, often because of issues of funding and resources, and the limited amount of time teachers can devote during the school day to learn about and practice technology (Little and Housand, 2011). "Local teacher organizations' contracts and leadership more or less constrain the use of teacher selection to develop school learning community, and, more generally, the ability of school administrations to establish conditions of teachers' works that support collaborative practice" (McLaughlin & Talbert, 2006, p. 61). Online communities for busy teachers, whether new or seasoned professionals, can be sustained and effective only if they are well understood by the participants, supported by the administration, and deemed relevant to practice (Najafi & Clarke, 2008). Ultimately, in this era of a growing reliance on technology, teachers need to embrace it, interweaving it into practices and routines, the curriculum, and beyond (Lock, 2006).

STUDY RATIONALE

This study is important for two reasons. It *explores* the *problems* beginning teachers face of having to navigate through the traditionally otherwise closed doors of teacher isolation, and surviving their early years in the profession through the context of online community interactions. My hypothesis is that social networking can open the doors and ease the transition for new teachers through the ongoing dialogue and professional development communities.

It *frames* exploration research through the construct of a learning theory for the 21st century, connectivism analyzing how new teachers discover new knowledge and exchange ideas. Both reasons are discussed in this section.

The Increasing Value of Social Networking

My research attempts to address the growing importance of social networking in fostering teacher training and induction. This issue has been identified in the literature in recent years. However, social networks are still relatively new, and the timing of past studies has proven to be a factor for concern in their conclusions. The researchers, Moore and Chae (2007), suggested in their conclusion that new teacher problems may be addressed in using social networking for support the future, with more success, when "incoming teachers in the next few years will have no such barriers" (p. 223). More recently, a study published by Pew Internet, Pew and educational researchers Purcell et al (2013) suggests that the time to re-explore teacher's use of social media and other Web 2.0 technology may be now. They found that 92% of teachers surveyed say the Internet has a "major impact" on their ability to access content, resources, and materials for their teaching, while 69% say the Internet has a "major impact" on their ability to share ideas with other teachers.

The conduit of social networking can present a wide variation of experiences for the online participant as the community struggles to remain vibrant and relevant. Lock (2006) argued that if the purpose of social networking is to create meaningful interactions between teacher participants, then, the network must be accessible and an integral part of the professional learning environment. It must not be an "add-on." Since teachers are, by profession, not often required to working online, they must learn to recognize the online community as the place to go for building and sharing ideas. The community must be vibrant; ongoing discussions must be posted daily. Long lags or entirely asynchronous communication without variations of communication, such as texting, chatting, or even face-to-face encounters, can diminish the opportunities for learning and decrease the number of future visits participants make to the site. According to Matzat (2013), hybrid communities, which combine both face-to-face and online social interactions, can be extremely beneficial to teacher professional development since they bridge the gap between nuisances of online socialization and more traditional face-to-face meetings.

The open environment afforded by online communication in a social setting such as a professional development community strikes a dissonant chord with the often-isolating culture of teaching in Lieberman and Miller's (1984) early research on teacher communication. They refer to type of discourse in which teachers typical engage as "the rule of privacy," which allows teachers to talk face-to-face publicly, perhaps in the teacher's lounge, about weather, sports, or even "complain in general about the school and students," but refrain from discussions about their teacher practices and what goes on in their classroom (p. 11). To be successful and vibrant, professional online learning communities must evolve into environments where their participants are expected to share individual ideas and personal professional issues in an open forum. "Learning from colleagues requires

both a shift in perspective and the ability to listen hard to other adults, especially as these adults struggle to formulate thoughts in response to challenging intellectual content" (Grossman, Wineburg, & Woolworth, 2001, p. 973). Time and resources must be given to support and facilitate the online community. Most importantly, the community must be allowed to grow organically, at its own pace, attracting and retaining members because each sees a value in it, not because it has been imposed on them.

The literature shows that online communities, when properly implemented, can benefit teacher participants. Generally, learning communities and communities of practice (Lave and Wenger, 1991) have demonstrated how collaborative learning can assist new teachers as they grow in the profession. The overall goal of these communities, according to Schlager and Fusco (2003), is to design a technologically based (both on and offline) support community for teachers that meets their needs socially, emotionally, and professionally; a program that is endorsed and support by the district and welcomed by its participants. Schlager, Farooq, Fusco, Schank, and Dwyer (2009) showed how teachers provided moral support and encouragement for each other in an online community study. Moore and Chae (2007) believe that new teachers today, because, in part, of their technological age upbringing, already have "considerable proficiencies" in using online resources, a positive, familiar, and comfortable attitude toward technology, and access to technology. The era of online professional development for teachers, is upon us, led, perhaps, by the induction process of new teachers.

The Importance of Connectivism Theory

The theoretical framework of my study attempts to explore new teachers' use social networks and online communities through the lens of connectivism. Boitshwarelo (2011) wrote,

> Connectivism is receiving acknowledgement as a fresh way of conceptualizing learning in the digital age. Thus, as a relatively new instructional framework, it is imperative that research on its applicability and effectiveness in a variety of educational contexts is advanced. In particular, a high premium should be placed on context-specific research that is aimed not only at developing general principles but also at improving practice in local settings (p. 161).

Online communities, such as PD and communities of practice (CoP) as described by Wenger (1998), represent examples of conduits that embrace the connectivism learning theory. While the concept of a CoP is influenced by constructivist perspectives, especially within social learning, it also fosters social constructivist theories of situated and distributed cognition (Boitshwarelo, 2011). For the individual professional, a CoP can represent a node or connection in their network of knowledge. They can share with other professional their connections, and perhaps become aware of new connections in their own personal learning network.

Both social constructivist theories are essential to CoPs and connectivism. Situated and distributed cognition can be used to analyze community building with Web 2.0 technologies. On common ground with connectivism, social constructionists believe that the world is shaped by dialogue and discourse we share (Gunawardena, Hermans, Sanchez, Richmond, Bohley, & Tuttle (2009). Situated cognition is a fundamental tenet of CoPs (Boitshwarelo, 2011; Brown Collins, and Duguid, 1989; Lave and Wenger, 1991). Learning occurs through social interactions, using specific tools and activities. Knowledge becomes a product of the event and activity and cannot be separated from it (Brown et al., 1989). Distributed cognition is of greater relevance to connectivism. It states that no one person or device has all the information required to complete a task or solve a problem (Hutchins & Kirsh, 2000; Winn, 2003). Instead, the knowledge spans across various knowledge from several people and tools (Stahl, 2005; Hutchins & Kirsh, 2000). Hence, "knowledge is distributed among a community of people and devices" (Winn, 2003, p.341). The manifestation of connectivism as exhibited in CoPs provides a basis for exploratory research into how participants in these conduits allow the tenets of the connectivism learning theory to take its course.

PROBLEM CONTEXT

The challenges new teachers experience within their first year of practice, including isolation (Dodor, Sira, & Hausafus, 2010; Ingersoll, 2002; Lortie, 1975) and transition shock (Lortie, 1975; Rogers & Babinski, 2002) have been researched for well over a quarter century. Most research concludes that the culture of teaching, the nature of the profession, and even the structure of the school (Cookson, 2005) are, in part, to blame for the lack of opportunities teaching adults have to learn in collaboration with others. Several researchers in the current literature have experimented with the uses of online teacher professional development communities (Lock, 2006; Marzat, 2012; Schlager & Fusco, 2008).

Those who have specifically looked at structuring community around the needs of new teachers report outcomes such as, sample size too small (Taranto, 2011), too early in the use of technology (Moore & Chae, 2007), or the duration of the study was not long enough to draw conclusions on teacher retention (Fontaine, Kane, Duqette, & Savoie-Zajc, 2012).

Within the context of these studies, communities of practice (CoPs) have been the most commonly used framework (Wenger, 1998). The CoP structure serves as "an example of socially situated theory of learning where learning is seen as social participation and consists of four aspects: learning as community, learning as identity, learning as meaning, and learning as practice" (Concole, Galley, and Culver, 2011, p, 123). Moreover, according to Schlager and Fusco (2003), communities of practice are not only a context for the "inextricably entwined" context of professional practice and development, but are also a *social network* of individuals. The social aspect of community interaction, coupled with the individual "node" one develops from the experience, make the CoP a conduit for connective knowledge, or connectivism (Downes, 2012).

Connectivism integrates connective knowledge (Downes, 2012), social constructivism (Vygotsky, 1978), and social network theory (Barabasi, 2002). As a learning theory for the 21st Century, technology is a focal point in the learning process. Connectivism posits that learning occurs through connecting ideas, concepts, and perspectives in one's personal learning networks, which comprise various information sources and technologies, such as the Internet, databases, search engines, and online resources. Siemens (2004) describes the central tenets of connectivism as:

- Learning is a process of connecting specialized nodes or information sources.
- When specialized nodes and information source are composed of digital, electronic, online resources, learning and technology are inextricably linked.
- Learning, and consequently knowledge, rest in diversity of opinions.
- The ability to perceive connections between numerous perspectives, opinions, and concepts is central to learning.

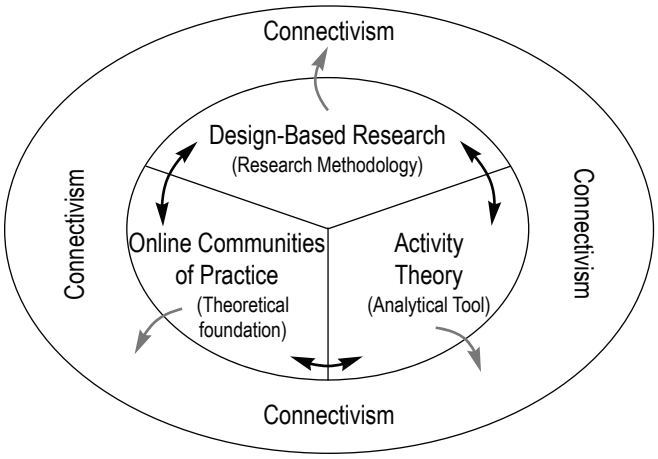

Figure 2. Boitswarelo's Theoretical Synergies Framework for Connectivism (Boitswarelo, 2011).

- Developing and maintaining connections is necessary to facilitate continuous learning.
- Evaluating information prior to engaging with that information is a meta-skill that is applied before learning begins. These evaluative decisions regarding the worthiness of learning specific information represent a learning process.

The process of learning requires building a personal network of strong, connected links between concepts and ideas (Dunaway, 2011). Interacting socially with others across technological networks helps to add nodes and strengthen the connections.

Boitswarelo (2011) states that "connectivism by its very nature is about connecting people for learning purposes and reducing isolation between stakeholders involved in the learning process." (p. 172). To advance the research agenda of connectivism, his model, a framework of synergies for conducting developmental research in connectivism, integrates three constructs, which address issues of culture, complexity, and context. Through his model, he is able graphically represent the constructs' relationship to connectivism. He addresses culture through designed-based research (DBR), which is qualitative educational ethnological research. He employs activity theory (AT) to address complexity. AT includes the relative theories of constructivism, cognitive, and social research. The context of Boitswarelo's observations is centered on communities of practice (see Figure 2). Using his framework, Boitswarelo maps learning ecologies of his research to connectivism using one of his education studies.

My study modifies Boitswarelo's model in a practical attempt to demonstrate how the three specific constructs I chose have a direct influence on aspects related to beginning teacher success: learning, knowledge, social capital and collaboration. I have chosen to incorporate connectivism as the DBR to address the complexity of my social research, NTC as its context, and Netnography to address cultural aspect of the research. It is through the methodologies and framework outlined in the model shown in Figure 3 that I will structure my research. To date, no research

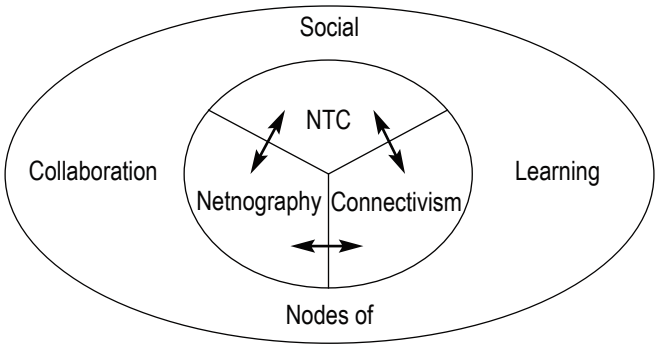

Figure 3. Modified **Theoretical** Synergies Framework.

in the area of beginning teachers' use of an online community to bolster their confidence and collaborate on practical work ideas and strategies has been structured on the two items that are designed for online community research: netnography and connectivism.

Connectivism, as with all theories, is not without critics. Some argue that the principles are not sufficient to consider it an entirely new learning theory. Driscoll (2000) regards a new theory as one that reinterprets previous findings and describes its irregularities within the new theory. Verhagen (2006) argues that connectivism is nothing more than pedagogical perspective. Kop and Hill (2008) suggest that it lacks enough empirical research supporting the hypotheses, and hence, has not moved from theory to education application research. Despite these objections, however, this new learning theory, at its core, is tethered to the strengths of other, more established anchor theories like constructivism and social learning, in an analytical design customized for online social networking. These theories will help to support the nascent connectivism theory in my research.

PROPOSED STUDY

I propose to conduct a netnographical study — a specialized form of ethnography adapted to the unique CMC environment of today's socially networked world (Kozinets, 2010) — as a nonparticipant/observer (Creswell, 2013), or complete observer (Norskov & Rask, 2011) — on the NTC eMSS program's online community. While distinctions between online social life and the social worlds of 'real life' are seemingly apparent, the two worlds are increasingly blending into one "world that includes the use of technology to communicate, to commune, to socialize, to express, and to understand" (Kozinets, 2010, p. 2). Hence, the nascent methodology, netnography, provides guidelines for "researching online communities and cultures, and other forms of online social behavior" (p. 3).

Ethnography focuses on qualitative research of culture-sharing individuals to learn more about their behaviors, beliefs, language and shared patterns (Creswell, 2013). Unlike grounded-theory, ethnography requires that people inhabit a specific location or visit it often enough to share culture. Typically, an ethnographer studies the culture as a participant/observing immersed in daily interactions observing and interviewing people. Ethnography evolved from the roots of comparative anthropology in the early 20th century through research by anthropologists, such as Margret Mead.

Netnography is a technologically based offshoot of ethnography that observes the behaviors, beliefs, and language and share patterns of people in online communities. As a method, netnography can facilitate, at a lower cost than ethnography, observations in a more online-culturally acceptable way, remaining unobtrusive and less evasive (Kozinets, 2010). This does not mean that the researcher must "lurk" online unannounced, but may not be invited to participate until trust and a deeper understanding of the researcher's intentions are established. To date, most research performed through netnography has been focused on marketing research. However, Kozinets (2010) sees the methodology as adaptable to other communities.

According to Kozinets (2010), the experience of netnographically studying a community online diverges from ethnography in several distinctive ways. First, unlike face-to-face encounters, online access, approach, and potential inclusion is, by nature, somewhat anonymous. Participation and observation take on new meanings online. Identities can be altered or masked (Angrosino & Rosenberg, 2011). Second, gathering and analyzing potentially large amounts of data have implications for challenges and opportunities. Lastly, "there are few, if any, ethical procedures for in-person fieldwork that translate easily to the online medium" (p. 5)

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Special Education for English Language Learners: Assessment and Diagnosis

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ABSTRACT

The number of English Language Learners (ELLs) in America's public school systems represented in special education has been disproportional for decades. Both over and underrepresentation of ELLs in special education is problematic since either can cause reduced access to the most appropriate academic environment. This paper examines these disproportions as well as the challenges that aid in creating them. A clearer understanding of the factors to be considered before referring an ELL for a special educational assessment will be addressed, as well as how to properly proceed with the referral. Furthermore, careful attention will be given to the protocol that should occur when new students who bring cultural and linguistic diversity to the educational setting are enrolled.

Census reports are showing increases in the English Language Learner (ELL) student population by a rapid rate in the United States. These unprecedented numbers bring a unique cultural and linguistic identity to the classroom (O'Brien, 2011). Educators need to become aware of these cultural and linguistic identities as a first step to overcoming the disproportionality in special education that is prevalent today. The fact that ELL students have been misrepresented in special education programs is testimony to the need for a clearer understanding of factors to be considered before educators consider referring an ELL student for special education services (Case & Taylor, 2005).

Overrepresentation in special education of ELL students has been a long-standing problem in schools (Sullivan, 2011). Previous research (Yates & Ortiz, 1998) also concludes that overrepresentation of ELLs in special education has been a prolonged issue. Current research, however, states that educators are doing a better job as a whole of identifying language obstacles from actual learning problems. Although statistics still show an overrepresentation of ELLs, the smaller number may be due to an underrepresentation of students that actually need the services, therefore creating a varied, adverse effect. Evidence was found by Zehler, Fleischman, Hopstock, Pendzick, and Stephenson (2003) that there was beginning to be underrepresentation of ELLs in special education. This is a change from the evidence of overrepresentation found with the research in earlier years.

Both over- and under-representation of ELL students are extremely important when considering disproportionality. Unfortunately, the interest has focused mainly on the over-representation in the past. Although this is indeed an area of concern, both over- and under-representation are questionable if they are the direct cause for reduced access to the most relevant forms of educational services. Whether such placement causes inappropriate placement in special education programs for students who do not need this particular support, or lack of support for students who would benefit and need this education provision, this is cause for reform. Allowing this to continue should cause great concern for educators and researchers alike (Linn & Hemmer, 2011).

Another burden is when ELL students are indeed found eligible to receive specialized services but with a lack of appropriately trained and/or bilingual special educators that can accommodate their particular needs. "English language learners who need special education services are further disadvantaged by the shortage of special educators who are trained to address their language-related and disability-related needs simultaneously" (Ortiz, 2001, p. 1). It has become vital to train teachers of students in special education who are also ELLs on best practices for this population at the very least.

The majority of the time, ELL students without disabilities will in fact demonstrate an increase in English language proficiency if appropriate instruction, modifications, and if needed, interventions are present (Downer, Grimm, Gamagami, Pianta, & Howes, 2012). If language proficiency and general progress are delayed, several factors should be taken into account before an assessment is warranted in determining learning obstacles for a student whose first language is not English. Educating school teaching staff to better understand when it is appropriate to refer an ELL to special education and then be able to distinguish a learning disability from language differences is essential.

This report is intended for school administrators, teachers, and parents to aid in knowing and providing appropriate services for our ELL population. These services include best instructional and environmental practices, when a referral for special education is warranted, and the guidelines for the actual referral and assessment. This research-based document will assist in avoiding (a) unnecessary diagnoses of ELLs in special education as well as (b) neglect of diagnoses for ELLs who in fact need these specially designed services.

THE ROLE OF CULTURE

Research by Taylor and Whittaker (2009) has confirmed the assumption that it is crucial for teachers to not only understand the cultural differences that students bring into the classroom, but also embrace these differences. It is of utmost importance to make these students feel valued and their families feel welcome. In a study by Roxas (2011) research was conducted on how to foster a sense of community for students new to this country. Roxas concluded that in order to make students and parents feel as though they are part of the educational community, an inviting, multicultural classroom space is vital. As the sense of community increases, students' motivation will also increase, therefore guiding them to do well academically, according to Roxas. Teachers must gain background knowledge of their students in order to effectively teach them. One way to accomplish this is through classroom meetings that encourage dialogue, individual expression, and concerns that can be addressed in an open but immediate manner (Roxas, 2011).

It is equally necessary and valuable to take into consideration the cultural background of all parents/families with which a student resides. The particular customs or beliefs amongst our culturally diverse families need to be realized when it comes to the education of their children. For example, the fact that parents actually may disagree with a school's concerns or recommendations, or actually express concerns themselves, may conflict with their belief system in regards to group harmony. As a result, some parents may choose to remain silent during important meetings or conferences and simply appear to agree with all recommendations by the educational system (Artiles & Ortiz, 2002). According to Westby (2005), culture can influence the ways in which language proficiency develops as well as how it is actually used during interaction with others.

THE ROLE OF LANGUAGE

Research suggests that a child new to the English language may develop basic communication skills (BICS) after 1 to 2 years, but it may take 5 to 7 years to acquire cognitive academic language proficiency (CALP) (Cummins, 2008). When appropriate strategies are not implemented to improve a student's lack of proficiency in the language of the classroom setting, children are not provided with an opportunity to understand what is being said or express what they need to say. Unable to work at full capacity, learning struggles then become similar to that of children with disabilities (Barrera, 1995). Schools must recognize that with research-based instruction and practices for ELLs in place, most students will need a minimum of four years before they are ready to meet the full demands of general education without scaffolding and support (Taylor & Whittaker, 2009).

THE ROLE OF THE LEARNING ENVIRONMENT

Given the abundance of factors that influence language acquisition, all teachers must plan instruction that is comprehensible and meaningful in a low-anxiety environment (Taylor & Whittaker, 2009). The best instructional strategies that should be evident in an ESL or general education classroom (e.g., sheltering content for students whose first language is not English), should include the following evidence-based practices:

- Use manipulatives to reinforce new concepts
- Use thematic instruction
- Use cooperative learning
- Provide visuals-gestures, props, graphic organizers, charts and a labeled environment
- Incorporate hands on activities, whole language activities and learning stations/centers with varied ability levels
- Provide a variety of texts and resources or curriculum topics at a range of reading levels
- Allow plenty of "wait time" before allowing students to answer/yell out answers
- Provide models of completed homework assignments, projects, etc.
- Ensure tests and assignments are written in clear concise language and are easy to read
- Modify if need be
- Recycle new and key words
- Check for comprehension (e.g., use questions that require one-word answers, props, or gestures)
- Build upon cultural realities. Create an inviting classroom space that is multicultural
- Ensure students have all necessary materials (e.g., binders, notebooks, textbooks, handouts, etc.)
- Provide student with a peer mentor
- Seat the student near the teacher.
- Print clearly, do not use cursive, and give instructions orally (use pictures, photos, or drawings when possible)
- Print key words, page numbers, homework and deadlines, etc. on the board
- Provide reading lessons rich with vocabulary, phonics, illustrations, guided instruction, visuals, and multi-sensory opportunities

- Opportunities to share and speak in a non-threatening environment (Batt, 2008; Birman, 2007; Boyle & Pergoy, 2001; Brooks, Adams, & Morita-Mullaney, 2010; Cummins, 2008; Freeman & Freeman, 1998; Hansen-Thomas, 2008; Lightbown & Spada, 2011).

It is important to note that typically, developing ELLs who are exposed to warm, sensitive, well organized, and cognitively stimulating interactions with adults will progress in their new language at an acceptable rate (Downer, et al., 2012). By avoiding unnecessary remedial classes and paying careful attention to language development, decoding skills, concept expansion, and thinking process that are so crucial to learning to read, the classroom teacher can help the ELL student become truly proficient. This proficiency can happen if research-based teaching of ELLS occurs.

PROTOCOL FOR REFERRAL

Often, a child is referred too soon for special services without adequately providing the above recommendations and therefore not demonstrating the adaptations made or awareness of what that particular child needs in his or her school environment (Brown, 2008). Another worrisome situation occurs when referrals are actually placed on hold to allow for the child to adapt to their new language and culture *without* providing evidence based practices in the learning environment. According to Brown (2008), culturally sensitive responses to intervention (RTI) when an *appropriate* learning environment alone is not allowing the student to make adequate language gains is also essential. The RTI approach provides assurance for several reasons. Since the universal screening and progress monitoring called for in the RTI process actually allows for comparison of students to other "true" peers, results are more valid. Also, an effective RTI model requires collaboration among *all* educators, therefore giving more opportunities for dialogue, peer coaching, and integrating best practices (Brown, 2008).

Some factors that may help in determining whether or not a child is lagging behind academically because of language alone, or because of the possibility of a disability, are listed below.

Ralabate and Klotz (2007) list the following as common errors in English language due to language unfamiliarity:

- Words not structured correctly
- Words not verbalized correctly
- Words with incorrect meaning
- Errors in use of plurals
- Incorrect word order (i.e., misplaced verbs or articles)
- Poor subject-verb agreement
- Incorrect verb tense
- Errors in use of articles "the, those, these, a" with nouns
- Limited use of age-appropriate vocabulary
- Incorrect use of prepositions
- Confused placement of words or phrases.

Ralabate and Klotz (2007) list the following as common errors in English language due to learning disabilities:

- Confused sequencing when relating an event
- Lack of interrelatedness of symbols or objects
- Poor organization or sentence structuring
- Delayed responses or reactions
- Poor topic maintenance
- Difficulty maintaining attention
- Poor memory.

A formal referral for a special educational assessment may be justified after it has been determined that a student's academic performance and/or behavior cannot be explained solely by culture, language, or learning environment alone. If a peer analysis also shows signs of learning struggles beyond the realms of these factors, then a referral should not be postponed. A peer analysis can be critical in deciding if the student's performance is anomalous. Optimal peer groups are typically ELLs with the same language background, same amount of time in the program, and in the same school grade. It is vital to search district data and find as large a peer group as possible (Office of Superintendent of Public Instruction, 2009).

THE ASSESSMENT PROCESS

After it is determined that an assessment is in the best interest of the student, the following protocol should be completed once the assessment process begins:

- Create a process with a multi-disciplinary team: Special Ed "best friend," content and/or grade-level teacher, administrator, ELL staff.
- Get approval for the process and communicate it often to all staff.
- Avoid an overwhelmingly complex process if the majority of referrals are based on simple misinformation.
- Parents need to be contacted early in a language they understand regarding the teacher's concerns.
- Parents need to be educated about language development and differences between siblings, and the role of first language literacy. (Office of Superintendent of Public Instruction, 2009)

- Information about the student's language must be considered in determining how to conduct the evaluation to prevent a student from being misclassified. An ELL student must be assessed in the student's proficiency in English and the native language proficiency in reading, writing, speaking, and understanding. (Massachusetts Department of Education, 2001)

It is important to note that cultural bias is evident in the special education assessment process in America's public school systems as stated by Lynch and Hanson (2009). Although this is not intentional, the instruments designed for this purpose long ago are still used today. These materials were created with native English speaking students in mind. These particular assessments are not always applicable for ELLs or their families, nor are they suitable for those with different cultural practices. Assessments that are not available in the student's native language cannot be assumed to be appropriate. "If formal assessments are used, then they should be conducted in the child's primary language, and missed items especially related to vocabulary in one language my need to be repeated in the second language" (Lynch & Hanson, 2009, p. 458).

Findings from the Minority Research Institute concluded the following:

- Language proficiency is not always taken into account in special education.
- Testing and assessments are completed primarily in English.
- English-language obstacles that are normal for second-language learners are misinterpreted.
- Home data is not considered during the assessment process.
- The same English-dominant tests are used with most children.
- Individual education plans for students who do need the services have few, if any, accommodations for bilingual children.

These pitfalls need to be avoided during and after the assessment process so as to provide ELLs and their families the most valid data possible when results are reviewed to determine eligibility for special services.

OUTCOME OF ASSESSMENTS

Once the assessments are appropriately completed, they must demonstrate that if a disability is found, it is evident in the dominant language. Descriptive data, not test scores, should be the main factor in deciding if the student qualifies for special education services. Therefore, standardized test results used must be cross-validated with performance-based measures. If both performance indicators present as low and parents are also concerned about their child's communication skills or academic progress, then the student most likely has a disability (Artiles & Ortiz, 2002). Proper placement in special education services along with continued exposure to best practices in a language rich, general education and English as a second language (ESL) environment should also continue. Since it is in the student's best interest to receive special education and ESL services, these service providers along with the general education teacher should work in a collaborative manner to address the student's educational objectives and goals for adequate progress.

CONCLUSION

To best serve all students, it is logical to ensure that they are placed in the most appropriate environment for their particular needs. Often, the complication of language, culture, and assessment for our ELLs can hinder efforts to achieve this proper placement. Appropriate placement hinges on effective evidence based instruction, culturally sensitive intervention models, and effective assessments if needed, to determine if slow rates of academic progress are a product of language issues, learning disabilities, or other factors. Children truly in need of proper ESL or special education services that are not exposed to the appropriate practices are forced to have valuable educational service time wasted. Furthermore, a child who is capable of developing English skills at an appropriate rate and therefore capable of making steady progress is failed of this opportunity if best practices are not in place. This raises the chance for them eventually being placed in special education unnecessarily because of the lack of progress being made.

Research has shown that (a) some placement errors can be attributed to testing limitations, and that (b) testing in the student's native language yielded better results (Artiles, Rueda, Salazar, & Higareda, 2005). The ability to properly assess can be a problem if there are limited staff members qualified to test the student who speak the students' native language. Failure to complete assessments in the ELL's native language and misinterpreting the data are the main reasons ELLs are placed into or denied special education services inaccurately. Another factor that can influence the services received by ELLs is the views of the team and administration making the decision, given the pressure on schools to meet particular goals on high stakes testing. Policy analysis is the next logical step in this area of need. Research needs to be examined between the national and state initiatives and the representation patterns of ELLs in special education (Linn & Hemmer, 2011).

It is disturbing that research from a decade ago had similar concerns as more recent studies of today. Although one variable appears to be less overrepresentation and more underrepresentation, this still creates disproportionality. Reasons for disproportionality from previous research to current research (Zehler, et al., 2003) appears to be evident because of similar factors. These factors include inconsistencies in pre-referral strategies, inadequate classroom instructional practices and assessment protocol, as well as inappropriate assessment materials. These issues still need to be addressed in many of our schools today. As previously stated, proper identification, classroom instruction, intervention if needed (Downer, et al,. 2012) and valid identification of English language learners in need of special education services across America is more important now than ever before. With high stakes testing required for graduation, accountability factors for teachers and students alike, and college and career readiness skills more demanding and competitive than ever before, we cannot let this special but growing population down any longer.

Although best practices and cultural sensitivity *should* be an easy fix, it is important to be consistently implemented. The challenge lies with culturally and linguistically appropriate assessment materials and the trained/bilingual staff needed to facilitate the assessments. After decades of challenges in this area, the culturally sensitive and dominant language tests should be created immediately. Although there is still the challenge that lies with the lack of appropriate staff, recruitment strategies should be at the forefront of this plan, which is in immediate need. Translation services should also be sought. It will be a disgrace if a decade from now more current research continues to show disproportions of ELLs in special education. As Ludwig Wittgenstein (n.d.) stated, "The limits of my language are the limits of my world" (1).

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Educational Resources

The Hierarchical Structure of Learning

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ABSTRACT

This paper explores the strengths and limitations of Robert Gagné's (1916-2002) views with respect to learning. Gagné supports a theory of learning based on the concept of a hierarchical grouping of (subordinate) topics, the mastery of which results in the development of knowledge. It is Gagné assertion that not only is command of these topics important, but the sequence in which they are learned is critical as well. Gagné's theories have implications in today's classrooms, from the creation of curriculum to the assessment tools used for evaluation. A critical evaluation of Gagné's opinions serves to enrich educators' notions on the nature of learning.

Theories surrounding the development of knowledge have been the focus of philosophers, educators and psychologists for centuries. Many suggest that the acquisition of knowledge is built upon increasingly complex substructures. These sub or base structures fall under the constructs of declarative and procedural knowledge as defined by Anderson (1994), or are viewed as subsets of tasks that need to be learned before more complex sills are acquired (Gagné, 1973a). Anderson (1994) defines declarative knowledge as the subset of skills that contribute to larger, more complex skills, which in turn contribute to procedural knowledge. Gagné (1973b) explores similar ideas in his writings on the importance of subordinate intellectual skills on future knowledge development. These writings form his theory of "hierarchy of subordinate knowledges" (p. 160).

Robert Gagné (1973a) suggests that learning is a fluid process (p. 107). He recognizes the importance of applying principles of learning to curriculum development. However, he asserts that current rules applied to curriculum may need to be changed as we continue to study the processes of learning. Gagné rejects the prior views of learning as occurring via connections between stimuli and responses (p. 108). In using this method of instruction, Gagné states that a teacher faces the challenge of creating a lesson that presents the stimulus and response at the same time as well as provides ample opportunity for repetition (p. 109). In Gagne's view, the limitation of this method is that it does not account for the role of memory in learning. In the stimulus-response method, memory or retention is thought to occur as a direct result of repetition, not as a result of cognitive associations between prior knowledge and new material. In the stimulus-response model, the greater the repetition of the material the greater the retention. Retention via this method is thought to occur through strengthening the connections between the stimulus and response.

Gagné (1973a) supports a theory of learning that challenges the stimulus-response model (p. 111). He rejects the need for repetition in exchange for the view that learning occurs in a sequence whereas baseline knowledge must first be attained before a new skill is acquired. Gagné (1973a) thus states that "The most dependable condition for the insurance of learning is the prior learning of prerequisite capabilities" (p. 111). If such conditions for learning occur, it is Gagné's view that new skills can be learned without repetition. He supports his view that a student will find success if he has the prerequisite skills in stating that "any particular learning is not at all difficult if one is truly prepared for it" (Gagné, 1973a, p. 111). By being prepared for this learning, Gagné infers that the student has obtained the prerequisite knowledge required in order to learn the new topic. Thus, an individual learner cannot progress through the hierarchy of knowledge until subordinate topics are mastered. The key issue here is that if there is a gap in the knowledge of a subordinate skill, then acquiring the knowledge of the superordinate topic will not occur (Gagné, 1973b, p. 161).

Another way in which Gagné (1973a) challenges the practice of repetition as a memory aid is by exploring the role of coding and retrieval on memory (p. 113). One can see here how his ideas are influenced by the information processing theory of learning. Gagné asserts that it is not enough to simply practice or repeat things once they have been learned. He posits that, in order to truly retain and remember information, it must be retrieved from memory (p. 113). Gagné also explores how cues can affect this retrieval of information. The cues he refers to include grouping information into categories in order to aid retrieval. This is significant as Gagné posits that "remembering is markedly affected by retrieval at the time of recall, more than it is, perhaps, by events taking place at the time of learning" (p. 113).

Gagné (1973a) rejects the view of student as a passive recipient of instruction (p. 113). He suggests that education should arouse innate abilities by pulling from prior knowledge to advance to the next skill. The process by which performance or skill is broken down into the subordinate topics of this prior knowledge is defined by Gagné and Medsker (1996) as "Task Analysis" (p. 154).

Gagné's (1973b) ideas on the role prerequisite capabilities plays in knowledge acquisition mirror similar ideas by Anderson (1994) with respect to the importance of declarative knowledge as the building block of knowledge. Both men suggest that skill development occurs as a systematic process starting with smaller units or skills, (declarative knowledge or prerequisite capabilities) and building up to more complex understanding and skills (procedural knowledge or new skill). Therefore, in addition to applying the task analysis theory to learning academic concepts, Gagné & Medsker (1996) also relate it to the acquisition of motor skills (p. 100). Gagné and Medsker's (1996) position on this topic is that in order to learn a new motor skill, component motor skills must have previously been learned. If the component skills have been well-learned, then it is simply a matter or incorporating them into the new motor skill (p. 100). However if the component skills are not well-established, more practice will be needed before aptitude of the more complex skill can be achieved (p. 103). This is very similar to Anderson's (1994) example of the procedural knowledge inherent in driving a manual transmission. Anderson (1994) posits that, in addition to knowledge of the prerequisite motor skills for driving (declarative knowledge), repetition and practice (forming procedural knowledge) are needed to make the process of driving a manual transmission an automatic skill. Gagné and Medsker (1996) build upon Anderson's (1994) assertion that repetition and practice are important. They state that repetition "must be accompanied by (1) intent on the part of the learner to achieve an improved performance, and (2) informative feedback to the learner." (Gagné & Medsker, 1996, p. 103). These authors agree to the theory that if these two conditions are not present, the motor skill will not be mastered (Gagné & Medsker, 1996, p. 103).

In addition to the role of practice and feedback on the acquisition of new skills, Gagné (1973a) values the role of retrieval in the learning of a new task (p. 114). He therefore suggests that retrieval strategies should have a place in instruction (p. 114). Gagné links the role of these strategies to the network of superordinate categories created via the process of learning. In his view, the process of searching through these categories aids in retention. Although he rejects the need for pure repetition to enhance learning and retention, he does advocate "periodic and spaced reviews" (p. 114) of material. He does not provide a clear explanation of how or why these reviews improve learning, however, he values their benefit in requiring a student to "exercise his strategies of retrieval" (p. 114).

Gagné's ideas have significant implications in higher education. Instructors in this field are often challenged, especially in first year courses, by students who come from different backgrounds and experiences. It is very difficult for an instructor to know what prerequisite capabilities students possess. This becomes problematic when instructors must attempt to find the most common point of base knowledge before proceeding. Although the value of doing so is evident, some students who already have background knowledge of the subject area will become bored, while the information may still be too advanced for others. In fact, Gagné (1973a) advocates the use of diagnostic testing to precisely determine which prerequisites for learning are present in each individual (p. 114). Although this appears logical, when reading Gagné's work, it is also impractical when trying to apply it to practice. The use of diagnostic tests may be of value to group students by ability in the lower grades. However, at the university level, aside from basic skill assessment (at some universities), diagnostic testing is not used to group students within or across courses.

One of the major limitations of Gagné's theory in the university setting is that instructors would first have to create a hierarchy of skills that are prerequisites to the skill they intend to teach (a task analysis). The instructor would then have to develop a method to assess those skills and then ensure that all students have them before moving forward. This process would be incredibly time consuming and there is little evidence to support its effectiveness.

Gagné (1973b) attempts to provide evidence to support his theory of learning by a variety of experiments designed to investigate the utility of programmed learning materials (p. 159). In these experiments, the learning programs provided were designed to convey new material, yet assumed that subordinate knowledge was present. The experiments measured task performance, transfer of knowledge, and subordinate knowledge. The results of these experiments supported his hypothesis that students cannot learn a topic if their knowledge of a subordinate topic is incomplete (p. 163). Therefore, Gagné concludes that, with respect to individual differences, the most critical variable in learning is the possession or lack thereof of subordinate knowledge. He suggests that intelligence, on the other hand refers to the speed at which a learner moves up the hierarchy (p. 163).

Gagné (1973b) goes on to make an important statement that more than one hierarchy of subordinate topics can be utilized to learn a final topic (p. 165). With this assertion, Gagné reiterates his idea that learning takes place by moving up this hierarchy of knowledge. He does not attempt to specify the subordinate topics for specific subjects

because they may not always be relevant. His theory primarily focuses on the overall concept that learning is acquired via hierarchical steps.

Gagné's (1973b) views expose the interesting possibility that several, equal subordinate topics may play a role in moving a student to the next level of knowledge acquisition (p. 163). Gagné puts forth a hypothesis that topics higher up the hierarchy cannot be learned if any of the subordinate topics are not mastered (p. 163). This leads one to ask what might happen if only one out of two subordinate topics are mastered. In other words, can one topic have two subordinate topics that weigh equally? Thus, if only A is learned, can C be learned based solely upon the knowledge of A or will B also have to be mastered in order to move up to C (see Figure 1)? In Gagne's model it appears that C can only be learned if both A and B are mastered, but how does one decide the weight of these two topics (A & B)? The assumption is that they're equally weighted, but are they really? Can two separate topics have the same weighted influence on subsequent knowledge? If so, how would one measure the weighted values (if any) on these influences?

In Gagné's (1973b) figure of a learning program, the arrows only go in one direction (Figure 17, p. 162). Does this imply that knowledge can only travel in one direction? Might it be possible to master a more "advanced" topic in the hierarchy allowing a learner to attain an improved understanding of a subordinate topic?

In relation to his hierarchy of learning program, Gagné (1973b) creates two categories of variables involved in the process of learning mathematics (p. 161). The first of these is instructional variables which are presented when a novel topic or principle is being learned. What is interesting here is that Gagné finds a use for repetition by adding it to this category. As indicated earlier in this paper, Gagne (1973a) stated that if subordinate tasks are solid, repetition is not needed for mastery (p.113). However, in discussing his mathematics learning program, Gagné (1973b) advocates for the role of repetition in learning subordinate tasks (p. 164). In this situation, he suggests that providing repetition (via repeated, guided examples) of subordinate topics aid in the acquisition of the target skill (p. 164).

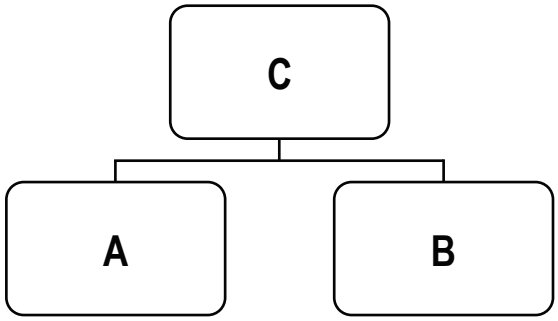

Figure 1. Relationship of subordinate categories (A&B) to learning outcome (C).

The second category Gagné (1973b) addresses is that of topical order (p. 161). Topical order refers to the appropriate sequence in which subordinate topics must be learned in order to accomplish the learning goal (p. 161). When moving up the hierarchy, mastery of subordinate topics must be achieved through the "definition of terminal performances, activation of recall, and guidance of thinking" (Gagné, 1973b, p. 162). Gagne advocates utilizing methods from both categories when creating a learning program for mathematics.

If one were to use Gagné's theory to direct his teaching methods, he would have to be comfortable with some certain assumptions. In Gagné's (1973b) hierarchy, knowledge is broken down into subordinate skills (p.162). This makes the assumption that one can correctly identify the true subordinate topics necessary for learning a new skill. Identifying these topics may be more easily accomplished in simpler tasks, but for complicated tasks, identifying the subordinate topics could prove to be more difficult. Uncontrollable variables such as individual learning styles, level of maturation and recall play important roles as well. One could also argue that at some point, all subordinate tasks are novel. How does one therefore determine the most basic task in a hierarchy of items? Also, how is that first, novel step learned? Advocates of discovery learning might suggest that learning this first step is where discovery learning has its greatest impact.

Although Gagné is not an outright sponsor of discovery learning, he did perform a study which supported the role of guided discovery on knowledge acquisition in mathematics (Gagné & Brown, 1961). The purpose of this study was to measure the effectiveness of programmed learning materials on understanding how to state and use formulas for summation within number series. In this investigation, the subjects included boys who were between 14 and 16 years. They were divided into three groups (of 11 students each): Rule and Example (R & E), Discovery (D) and Guided Discovery (GD). The R & E group was given a formula to solve the problem, the D group was left to attempt to determine the formula on their own, and the GD group was given successive steps to follow which helped guide them to the correct formula. The subjects were trained in their method over two consecutive days and at the end of the second day performed a test measuring time and accuracy in determining a rule for a novel number series. Although all groups improved, the results showed that the most effective condition at improving time scores for problem solving was the GD condition. The R & E condition was the least effective and the D group fell in between. Given that the steps created for the GD condition followed

a hierarchical path, these data support Gagne's view that learning new skills is based upon the mastery of subordinate skills. However, as the study was constructed to evaluate the discovery component of learning as well, the results supported the role of both structured and discovery learning in understanding number sequence.

In later writings, Gagné (1973b) expresses confidence about his theory with his assertion that, irrespective of ability scores before participating in his learning program, if given enough time, the performance of all learners will be independent of these scores (p. 163). This is a powerful statement advocating the value of teaching mathematics by his suggested method.

Gagné's theories have implications in today's classrooms. In his theory, if a student is not taught the prerequisite skills, he will not be able to move on to the next skill. If one subscribes to the ideas inherent in Gagné's theories, he would structure his classroom in such a way to ensure subordinate skills were mastered. The task therefore in teaching in accordance with Gagné's ideas is to identify these skills and to create methods to teach them to students in the appropriate order.

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Pieces, Parts, and Quotients: The Language of Fractions and Fractional Numbers

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ABSTRACT

Rational number is a key component of mathematics curriculum, yet it represents a great challenge to teachers and students alike. The introduction of rational numbers through fractions in grades three and four is especially difficult for students. This paper reviews the historical development of the concept and notation of rational numbers and fractions, as well as the role rational numbers play in mathematics as a discipline. In addition, the teaching and learning of rational number are reviewed. Research describing how children demonstrate their knowledge of rational numbers and the language they use to describe rational numbers is discussed. Research focused on the methods used to teach rational numbers, including the language that is used, is also discussed.

IMPORTANCE OF THE PROBLEM

In 2008, the National Mathematics Advisory Panel (NMAP) published recommendations for better preparing American students in mathematics with a focus on preparing them for entry into algebra at the secondary level of their education. One of the recommendations was fluency with rational numbers. Panasuk (in press) claims that to be specific and adhere to correct and precise mathematics terminology is vital to teaching and learning about rational numbers. She asserts that "the definition for a rational number and the definition of a fractional number have too often been perceived and used incorrectly"(in press).

She explains, "A fractional number is *a number* that symbolically expresses part of a whole as a quotient of two integers, and can be presented in the form *a/b*, where *b* is not zero. Quotient is a result of division, and integers are all whole numbers including zero (….-2,-1,-0,1,2,3….). So, the quotient of integers is a number (e.g., 3/5, 6/7, 11/9)" (in press).

Panasuk further claims that "Unfortunately, the term 'fractional numbers' has been incorrectly replaced by the term 'fraction' in many textbooks, and thus in many classrooms. Fraction is not a number, it is region or a section or a portion of a whole. A portion cannot be a number" (in press). She gives this definition of a rational number, " A rational number is *any* number that can be presented in the form *a/b* , where *b* is not zero. This would mean that any non-terminating repeating decimal (e.g., 2.53535353…..) is a rational number, as well" (in press). Panasuk compares these definitions by stressing that 'every fraction is a rational number, but not every rational number is a fractional number" (in press).

The multifaceted concept of rational number itself and its sub-concept, fractional number, inevitably lead to certain difficulties when teaching and learning the concept. Both empirical and anecdotal evidence show that success in the teaching and learning of rational numbers has been hard to achieve. Scores from standardized testing reveal that many American students, even high school students, seriously lack knowledge in rational numbers (Siegler, 2013). The National Assessment of Educational Progress indicates that children in grades four, eight and twelve all have difficulty with mathematical tasks involving rational numbers (National Center for Educational Statistics (NCES), 2009; NCES, 2003). In 2003, 40% of eighth graders were unable to identify an equivalent ratio, and 33% of eighth graders were unable to locate ¾ on a number line (NCES, 2003). In 2009, only 55% of fourth graders could correctly match a visual representation of a rational number to a given rational number (NCES, 2009). These results provide support for anecdotal evidence from teachers in the United States who claim that their students continue to struggle with the tasks related to rational numbers

In addition to the complexity of the rational number concept itself, it has also been suggested that the cause of such struggle may be the students' level of cognitive maturity. Brown and Quinn's (2007) review of the research on rational number instruction summarized that rational numbers have been taught before students are cognitively ready for symbolic manipulation of rational numbers. If algorithms related to rational numbers are taught prior to the student being cognitively ready, it is possible that students merely memorize the algorithm rather than develop conceptual understanding. Furthermore, Brown and Quinn (2007) indicate that even those students who were taught under "optimal conditions" did not demonstrate understanding of rational number operations, and argue that instruction on rational numbers should be delayed until students have developed more formal reasoning skills (p. 23).

Bezuk and Bieck (1993) assert that ordering rational number and the concept of equivalence taught during the late elementary grades are often presented in a "superficial" and "meaningless" manner (p. 119). It is possible that curriculum materials or methodologies force teachers to assume that the elementary school children are operating at a "formal operation" stage of reasoning (Piaget, 1969). However, Brown and Quinn (1993) state that most late elementary school students are at the concrete operational stage (Piaget & Inhelder, 1969). According to Piaget and

Inhelder (1969), at the concrete operational stage, children are in a process of development of reversible reasoning, conservation of matter, skills in seriation and classification, and thus likely are able to understand the part-whole relationship, however it is unlikely that they are able to comprehend proportional reasoning (pp. 102-104).

Regardless of research findings, the Common Core State Standards in Mathematics (2012), which have been adopted by many states, suggest introduction to fractions and fractional numbers in grade three, which seems well before these students can be expected to possess formal reasoning skills. By the end of grade three, students must be able to recognize a fraction as a part of a whole as well as recognize a "fraction as a point on a number line" (Common Core, 2012, 3.NF.A.2). Panasuk (in press) suggests making a clear distinction between 'fraction' as a portion of a whole and 'fractional number' as a symbolic representation of the portion of whole, which in turn can be represented as a point on a number line. Such categorization might help the teachers and the students to comprehend the nature of both concepts as well as the difference between part of a whole or a region of a figure and its numerical representation as a point on a number line.

By the end of grade four, students must be able to "compose and decompose fractions" (Common Core, 2012, 4.NF.3.b), perform some basic computation with fractions, and compare two fractions to determine which of them is larger (Common Core, 2012, 4.NF.A.2). Apparently, the Common Core State Standards use the word 'fraction' to refer to both the "part-whole" concept (i.e., partitioning an object) and to the division concept (i.e., arithmetic operation with symbols).

The issue with teaching and learning rational number concept and its sub-concept, fractional number, is not unique to the United States. Siegler (2013) describes the global aspect of this educational problem, noting that teachers in Japan and Korea report that their students also struggle to comprehend rational numbers. Stafylidou and Vosniadou (2004) investigated the learning of rational numbers in Greek children aged 10 through 15, and found that slightly more than half of first year Lyceum students (i.e., about 15 year olds) could correctly order rational numbers. Furthermore, Siegler (2013) states that many students at the college level still have a poor understanding of rational numbers. Even adults who appear to demonstrate knowledge of rational numbers may, in fact, lack conceptual knowledge of these numbers (Bonato, Fabbri, Umilta, & Zorzi, 2005). The research indicates that it is challenging to teach rational number skills and promote conceptual understanding. Whether rational numbers are introduced too early in the curriculum, or there is a problem with the language used in the classroom, both need to be examined. The following section is focused on the language used to teach the concept of fractional numbers, and analysis of possible conflict between the nature of the concept of fractional number and the concept of fraction.

STATEMENT OF THE PROBLEM

Over the long history of the development of, and the learning and teaching of mathematics, it has been generally assumed that success in learning rational numbers in general and fractional numbers in particular, has been linked to success in learning the subject, including the learning of algebra and calculus. The roots of the word *fraction* have its origin in Latin verb *frangere*, meaning 'to break'. The Online Dictionary of Etymology states that the origin of the English word *fraction* comes from Anglo-French word, *fraccioun* meaning 'a breaking, especially into pieces'. Outside the mathematics classroom, the word *fraction* is casually used to describe a part of an object (e.g., half pound of apples) or a piece of something (e.g., a fraction of time). In elementary mathematics classroom, the word *fraction* is used to address both a portion of a whole object or a set of objects or a fragment of something and a number, which is a result of division of two whole positive numbers.

It is very likely that when the young children are reasoning about partitioning and fractional quantities, they are thinking either of sets divided into equal groups, or a whole item divided into equal pieces. However, this everyday language does not capture clearly enough the essence of the idea of fractional number, which is a symbolic representation of division. This conflict between the everyday language and the mathematical meaning of the result of division may greatly affect students' development of the concept of the fractional number in grades three and four. Also, it can be particularly challenging to teachers who have been influenced by the methods espoused by some authors who encourage informal and colloquial (i.e., not rigorous and accurate) talk about mathematics in the classroom.

The current curriculum introduces the rudiments of the rational numbers in grades three and four. Given the complexity of the concept of rational number and its sub concept, fractional number, combined with the fact that many textbooks and methodological recommendations use informal colloquial language that opens for misinterpretation and breeding misconceptions, it seems imperative to investigate the issues related to the language (i.e., terminology) used by teachers to introduce fractions and fractional numbers One can hypothesize that the use of informal rather than rigorous language may have detrimental effects on student learning and comprehension of fractional numbers and fractions. Thus, it is important to study the nature of the language related to the concept of fractional number and how it is integrated in

teaching. It is also important to understand the degree to which the language used to teach the concept of fraction and the concept of fractional number promotes student learning of the fractional number concepts, including definition, magnitude, and equivalence.

PERTINENT AREAS OF RESEARCH

This paper will address the historical development of the concepts of fractions, fractional numbers, and rational numbers, and their importance in mathematics. In addition, this paper will discuss the definitions currently used in standards documents and textbooks, as well as the type of language that is used in these definitions and the assumptions that are made by the writers of the definitions. Finally, I will discuss the issues related to fractional number learning and teaching in the elementary school. Theories related to the cognitive development of children and their connection to mathematics will also be reviewed (e.g., Geary, 1994; Vygotsky, 1978), along with the methods used to teach children about fractional numbers (e.g., Empson, 1999; Sgroi, 2001; Sheffield & Cruikshank, 2001).

STRAND ONE: MATHEMATICAL DEFINITIONS

The importance of accurate and clear definitions in mathematics cannot be understated. Without clear and precise definitions of terms, students and teachers are both placed at a disadvantage. According to James Milgram (2005b), there are two key components of mathematics, the first is "precise definitions of all terms, operations and the properties of those operations" (p. 3). This can be clearly seen in the case of fractional numbers, which Milgram claims are frequently not defined for students, instead being "fuzzily" defined as part of a whole, a ratio, or a quotient (p. 28). Milgram asserts that it is the reliance on these ill-defined ideas that lead to students procedural misconceptions. Therefore, the first part of this paper will focus on the definitions of fractions, of fractional numbers, and of rational numbers. While it is recognized that the uses of the fractional notation used with irrational numbers is important, it is outside the scope of this paper, with the exception of when and how they are included in the definition of fractional numbers.

The Importance of Definitions

Milgram (2005b) argues that several fundamental questions should be considered regarding how a definition is stated, including,

- What type of item or concept is included within the definition?
- What is excluded by the definition?
- How might subtle changes in phrasing or choice of words impact the precision of the definition? (p. 3)

With these questions in mind, one can investigate the definition offered by Panasuk (in press), as well as definitions offered in number theory textbooks and math textbooks available to the teachers of elementary students.

Tobias Dantzig (2007), in his book, *Number: The Language of Science,* describes the power that words have to evoke images in our minds. He notes that the way in which words are associated with numbers, especially with the natural numbers, can evoke especially strong images. According to Dantzig, these numbers "appear to us so rooted in firm reality as to be endowed with an *absolute* nature" (p. 101). And yet, these seemingly absolute entities become, in the hands of mathematicians "a set of abstract symbols subject to a system of operational rules" (Dantzig, 2007, p. 101). It could be argued that the same comments can be made about the fractional numbers. Even a very young child may be able to recognize the process and the product of partitioning of a whole (e.g., one half taken from an apple or a sandwich cut into two equal pieces) and even have a mental image associated with the whole and the parts. . However, **only** the formal "systematic learning" under teacher supervision (Vygotsky, 2004, p. 351) of the formal definition of the fractional numbers would allow the students to relate these physical entities (i.e., parts and a whole) to a symbolic representation of the partitioning and operate upon it, make observations and use rigorous, precise language to generalize the patterns.

McCrory (2006) suggests that pre-service elementary math teachers may not have solid prior knowledge about some mathematical concepts to be able to comprehend formal definitions of these concepts. McCrory describes the tension that exists between the need to describe terms in a precise and mathematically correct manner, and the need to present pre-service teachers with the information that will allow them to teach their students mathematical concepts. Milgram (2005b) notes that such tension exists in mathematics texts directed toward elementary students as well.

McCrory (2006) describes two different approaches to definitions that are used in textbooks for elementary mathematics teachers. In the first approach, the author of the textbook presents a formal definition of a concept or an operation, and then works from that definition to show how it is applied. In the second approach, a "teaching sequence" (p. 22) is provided, in which the formal definition is constructed through a series of activities and experiences. McCrory claims that the second approach is more similar to what teachers actually use in the elementary mathematics classroom. Thus, it can be claimed that the presentation of definitions, either as *a priori* knowledge, or as the formalization of knowledge generated through experience, may

represent a specific stance on the learning and teaching of mathematics.

Historical Development of Fractional Numbers

Smeltzer (2003) observed that "from earliest civilizations onward man has attempted to deal with 'parts' in his reckoning" (p. 100), and, due to the inability to evenly divide one whole number by another, the fractional numbers were born from necessity. Long before the Greek empire, the Babylonians and later the Egyptians worked on problems related to commerce as well as problems in geometry (Eves, 1990). The sexagesimal system of the Babylonians led to all of their fractions having denominators which could be expressed in the form 60^n. Smeltzer (2003) discusses that this Babylonian notation for fractional numbers was favored by scientists, including Leonardo of Pisa, through the Middle Ages. One of the most frequently cited examples of early use of fractional numbers is Egyptian unit fractions. Unit fractions are those fractions which have a numerator of 1, and a denominator that is a whole number greater than 1.

Many authors suggest that the Egyptians used unit fractions because of the Egyptian hieroglyphic notational system (Smeltzer, 2003; Long & DeTemple, 2000). The use of a single symbol to represent a numerator of one could be paired with any number symbol to indicate a fractional number. Eves (1990) describes many of the problems Egyptian posed to be practical—related to amounts of wheat or heads of cattle—or geometric in nature. This supports Smeltzer's assertion that the fraction notation arose out of necessity. Long and DeTemple (2000) further note that the Egyptians had special symbols for certain commonly used fractional numbers, such as ½ and ⅔.

Smeltzer (2003) argues that sums of unit fractions rather than fractions with numerators other than one were not only easier to write, but were also easier to visualize, and describes some mathematicians expressing answers such as 4 ½ ¼ rather than ¾ (p. 101). Smeltzer goes on to describe that the "evolution of the concept of more general composite fractions and of abstract fractions as numbers in their own rights, developed only slowly from the earlier limited and concrete ideas" (p. 101). Regardless of the needs, the development of fractional numbers was hindered by a lack of consistent notation. While the Egyptians focused on unit fractions, the Babylonians had a sexagesimal notation, and the Greeks used a notation that had one accent after the numerator and two accents after the denominator, e.g., writing 1'5" to indicate 1/5 (Cajori, 1928). The use of a horizontal line segment (aka fraction bar) to represent a fractional number had not been consistently appeared until the fourteenth century, and even then, arguments regarding notation existed as late as the time of Augustus DeMorgan (1806–1871), a famous British mathematician (cited in Cajori, 1928).

Definitions of Fraction and Rational Number in Various Documents

Current standards documents. The Massachusetts 2011 Frameworks, which incorporate the Common Core State Standards, define a fraction as: "a number expressible in the form *a/b* where *a* is a whole number and *b* is a positive whole number. (The word fraction in these standards always refers to a nonnegative number.)" (p. 163). This definition may be compared with the definition offered for a rational number on page 167, "a number expressible in the form *a/b* or *-a/ b* for some fraction *a/b*. The rational numbers include the integers." The National Council of Teachers of Mathematics provides even less guidance on what, precisely, a fraction or a rational number is. In chapter 5 of its *Principles and Standards for School Mathematics*, the NCTM states that in grades three through five, students should "build their understanding of fractions as parts of a whole and as division" (para. 2). These definitions are not only mathematically inadequate, redundant, and inconsistent; they are confusing to the elementary mathematics teachers who are introducing children to their first formal experience with fractional numbers. Meanwhile these standards are the major documents a teacher might first look for both definitions and directions. If these definitions and directions are vague and inadequate, the "standards create a chaos and disarray" (Panasuk, in press).

Number theory textbooks. It seems logical to carefully examine the textbooks on Number Theory written by mathematicians in relation to definitions of fractional numbers and rational numbers. Davenport's (1982) discussion of rational numbers is logical and consistent, but also lacks a precise description of fractional number. Davenport describes the rational numbers as the outcome of removing "the limitation on the possibility of division… by enlarging the natural number system through the introduction of all positive fractions, that is, all fractions *a/b*, where *a* and *b* are natural numbers" (p. 14) as well as the elimination of any limits on subtraction, which is done by extending the natural numbers to include zero and the opposites of the natural numbers. Having done this, the "system of rational numbers" is attained (p. 14). Davenport's description demonstrates the extension of the natural numbers domain, but seemingly nothing else. It is long, convoluted and vague enough for a teacher, who needs clarification, to be confused. Like Davenport, Hardy and Wright (2006) do not provide a precise definition of either fractional numbers or rational numbers in their Number Theory textbook. In spite

of telling the reader that "the theory of numbers is occupied… with rationals, as relations between integers" (p. 38), the authors never supply a definition of the term rational number. It is remarkable to note such obvious lack of definitions in the textbook on number theory, a field which is supposed to define different domains of numbers and prove theorems about relationships between numbers and number domains.

Feferman's (1964) work on number theory provides the most complete discussion of the rational numbers, giving both an algebraic and a geometric interpretation of the concepts underpinning the development of the rational number system. Feferman describes the algebraic origin of the rational numbers in terms of determining which values of x and y are solutions of the simultaneous equations, $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$. These equations can be reduced to a single equation of the form $bx = a$, and Feferman extends the domain of the integers, **D**, to create a new domain, **K**, which, for all $b \neq 0$, there is a solution to this equation $bx = a$. In other words "for any $u \in \mathbf{K}$, there exist $a, b \in \mathbf{D}$ with $b \neq 0$ and $bu = a$" (p. 184). Thus, as with Davenport's definition, the rational numbers can be viewed as the result of division.

Having completed the algebraic motivation for defining rational numbers, Feferman goes on to discuss the geometric motivation for the creation of rational numbers. He asserts that this "has to do with the attempt to apply to process of counting to the measurement of straight line segments" (p. 184). In the geometric approach, a line segment is divided into unit segments of length b. The length of this line segment, if it is a, can be described as a/b units long, with b, the denominator of the fraction, acting as the referent unit. Feferman notes that "an appeal to intuitive geometric notions will be quite helpful occasionally" (p. 186).

One may ask what the definitions provided by Davenport (1982), Feferman (1964), and Hardy and Wright (2006) include and exclude, as well as what the impact of changes would have on this definition. One commonality in all of these definitions is that the concept of integers must be fully developed as a number system prior to any discussion of rational numbers. Perhaps the authors of these number theory texts may think that it is not necessary to define fractions and fractional numbers as a separate entity. Only Feferman (1964) discusses the evolution of the concept of the quotient a/b, from a geometric perspective which, as will be discussed below, is an important venue associated with the definition of fraction in textbooks for teachers of elementary school mathematics.

Textbooks for elementary school teachers. McCrory (2006) states that there are three major approaches to defining fractions in textbooks for elementary school teachers. The first method is the fraction as a part of a whole, and the second is the fraction as a point on the number line. The third is most similar to Feferman's (1964) algebraic definition, in which fractions are defined as the solution, a/b, to the equation $bx = a$. Apparently, a/b is a number, not a fraction. Panasuk, (in press) asserts such "free lance language is not only imprecise, it creates inevitable confusion in both the elementary teachers and elementary students".

I reviewed five texts for prospective elementary math teachers and found that most of them defined a fractional number as the solution to the equation $bx = a$. Two of the texts addressed the need for fractional numbers as a solution to this type of equation prior to providing the definition (Long & DeTemple, 2000; Billstein, Libeskind, & Lott ,2001). Both Long and DeTemple (2000) and Billstein, Libeskind, and Lott (2001) provide a definition that references the integers, rather than the whole numbers. Four of the texts discussed the historical development of fractional numbers, mostly focusing on the Egyptian use of fractions. Bennett and Nelson (2001) assert that fractional numbers historically represented a quantity less than one whole unit. This statement may serve to reinforce the concept of a fraction as a part of a whole, a special case of partitioning of an object or a set of objects or a thing (Panasuk, 20xx).

Only two of the texts reviewed, Wu (1998) and Milgram (2005a), define a fractional number as a point on the number line. It appears that Wu has constructed his definition so that it can be presented, directly from his text, to fifth and sixth grade students. However, Milgram explains his motivation in more detail, focusing on the need to join the two concepts of *part of a whole* and *division* that are both critical in the development of the rational number concept. He states "a part-whole definition *when properly formulated* implies the division interpretation *when the latter is given a precise definition*" (p. 230).

It is interesting to note that texts for preservice elementary teachers have a much more extensive use of concrete visual representations than texts for undergraduate mathematics students. Texts written for undergraduate mathematics students are much more symbolic and formal in their approach. However, this is in keeping with two of Milgram's (2005b) assertions—first, that "definitions need not be entirely verbal" (p. 5), and second, that some children may need to see numerous visual examples, while others may require a verbal definition. Milgram (2005a) refers to the definition he provides of rational numbers as not what is taught to mathematics majors, but interesting in its own right.

Modern Definitions versus Historical Representations

All of the textbooks reviewed, whether directed at prospective teachers or at students of higher mathematics,

use an algebraic, rather than geometric, approach in their definitions of rational numbers. Feferman (1964) addresses this issue, but justifies the use of the algebraic approach, rather than the geometric, saying, "the going is smoother if we follow" the algebraic course (p. 186). So although the geometric approach, which restricts the notation of fractions to whole number numerator and denominator, and thus restricts us to the fractional numbers rather than the rationals, is historically more accurate, the algebraic definition is more readily generalized. It is not surprising, then, that most of the texts reviewed focus on this definition of the rational numbers, even though it frequently presumes that the reader has a familiarity with the integers.

Texts which discuss the historical representations of fractions usually focus on the notational aspect, with a special focus on Egyptian notation (Long and DeTemple, 2000; Billstein, Libeskind & Lott, 2001). These texts do also address geometric representations of fractions, either using the area model, or partitioning a line into equal segments, but none of them link the geometric method with historical models. Additionally, there is no discussion of what types of problems may have motivated the Egyptians or other ancient societies to create notation for fractional numbers.

A Key Component of Mathematical Knowledge

Both the fractional numbers and the rational numbers are critically important in the development of skill in higher mathematics as well. Siegler (2013) has made the assertion, along with other researchers (Ni & Zhou, 2005) that theories of number learning cannot include only whole numbers, but must also include rational numbers. Research is being done that supports Wu's (2001) assertion that rational number understanding is important for success in algebra (Siegler, 2013; Booth, 2012). As Wu (2001) notes, rational numbers, unlike whole numbers, have arithmetic operations which can be meaningfully and symbolically generalized, and so they offer an opportunity to introduce students to the ideas of generalization that are so important in the study of algebra. It is likely that students who have strong learning of fractional numbers and rational numbers will be better prepared to succeed in the study of algebra.

As students progress through the curriculum suggested by the Common Core State Standards, knowledge of rational numbers continues to be important. Bennett and Nelson (2001) note that, although children in the elementary grades work primarily with rational numbers in which both the numerator and denominator are whole numbers, they progress to rational numbers with integer numerators and denominators. They also discuss representations of irrational numbers in a format that involves a numerator and a denominator. Although these numbers do not represent parts of wholes, they certainly represent quotients, and so it is critical that students have an understanding of the concept of fractional numbers and rational numbers if they will be able to understand irrational numbers and the representation of irrational numbers.

Summary

The definitions discussed here indicate the importance of Milgram's (2005) emphasis on clear and precise language in mathematics. This is important for the study of fractional numbers and rational numbers, not only in the elementary grades, but also as students progress through mathematics. There is a need for clarity and precision in definitions of fractional numbers and rational numbers in three areas: standards documents, curriculum materials, and texts directed at preservice elementary mathematics teachers. By the time students reach algebra, it is assumed that they will have conceptual knowledge of fractional numbers and rational numbers. Milgram (2005a) states that most students will not study rational numbers again after middle school, and what they learn about rational numbers "must serve them for the rest of their lives" (p. 228).

STRAND TWO: LEARNING AND TEACHING FRACTIONAL NUMBERS

The first strand of this paper investigated the development of the concept of fraction and the concept of rational number, as well definitions often used in classrooms. This strand will examine research focused on students learn fractions and fractional numbers. This includes the cognitive restructuring needed to effectively learn fraction and fractional number concepts, and the language used by students in their discussion of fractions. Consequently it is also valuable to investigate the challenges involved in the teaching of fraction concepts and fractional number concepts. Therefore, this strand will also examine the conflicting research on teaching of fractions and fractional numbers, and the issue of semiotics and language use in the teaching of fraction and fractional number concepts.

Like other mathematical concepts, children's knowledge of rational numbers develops over time, and appears to be related to the cognitive developmental level of the child (e.g., Piaget, Inhelder, & Szieminska, 1960; Geary, 1994). Geary (1994) defines two types of concepts, *biologically primary*, which the human brain is structured to learn, and *biologically secondary* which are developed by humans due to cultural need, and which children are unlikely to learn through simple social interactions (p. xvii, 269). Biologically primary concepts may be compared to the *spontaneous concepts* discussed by Vygotsky (1978). He defines these spontaneous concepts as those that the child can learn

about naturally, through interaction with the environment. For example, preschool aged children may develop the ideas of counting and of addition through counting toys or cookies, forming and reforming groups, without requiring formal instruction (Geary, 1994). The items children count (e.g., cookies, toys, pizzas) are often culturally bound, but the process of learning to count occurs naturally.

Geary (1994) argues that biologically secondary skills are culturally bound, and are unlikely to be learned solely through social interaction and interaction with the environment. Similar to scientific concepts, which require a teacher to present to a student (Vygotsky, 1978), biologically secondary skills are also developed through more formal learning. As has been discussed earlier, the mathematics associated with fractional numbers has developed slowly, and in many cases still represents a challenge for teachers and learners. This supports Geary's (1994) argument that skill with fractional numbers, including ordering, comparing, and working with the numergraphs, or symbols, of fractional numbers, is unlikely to develop without formal instruction. However, it is possible that students may have already acquired some informal notion of fractions prior to receiving formal mathematics instruction on fractions and fractional number through their interaction with family members.

Challenges in Learning Fractional Numbers

Researchers assert that very young children develop an informal notion of fractional numbers that is related to sharing (e.g., Siegler, 2013; Sgroi, 2001). Children may be familiar with sharing cakes or cookies with family and friends. However, the shared portions may differ in size, or there may be a part left over. This issue was observed by Piaget, Inhelder and Szieminska (1960), where young children were asked to share a clay "cake" between two dolls, the children frequently left a portion unshared, or gave unequal portions. Sgroi (2001) echoes Piaget, Inhelder and Szieminska's (1960) observations when she notes that young children may use the word half as another word for part, regardless of the relative sizes of the parts or even the numbers of the parts. Very young children are likely to call quarters "halves," or to ask for a larger half. Sheffield and Cruikshank (2001) note that children must learn that there is no such thing as "the bigger half" (p. 233). Mack (1990) refers to this as "informal knowledge," which she defines as "applied, real-life circumstantial knowledge constructed by the individual student that may be either correct or incorrect and can be drawn upon by the student in response to problems posed in the context of real-life situations familiar to him or her" (p. 16).

So, while an argument can be made that some knowledge about fractions may be gained through everyday experience, it seems clear that formal instruction is needed for other types of knowledge, and that, in order for the student to be successful in mathematics learning, the two types of knowledge must be reconciled (Smith, 2002). It seems likely also that formal instruction is necessary to provide students with accurate, mathematically correct definitions of fraction and fractional number. Further, the teacher must provide an appropriate environment for this formal learning to take place (Ball, 1993; Streefland, 1993).

Language of fractions: convention, not invention. Educators observe that children often come to school using words such as "half," "part," and "whole," but it is unlikely that they know other terms associated with fractions and fractional numbers, such as "quarter" or "fifths." Ball (1993), in her work with third graders on a cookie sharing problem describe situations in which the students refer to one-fourth of a shape as "half." In one scenario, a young girl is sharing twelve cookies among five people. Noting that the student is calling the pieces of a cookie divided into five parts "halves," Ball states, "terms we use for fractional parts is a matter of convention, not invention," (p. 180) before telling the student that these parts are called fifths. The student agrees that this term makes sense because there are five pieces. Ball goes on to assert that students will not, without formal instruction, "discover" these correct terms (p. 180). Even after being told the correct term, the students Ball (1993) worked with maintained the idea of "half" as the generic word for portion, as one student did when discussing the result of the twelve cookies for five people sharing problem.

Based on this study, it appears that students do not arrive at school with a clearly developed notion of fraction or of fractional number. These students must be guided by a teacher to correct language use. They also need numerous opportunities for practice, and may need repeated reminders of correct terminology. Ball's study also provides strength to Geary's (1994) claim that fractional numbers are a biologically secondary concept, that is to say, they are concepts that require a teacher to formally present to the student.

Partitioning: another challenge. Pothier and Sawada (1983), describe four stages of partitioning of geometric shapes into equal parts. These are described as follows: (a) *Sharing*, in which children often use the word "half" to describe any fractional part, including fourths and thirds. Students in this stage may use line segments to partition a circle or a rectangle, but these partitions are not always equal. (b) *Algorithmic Halving*, in which children can partition areas into two equal pieces, and use the same type of partitioning line (e.g., a diagonal line) to further partition the shape into four, eight, or sixteen parts. In contrast with the Sharing stage the parts may not always be equal, but the entire figure is used. *(c) Evenness*, in which students are generally able to construct equally sized partitions of halves,

fourths, sixths, and other even-numbered denominator fractions. This stage is marked by students' consistent recognition of the need for equally sized partitions, although they are not yet able to construct them for fractions such as one-third or one-fifth. *(d) Oddness,* in which children realize that the halving strategy is inefficient and seek another way to divide a whole into the required number of equal parts.

Pothier and Sawada generated the stages through observation of 43 children in grades Kindergarten through three. They provided a problem similar to the one provided by Piaget, Inhelder and Szieminska (1960), in which children were asked to divide a cake, either rectangular or circular, amongst a given group of dolls, "so that each one has as much" (Pothier & Sawada, 1983, p. 308). Children were asked to partition the cake for a different number of dolls based on their age, that is, younger children were asked to produce fewer partitions than older children. One interesting outcome of their observations is that children do not become focused on producing equally sized partitions until stage three, and that prior to this, they will declare their partitioning satisfactory even if equally sized parts are not obtained. Because of this, Pothier and Sawada characterize stage three, *Evenness*, as a "breakthrough" (1983, p. 313). This is likely the stage that Ball's (1993) third students, who made statements such as "pretend it's equal" in regard to their partitions, were in.

The move to stage four represents another cognitive leap for children, one that Smith describes as only arriving "after substantial struggle" (2002, p. 6). Pothier and Sawada (1983) stated that this level of partitioning skill was suitable for small prime numbers, such as three and five parts. They observed this skill in children as young as eight and a half years old. They do note that some study participants were able to construct three or five partitions, but not both, and suggest that some children will accidentally select an appropriate partitioning line, and thus may appear to have reached stage four, when in fact they have not.

Empson (1999) developed a case study of 19 first-graders over a five-week instructional session on fractions. The lessons that Empson and the first-grade teacher prepared were centered around real-life sharing situations, such as the sharing of cookies, pizza, and pancakes. Empson's interviews with the children prior to instruction appear to reflect the stages described of Pothier and Sawada (1983), as the children seem comfortable with halving items to share them, but are more concerned when items must be shared with three people. Fourteen of the interviewed students used a repeated halving procedure in their unsuccessful solutions to the problem 2÷3 (p. 298). During a class discussion of a problem about three friends sharing candy bars, one child stated, "you can split it in half or in quarters, but you can't split it in threes" (p. 309). Similar to the third graders in Ball's (1993) study, the students in Empson's (1999) study persist in their use of the word "half" or "quarter" as a generic portion term for a number of lessons, even after being told the correct term.

Based on these studies, it appears that students use of the word "half" as a generic portion term not only indicates an incomplete notion of fraction, but that such use may impede them in gaining more correct and complete knowledge. Students are also challenged by the notion of sharing with more than two people, and of exhausting the item that is to be shared.

Impact of whole number understandings. It has been discussed that many children can effectively partition regions, and may appear to have an understanding of fractions as parts of wholes, they may still be impeded in learning by their knowledge of whole numbers. Streefland (1993) and others have discussed the impact that children's prior knowledge of whole numbers has on their learning of fractional numbers, especially with regards to operations such as addition of fractional numbers. Streefland (1993) refers to this interfering type of knowledge as "N-distractors" (p. 319). That is to say, when students appeal to additive reasoning associated with whole number arithmetic, rather than to the proportional reasoning needed for fractional number learning, it is possible that they are being affected by N-distractors. Behr and Post (1992) note that "early in the learning of fractions children understand 1/5 to be less than 1/8 because 5 is less than 8" (p. 217). This misconception can impact students' ability to compare one fraction or fractional number to another and decide which is larger (c.f., Ball, 1993; Behr & Post, 1992; Mack, 1990). Siegler (2013) asserts that there must be a reorganization of student thinking about number in order for rational number learning to occur. If students are not able to differentiate between properties that apply only to whole numbers and those that apply to rational numbers, their learning will be slower and incomplete (Siegler, 2013). Empson (1999) asserts that discussion of the difference in size between fractional numbers such as 1/3 and 1/4 can help students to overcome some of the problems from conflicting prior knowledge of whole numbers.

Streefland (1993) also emphasizes the use of pictures and diagrams to offset the damage that may be caused by a purely algorithmic approach. The students can use the diagrams or pictures as a step between the concrete fractions as part-of-whole representation and the symbolic, and thus more abstract, fractional numbers. Streefland asserts that this will "support, to a great extent, the student's thought process" (p. 295). Furthermore, the fractional numbers "will become genuine symbols, while still retaining their reference function" (p. 298).

Fractions and fractional numbers have many constructs. In addition to the challenges of extending the number system, students must also grapple with the multiple constructs contained in fractional numbers (Kieren, 1993). The fraction may represent part of a whole, and the fractional number the outcome of division, but children will also need to deal with ratio, and with the fractional number as an operator (e.g., "three-fifths of sixty").

Toluk and Middleton (2004) assert that children in grades 4 and 5 are likely to view division as an operation restricted to whole numbers, while fractional numbers are used to represent a part of a whole. Additionally, they assert that many children at this level view division as restricted to the case of a larger number divided by a smaller number. Through an eight week teaching intervention, Toluk and Middleton observed the development of the concept of fractional numbers in four fifth-grade students. They classified the four stages of concept development as follows:

1. Initially, students have a conception of fractions as parts of wholes, and division as an operation restricted to whole numbers. Division and fractions are not linked.
2. Students develop the idea of fractional quotient, in which fractional numbers arise from partitive, or equal sharing, division problems. Problems which were stated only using symbols (i.e., numbers) were not solved using fractional numbers.
3. Students begin to see division as a fractional number, when the fractional number is less than one. For example, when 7 items must be shared equally among 8 people, children who had this schema would automatically respond that the answer would be 7/8.
4. Students view a fractional number as representing division. This is the reverse of the division as fractional number schema, but still represents a separate step.

Sharp, Garofolo, and Adams (2002) describe a group of fourth graders solving a problem in which a whole number (15) must be divided by a rational number (1 2/3) in order to give the correct amount of medicine to a dog. One correct student generated solution shows 15 circles each partitioned into thirds, the denominator of the rational number. The student has drawn rings around five thirds and then put a number inside the ring indicating the day. Nine rings are drawn and numbered. A solution using fractional numbers is also presented, although this was solution was only produced through a teacher guided process. A second problem, which did not result in a whole number solution, created more issues for the students, as many were unsure what to do with the partial dose (e.g., the remainder). A number of ideas were generated, including throwing away the leftover part.

Sharp, Garofolo, and Adams's (2002) study is important because it describes the challenges students face as they move between the stages proposed by Toluk and Middleton (2004). The student observed was able to solve a division problem using pictures without adult intervention. However, the use of fractional numbers to solve the problem required the student to interact with a teacher and discuss the method. When confronted with a problem in which creating a whole number of doses was not possible, students overall struggled with how to express the result. Sharp, Garofolo, and Adams (2002) report appears to provide additional support to the assertion made by Toluk and Middleton (2004) that students struggle with the connection between a remainder in a division problem and a fractional number.

Challenges of Teaching Fractional Numbers

Just as students are challenged in learning the concept of fraction and of fractional number, teachers also struggle to effectively teach students these concepts. There are many reasons for this, including teacher knowledge (e.g., Post, Cramer, Behr, Lesh & Harel, 1993; IES, 20xx), teacher attitudes towards mathematics (e.g., Ball, 1993), and even power structures within schools (e.g., Ball, 1993; Post, et al., 1993). While important contributors to the teaching environment, these are beyond the scope of my proposed investigation. Therefore, I will discuss the following issues, which also contribute to teachers' struggle to effectively teach: lack of consensus about a single best way to teach fractional number concepts (Gagne & Dick, 1983; Moss & Case, 1999; Behr, Wachsmuth, Post & Lesh, 1984) , challenges with semiotics in mathematics (Pinella, 2007), and the lack of discussion of language use in methodology textbooks (Sgroi, 2001; Sheffield & Cruikshank, 2001; Van de Walle, 1998) .

Teaching Sequences for Rational Numbers

There are many didactic methods for teaching children about the rational numbers. These may be divided into behaviorist or neo-behaviorist methods and methods which provide a concrete to abstract approach, seeking to link students experiences outside the classroom with their experiences inside the classroom. Both approaches have benefits and challenges.

Gagne and Dick's work (1983) on the conditions of learning has been strongly associated with the neo-behaviorist view. They suggest that instructors excite the learner's attention, test for prior knowledge, provide guidance in

learning the skill, and assess the quality of a student's performance of a given skill. These suggestions are part of commercial curricula reviewed by researchers (Moss & Case, 1999) and part of experimental curricula used in interventions (Moss & Case, 1999; Behr, Wachsmuth & Post, 1988).

Characteristics of rational number curricula may also include some features that indicate a misunderstanding of Gagne's (1984) work. Moss and Case (1999), in their review of commercial curricula, noted four problem areas in the teaching of rational numbers

- Focus on manipulation of rational numbers rather than conceptual understanding
- Instruction that focuses on the teacher as giver of information, and seems to discourage children from reasoning independently. Such instruction often focuses on rote performance tasks.
- A use of instructional methods and representations that made it easy for children to confuse rational numbers and whole numbers.
- Problems with notations, whether represented as fraction, decimal fraction, or percent.

The first two of these problems raise questions regarding the type of knowledge that students must develop about rational numbers. Gagne (1984) differentiates between procedural knowledge, in which a learner is apply to consistently apply a learned procedure to obtain a desired result, and cognitive strategies, which allow the learner to know when and how to use a procedure. A focus on manipulation of rational numbers, and repetition of rote performance tasks seems likely to develop procedural skills, but perhaps at the expense of cognitive strategies.

In contrast to methods which highlight rote skills, Sgroi (2001) recommends that informal instruction about fractional numbers begin prior to formal instruction in fractional numbers. She recommends the following progression (p 208):

1. Nonverbal experiences, such as working with sets of items to be shared fairly among classmates, or with pieces of paper to be divided into equal parts
2. Comparison of manipulatives. Students who are working with red circles cut into thirds and blue circles cut into fourths could describe the blue piece as smaller than the red piece
3. Introduction of mathematical language. At this point the students would be told that there is a word that describes the blue piece, and that word is one-fourth.
4. Introduction of symbolic language. At this point, the word one-fourth is linked to the symbol ¼

It should be noted that Sgroi (2001) does not specify an order in which activities should be done, or instruct teachers on how mathematical language should be used in the classroom. Similarly, Sharp, Garofolo and Adams (2002) recommend that mathematical language and notation be introduced only after children have developed conceptual understanding of fractions and fractional numbers through familiar, real-life situations. They recommend that teachers encourage students to use their own language and pictures, as well as develop their own procedures, for working with fractions and fractional numbers. Students should be asked to keep track of their work, ideally on paper, and eventually using fractional numbers rather than pictures. Teachers observing students' invented procedures are cautioned to ensure that students are not over generalizing. Like Sgroi (2001), however, Sharp, Garofolo and Adams (2002) do not provide straightforward recommendations as to what teachers should and should not do.

Researchers have also been challenged in determining which rational number concept should be utilized first in teaching children about rational numbers. Moss and Case (1999) demonstrated success with an experimental group of students who began their study of rational numbers with percents and decimal fractions, concepts that the researchers asserted children encountered in an informal setting. Students in the experimental group were more able, following instruction, to successfully explain whether 2/3 or ¾ was the larger fractional number using a picture (Moss & Case, 1999, p. 137). However, Behr, Wachsmuth, Post and Lesh (1984) found that students who worked with fraction circles and divided rectangles also showed growth in their ability to compare two fractional numbers. The researchers asserted that this gives evidence that the children were likely to possess "an understanding of the inverse relationship between the number of parts into which a whole is partitioned and the size of each part." (Behr, Wachsmuth, Post & Lesh, 1984, sec. 5, para. 4).

The studies cited above demonstrate the challenges faced by researchers as they compare the value of different types of manipulatives for use in the teaching of fraction and rational number concepts (Behr, Wachsmuth, Post & Lesh, 1984; Moss & Case, 1999). They also show that it is not clear whether it is best to begin with fractions, percents, or another representation in the study of rational numbers. In the face of results that have been described by Behr, Wachsmuth and Post (1988) as "equivocal," and with limited guidance from methods textbooks, it seems clear that teachers are likely to be challenged by both the choice and the sequencing of learning activities.

Challenges with Semiotics in Mathematics

Pinella (2007) discusses the impact of teaching methods on student learning. She notes that in mathematics education, *semiotics*, or the signs used to represent concepts, are presented to students, rather than the concepts themselves. It is therefore possible, she argues, that children will be able to manipulate the symbols without gaining any correct conceptual knowledge. As noted by Streefland (1993) in order for a student to demonstrate conceptual knowledge, he or she must be able to recognize the fractional number as both a symbol and a reference point, whether that is a part of a whole, a ratio, or a point on a number line.

Pinella (2007) argues that perhaps time is not the only issue, but the lack of a clearly planned path to transition students from concrete models of fractions to the more abstract rational numbers, "providing students with concrete models and then requesting abstract reasoning, independently of the proposed mode, is… a sure recipe for failure " (p. 97). She describes instruction on fractions as occurring within a variety of semiotic registers, or representations. The teacher is familiar with these representations, and has conceptual knowledge of fractional numbers, but the student has neither familiarity nor conceptual knowledge. When textbooks are designed with many different representations, the teacher may move between representations too rapidly. The teacher may believe that the student understands that the fractional number ¾ may represent a circle divided into 4 equal portions with 3 shaded, or may represent three items shared among four people. However, the student may see this representation as two separate numbers with the fraction bar between them.

According to Pinella (2007), when students progress through a curriculum too rapidly, it is possible that the students will form incorrect mental images of fractions and fractional numbers. For example, students working only with area models may conflate equal parts with congruent parts, thinking that all partitions must be the same size and shape. This is similar to concerns raised about models of instruction that introduce symbolic notation and algorithms too soon. It is the teacher's task to determine if the student has a correct mental image, but this takes time and effort on the part of the teacher (Empson, 1999; Ball, 1993; Lamon, 1999).

Lack of Discussion of Language Use in Methods Texts

Many of the methodology books reviewed suggested the use of manipulatives and pictures for initially teaching about the concept of fractional numbers. Both Sgroi (2001) and Sheffield and Cruikshank (2001) describe activities in which children are encouraged to fold pieces of paper into equal pieces. Sheffield and Cruikshank suggest coloring a square in different ways to demonstrate that there are many ways to shade one-half of the square. Piaget, Inhelder and Szeminska (1960) note that such regular shapes appear to be easier for children to work with in partitioning than irregular shapes in which a midpoint cannot be easily visually determined (p. 312). The paper-folding and coloring methods utilize what is often referred to as the area model for teaching about fractional numbers (Wu, 1998; Van de Walle, 1998). Sheffield and Cruikshank (2001) suggest demonstrating using length and volume models as well, while Van de Walle (1998) describes three models for representing fractional numbers: area, length, and set. One drawback to these activities is that students may develop a strong sense of the part-whole construct associated with fractions, but not extend and connect that knowledge to the fractional numbers, particularly those quantities greater than one (Ball, 1993). Panasuk asserts that pictures must be used "with great care" (personal communication, June, 2013).

All three methods texts focus on the activities that the teachers should use in class, rather than the language that the teacher should be using. Sgroi (2001) focuses on informal language related to the manipulatives being used. Children discuss fractions in terms of "yellows" and "oranges," and "flats" and "longs" (p. 214, 216). Sgroi (2001) does not clearly state how the teacher is to lead children to understand the connection between the manipulative, the numeric symbol, and the word used to describe a fractional number.

Van de Walle (1998) also focuses on visual models and informal language. When teachers work with students to develop the concept of fractional numbers, Van de Walle claims that the "two simple but important ideas" are "the top number *counts*. The bottom number tells *what is being counted* (1998, p. 244)". He claims that students do not have a reference point for terms such as "numerator" and "denominator," and thus the vocabulary will not make sense to them. Empson (2002) takes a similar view to children's language use, encouraging teachers to emphasize what children have gotten right, rather than what they have gotten wrong. In one case, she describes asking a young boy how his halves (which are actually fourths) are different from another student's halves. Empson claims that this can help "probe student's thinking and help them better articulate it" (2002, p. 38).

Leinhardt (1988) also discusses problems arising from students' lack of understanding of the symbolic notation for fractional numbers. Her interviews of fourth graders showed that, although many possessed the part-whole understanding of a fraction prior to instruction, they could not explain why there are two numerals in a fractional number even following instruction. While Leinhardt does not discuss either the language used or the activities used during instruction,

it is possible that using the word "fraction" to refer to the part-whole construct, while using the term "fractional number" to refer to the result of division, may impact student understanding of the structure of fractional numbers.

SUMMARY

Geary (1994) asserts that some models of teaching ignore the biologically secondary nature of certain mathematical concepts, claiming that such models assume "given an appropriate social context and materials, children will be motivated and able to construct mathematical knowledge for themselves" (p. 265). In the case of fractions and fractional numbers, this implies that students who are given manipulatives such as fraction circles or Cuisinaire rods, and are allowed to discuss with each other, and possibly the teacher, will learn skills without formal lessons. Geary asserts that this approach may be sufficient for some mathematical concepts, such as counting and simple arithmetic, but it is insufficient for other concepts. While Geary does not specifically include fractional numbers in this list, it is logical to assume that fractional numbers, which represent an area of mathematics that is substantially different from whole numbers, one that requires a restructuring of thinking about number, it is unlikely that simply providing a social context and manipulatives will be enough to produce correct learning about fractional numbers.

Van deWalle (1998) notes that the "explosion of procedural knowledge at about the fifth grade is generally not supported by strong conceptual knowledge of fraction meanings because the curriculum simply has not provided time for the complex development that fraction concepts require" (p. 238). This observation appears to be supported by the work of Empson (1999), Lamon (1999) and Smith (2002), all of whom refer to the amount of time that teachers must take to discuss concepts with students, and the time that must be taken by the student to consider topics such as the density of rational numbers on the number line. Empson (1999) asserts that how children learn about rational numbers and what they are able to learn about rational numbers are inseparable. It is likely that instruction that is both mathematically correct and developmentally appropriate will lead to learning, even with abstract concepts such as rational numbers.

CONCLUSION AND RESEARCH QUESTIONS

Given the complexity of the concept of rational numbers (i.e., zero and all positive and negative numbers written in the form a/b, where b is not equal to zero; e.g., 5 is a rational number), and its sub concepts, including fractional number and fraction (i.e., a portion of a whole or a part of a whole) it is imperative to focus on the correct and precise language to define the concept and describe its nature (i.e., properties). As has been shown, research on the specific teaching techniques has demonstrated not only that there is no single method that can be described as optimal but also that the use of the single word "fraction," and its necessary association with the part-whole concept inevitably leads to the loss of the essence of the concept of fractional number (Panasuk, in press). Panasuk claims that, since third grade students currently are introduced to the concept of partitioning of a whole and are expected to show and describe diagrammatically and verbally parts of a whole, the use of the word *fraction* should be delayed until later grade 4 when the use of formal language is expected. According to Panasuk, when partitioning geometrical shapes or sets of objects in grades 1–3 grades and describing the relationship between the parts and the whole, the word *part*, or *piece* or *portion* is a better use then *fraction*. Ostensibly, "one cannot draw on the same word to describe basically two different entities, partitioning and naming the parts of whole, and representing symbolically a relationship between two numbers compared by division, particularly with young students whose mathematics vocabulary is in its rudimentary stage, yet" (Panasuk, in press). Thus, the term fraction, even in the math classroom, would have a meaning more similar to the everyday language meaning of a part or portion of an object. When speaking of a *number* written in the form *a/b*, in which *a* and *b* are both whole numbers and b is non zero, Panasuk proposes to refer to *a/b* as a *fractional number* only and specifically. Panasuk argues that, "The *'fractional numbers'* are in line with the terms commonly used for "whole numbers," "irrational numbers," or "imaginary numbers" which refer to the symbolically notated numbers on which operations are performed.

Panasuk (in press) suggests that even regardless of the curriculum (i.e., textbooks or other educational materials) and the methodologies used in the classroom, it is essential that teachers use mathematically correct and precise language to accurately describe concepts. She says that, "while most of the educational materials available to the teachers emphasize the importance of correct explanation of the concepts, there is little attention given to the terminology that is acceptable and even less attention is given to the terminology which is not acceptable, and thus must be avoided" (in press) Based on the above, the following research questions are proposed:

1. What is the impact of the proposed language on fourth grade students' ability to order fractional numbers between 0 and 1 on a number line?
2. What is the impact of the proposed language on fourth grade students' ability to explain why two

fractional numbers are equal or equivalent, using a number line and/ or pictorial representation?

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A Situated Perspective of Teacher Learning and Efficacy in Data Teams

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In a society increasingly driven by creativity and innovation, individuals must be able to adapt and learn quickly. Teachers are charged with preparing students for this reality. As such, teachers have become a critical determinant of the future social and economic success of their students. Recent research indicates that the strongest factor in a student's educational outcome is their teachers' instructional practice (Black & William, 1998; Elmore, 2004; Hattie, 2009). Continuous growth in teachers' professional knowledge is critical to developing instructional approaches that prepare students to enter a society and economy that values quick learners.

Schools have attempted to stimulate teacher learning and creativity by breaking down barriers that have previously kept teachers isolated (Schon 1987; Talbert & McLaughlin, 2002) by directing teachers to engage in collaborative inquiry with their colleagues to improve teaching and learning in schools (Curry, 2008; Desimone et al., 2002; Mandinach, 2012; Mason, 2003). Frequently, conversations that take place during teachers' collaborative meetings are focused on searching for more creative ways of reaching their students based on analysis of student data. The point of this type of collaboration is to provide teachers with an avenue for turning raw data into information by giving it relevance, purpose and meaning (Horn, 2005; Empson, 1999; Mason, 2003; Murnane, Sharkey & Boudett, 2005; Wayman, Midgley & Stringfield, 2006). The trend towards collaborative data analysis has prompted policy-makers and school leaders to devise a variety of structures for teachers' collaboration (Boudett, City & Murnane, 2010; Curry, 2008; *Data Teams Toolkit*, 2010; Earl & Fullan, 2003; Guskey, 2002; Spillane, 2011). Data teams have been promoted as a way to advance teacher learning and improve student outcomes through the sharing of student data, instructional problems, and innovative teaching practices (Boudett, et al., 2010, Horn, 2005; Sgouros & Walsh, 2012).

Data teams are groups of teachers who engage in inquiry about instructional practices using student data as a guide (Boudett, et al., 2010). This type of structured collaboration creates a situation in which teachers can construct meaning from student data through conversation with colleagues. In a data team, teachers analyze objective facts through dialogue to create information that is relevant and useful to them as it relates to their students' learning. Contextualizing student data allows teachers to make reasoned judgments about how to proceed with their students in their unique situations (Biesta, 2007; Hodkinson, Biesta, & James, 2007; Horn, 2005). Data teams provide an avenue for teachers to reflect on their students, their instructional practices, and on how to improve student outcomes (Boudett, et al., 2010; *Data Team Toolkit*, 2010; *Data Team Strategies*, 2013). The product of the collaborative work of a data team is the teacher learning that is produced as teachers turn data into knowledge they can act upon in the classroom.

Teachers' professional learning is critical to student success. Data teams are a collaborative practice in schools that may further or stifle teachers' professional learning. While an increased emphasis on teacher learning through collaborative engagement with data might lead to more innovative teaching practices, the connection between teacher learning and data teams remains vague because so little is known about what teachers learn from engaging as members of a data team. This study aims to explore that connection through an explicit focus on teachers' situated learning, and the ways in which teacher efficacy develops as teachers engage in structured collaboration centered on student data.

STATEMENT OF THE PROBLEM

Throughout the history of education, teachers have analyzed data about their students to guide their teaching, albeit with limited access to data beyond their own classrooms (Brunner et. al., 2009; Datnow, Park & Kennedy-Lewis; 2012, Davies, 1999; Pella, 2012; Tyack & Cuban, 1997). Recent developments in technology have increased teachers' access to data, ranging from standardized test results at both the national and state levels to common assessments at school and departmental levels (Datnow et al., 2012; Goddard, Goddard & Tschannen-Moran, 2007; Huffman & Kalnin, 2003; Ikemoto & Marsh, 2007; Kazemi & Loef-Franke, 2004; Sgouros & Walsh, 2012; Spillane, 2011; Wayman, 2005). However, the impact of student data on instruction and student outcomes has been limited in contexts where teachers work in isolation (Brunner et. al., 2009; Firestone, Mayrowetz & Fairman, 1998). Researchers and policy-makers have responded to teachers' isolation by advocating for collaborative analysis of student data with the intent that teachers might learn more about their students and their own teaching practice through collegial dialogue.

Given the recent trend towards collaborative data analysis in schools, it is crucial for educators to understand how collaborative engagement on a data team impacts teachers' learning. While data teams have many advocates, little is known about how teachers are affected by participating in them (Little, 2012; Riveros, Newton, & Burgess,

2012; Rone, 2009; Ruffner, 2008; Sgouros & Walsh, 2012). This study will investigate what teachers engaged in data teams learn from their participation, and how their learning impacts their efficacy to make changes to their instruction. It will also apprise educational researchers and school leaders who seek to learn more about how teachers learn in collaborative settings. Studying teacher learning in such a context requires a lens that focuses on the social nature of learning if what is learned through teachers' collaborative efforts is to be more clearly understood.

JUSTIFICATION OF THE STUDY

Social Constructivism

The seminal works of Dewey and Vygotsky support the concept that people learn from engaging in social situations (Dewey, 1884; Vygotsky, 1938). Derived from these works, social constructivism is a cognitive theory that concentrates on the knowledge that groups construct through collaboration. Within social constructivism, social learning and situated learning focus on different aspects of what individuals learn from social interaction. Social learning theory focuses on what people learn from observing each other in social situations (Bandura, 2000). Situated learning is a perspective of learning that examines how actions, social interactions, and even material objects within a situation influence how and what people learn in a situation (Lave & Wenger, 1991). Lave and Wenger (1991) refer to collaborative groups as communities of practice, which often have complex layers of knowledge, skills, and behaviors that must be mastered in order for an individual to take on a significant role within the group.

Existing Research

Much of the existing research on teacher collaboration with data ignores what teachers learn from each other in situations like data teams. Utilizing social constructivism as framework to drive this study allows for a focused examination of what teachers learn from each other as they collaborate around student data. Most studies of on-the-job professional development initiatives focus on trying to describe the impact of a program on a variable such as student achievement, or have focused on what content knowledge teachers learned by participating in a workshop (City, Elmore, Teitel & Fairman, 2009; Desimone, Porter, Garet, Yoon & Birman, 2002; Parise, Leigh, Mesler & Spillane, 2010). Research about teachers' collaborative engagement with student data has primarily focused on how the collaboration is structured (Rone, 2009; Ruffner, 2008; Pella, 2012). Other studies have explored correlations between teachers' access to student data with student outcomes (Brunner et al., 2005; Datnow et al., 2012). However, what teachers learn from the on-the-job professional development experience of participating in a data team has been largely ignored as researchers have attempted to understand links between data teams and student achievement (Little, 2012; Rone, 2009; Ruffner, 2008). By focusing on what teachers learn from their collaboration, this study addresses the gap in the research on teacher learning in a data teams, a crucial gap considering its potential for transforming students' experiences (Fishman & Davis, 2006; Fishman et. al, 2003; Greeno, 2006; Hattie, 2009; Lave & Wenger, 1991; Sawyer, 2006).

This study will provide educators, policy-makers, and researchers with a nuanced view of the results of implementing a data team as part of the professional development activity of teachers. This study will inform efforts to develop more effective structures for on-the-job teacher learning through a clearer understanding of how teachers learn from their collaborative engagement on a data team (Fishman & Davis, 2006, Spillane, 2011). As more schools adopt data teams, attention should be focused on how teacher learning impacts their professional practice.

PURPOSE OF THE STUDY AND RATIONALE

The purpose of this exploratory qualitative study is to investigate what teachers on data teams learn. Like (Huizenga, 2011), I also want to know more about how teacher learning might affect their self-efficacy with regard to developing more innovative instruction. This study will employ a framework that incorporates situated learning to examine how teachers construct meaning from data, through their collaborative interaction (Lave & Wenger, 1991). No known studies of data teams from the perspective of situated learning have been conducted, and therefore little is actually known about the interactions that occur within a data team, and how data team conversations might open up or close down learning opportunities for teachers. Studying teachers as they interact with each other to discuss student data will reveal how they construct new knowledge within their field (Schon, 1987) and will lead to a greater understanding of the value of collaborative models of on-the-job professional development, which can guide future educators and researchers as they look to promote teacher learning in collaborative situations.

Any collaborative situation presents an opportunity to learn from others as well as from the material resources in a situation. Data teams have grown in popularity because of the hypothesized power of collaboration between teachers. Studying data teams using the lens of social constructivism allows the study of what teachers learn as a result of being engaged in collaborative work. Analysis of the discourse

within data team meetings will provide insight into how teachers learn from being participants in these situations, and will help to measure their learning by examining their participation over time. Transcripts of data team conversations will likely reveal more about the changing participation of data team members. As the participation of individuals within the data team changes, their situated learning can be more clearly understood. Since data teams are designed to create opportunities for teachers to learn and change, measurements of changes in the self-efficacy of teachers to modify their instructional practice are appropriate measurements of the type of change anticipated by data teams themselves (Bandura, 2000, Huizenga, 2011).

The fundamental proposition of this study is that data teams can be positive professional learning opportunities for teachers who participate in them, and that teachers' learning from their participation in a data team can begin to impact classroom instruction. A close study of what teachers learn as they participate in a data team and how teachers describe their self-efficacy to make instructional changes as they engage in this type of professional learning would provide researchers, school leaders, policy makers, and teachers with a clearer understanding of the value of collaborating in data teams.

THEORETICAL FRAME

Social Constructivism

Social Constructivism is the overarching perspective on learning that defines the lens of investigation in this study (Figure 1). Within this view there are two specific theories used to examine different aspects of teachers' learning, social learning and situated learning. Each of these theories relies upon investigating changes over time as evidence of learning. Social learning focuses on learning that occurs from observation, while situated learning focuses on learning that occurs through active participation in a community of practice. Hence, social learning is measured by looking at changes in individuals' characteristics, while situated learning focuses on changes in individuals' participation in the group. Analysis of the collected data within the context of these theories from the perspective of social constructivism will provide a clearer understanding of the professional learning of teachers on data teams because it will emphasize how participants changed as a result of their collaboration. Looking at shifts in the participation of members of a community of practice, analyzing changes or patterns in teachers' discourse, and investigating changes in teachers' self-efficacy all overlap and ultimately center on what the participants have learned as a result of their immersion in the situated activity of the group.

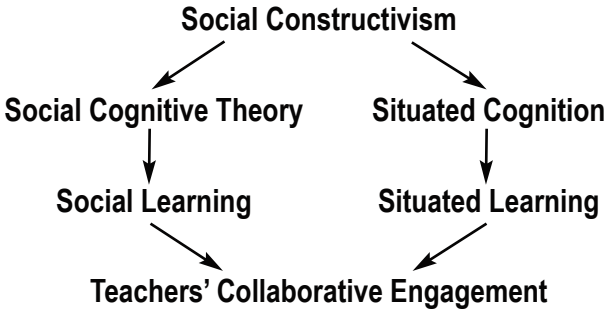

Figure 1. Theoretical framework

The perspective of social constructivism has its origins in the early philosophy, science, and psychology of learning. John Dewey emphasized the importance of the situation in the learning process, explaining that the "…environment is a necessity to the idea of organism, and with the conception of environment comes the impossibility of considering physical life as an individual, isolated thing developing in a vacuum." (Dewey, 1884). Furthering the idea that learning can be viewed from a social perspective, Vygotsky posited that "...human learning presupposes a … social nature…" (Vygotsky, 1938). His theory of the zone of proximal development relies on a conception of learning that emphasizes the role that situations have in stretching one's knowledge and skill levels (Vygotsky, 1938). The perspective that the social situation was a critical component of one's learning eventually developed into the theory of situated learning in the 1980s.

Situated Learning

Situated learning reveals the limitations of the Cartesian conception of learning that locates learning activity purely within the individual's own mind (Sawyer & Greeno, 2006). Rather, situated learning holds that learning occurs outside of our own minds, in the realm of our social activity. It argues that this learning can be measured, studied, and understood by looking at changes in the participation of individuals (Sawyer & Greeno, 2006). Building on the work of Dewey and Vygotsky, situated learning holds that learning is socially constructed within particular cultural and physical contexts; it emphasizes how people learn by engaging with others in work and community activity. Groups that collaborate because the members share a profession are referred to as communities of practice. Lave and Wenger explain that ".. there is no activity that is not situated. [Situated learning] implie[s] emphasis on comprehensive understanding involving the whole person…and on the view that agent, activity, and the world mutually constitute each other." (Lave & Wenger, 1991). This mutual constitution also includes the environment as part of the situation, capable of presenting learning opportunities. The situated

learning perspective values the dynamic nature of the social activity of group members in creating the situation. It therefore emphasizes the active construction of knowledge by group members, and therefore calls for dynamic ways of measuring learning as it happens.

Situated learning provides a lens for a closer study of learning in social situations by breaking down situations into measurable pieces to better understand the learning process. Learning within communities of practice can be measured by looking at changes in the practices of the group. Such a change might be caused or affected by changes in the self-efficacy of group members, or by changes in the groups' discursive patterns. The discourse of a group is certainly subject to changes in the participation and practices of its members, which might be influenced by their self-efficacy.

Investigating situated learning as it happens requires tracking concrete manifestations of learning, such as the evolving engagement of members within a community of practice, or changes in their discursive patterns. Examining changes in participation within a community of practice, combined with analysis of discourse will allow for a rich understanding of the learning process of a group as the learning unfolds and changes over time. The two theories can be useful in teasing out examples of teacher learning that may otherwise be difficult to identify given the overlap between observational and participatory learning that can take place in a collaborative group. Within the theory of situated learning, Lave and Wenger introduced the concept of communities of practice based on their research on how collaborative efforts in apprenticeships lead to learning in social situations. Changes in the practices of the individual or the group are taken to be indicators of learning, described by Lave and Wenger (1991) as shifts from "legitimate peripheral participation" to full participation in a groups' activity. Individuals assume different roles within any community of practice, and often their level of participation changes as their competence in participation changes over the time they are engaged in the community of practice. As individuals or the group change their practices, those changes are evidence of their learning (Lave & Wenger, 1991).

While post-hoc analyses are often employed to measure learning, the situated learning perspective stresses the learning process as it happens, which requires study of a group as it interacts with itself. A key aspect of understanding the situated learning of a group is to study its discourse. During analysis of a group's interaction, discursive patterns are analyzed to identify changes in established patterns of how the group interacts. Identifying changes in discursive patterns can reveal ways in which the groups' interaction has opened up or stifled learning opportunities. Grounding the analysis of the discourse of data team meetings within the perspective of situated learning frames how changes in the participation of members over time should be understood; it emphasizes that discursive patterns moving towards or away from full participation in the group can be used to identify learning within the situation (Gee & Green, 1998). Examining how participation of group members changes over time enables a deeper understanding of how and what a group has learned through their situated activity.

Social Learning

Engagement within a situation can lead changes in one's perception of one's agency within and across situations. One specific indicator of learning related to an individual's perceived agency as a result of their participation in a situation is self-efficacy. Self-efficacy is a self-reported measure of one's confidence in the ability to make meaningful change (Bandura, 2000). Bandura's work illustrated that through modeling and imitation, people learn from observing situations (Bandura, 2000). To learn from participating in a situation "social learning," those involved must be engaged in the work, which requires motivation (Blumenfeld, Kempler, Krajcik, & Sawyer, 2006). One crucial determinant of motivation in learning environments is competence, which can be dependent on self-efficacy (Schunk & Pagares, 2002). A collaborative experience, such as data teams, can enhance the efficacy of its members (Blumenfeld et al., 2006). Therefore, study of the changes in the self-efficacy of the participants in this study is critical to understand more about what teachers learned from their participation in a data team. Indications about participants' self-efficacy to make instructional changes not only clarifies what teachers learned from engaging as members of a data team, but also alludes to how their learning might begin to have effects for their classroom instruction.

Within any collaborative group, what is learned can be more accurately described and understood if it is viewed as arising from social interaction. Studying teacher learning in groups from the perspective of social constructivism allows the investigation of different aspects of the social engagement of the participants, and the materials at hand, and how those aspects of their situated engagement lead to learning. The strength of the situated learning framework is that learning can be studied and understood in-process, instead of individuals' learning within a collaborative group being understood exclusively as an output of activity.

RESEARCH QUESTIONS

1. What do teachers learn from their situated engagement as members of a high school data team?

2. To what extent do conversations open up or close down opportunities for teacher learning within their situated engagement on a high school data team?
3. How do teachers on a high school data team describe their self-efficacy to make instructional changes?

OVERVIEW OF THE STUDY

This exploratory qualitative study will examine the collaborative work of a high school data team consisting of 6–8 teachers over the course of one school year. The study seeks to investigate the nature of what teachers learn from engaging in a high school data team and how their collaboration might influence their self-efficacy. The theories of social learning and situated learning will be used to study the learning of members of a data team relative to: changes in teachers' participation within their community of practice, the opening up or closing down learning opportunities during discussions, and changes in the self-efficacy of teachers to make instructional changes. Three to five observations of data team meetings will be audio recorded, transcribed, and then coded for discursive patterns. Patterns identified in initial observations will be used to inform interviews of data team members. The data collected from the observations and interviews as well as artifacts from data team meetings, will be analyzed to draw conclusions about how and what teachers learn from engaging in a data team.

SIGNIFICANCE OF THE STUDY

This study serves a practical purpose for educators on the field. As schools utilize more collaborative practices, there is a need for a better understanding of how and why collaboration functions within a group of teachers. Little is known about how teachers learn from collaborating with each other on a data team from the perspective of social constructivism. As more teachers and schools engage in collaborative inquiry with student data, more should be known about how and what teachers learn from participating in a data team, and how they impact teachers in terms of their professional knowledge and self-efficacy to make instructional changes. The results of this study can inform teachers and leaders who are involved in data teams in schools.

Researchers will be interested in how data teams are approached in this study. Most of the data team research focuses on their implementation, or on establishing loose correlations between their existence and student achievement. The focus on teachers' learning in this study will allow researchers to more clearly understand what teachers learn from engaging as members of a data team, and how their learning might begin to affect the classroom. There have been very few studies conducted utilizing the theoretical frame of situated learning to investigate teachers' collaborative professional development (Riveros et al., 2012). Study of teachers' social learning and self-efficacy as a result of collaborative professional development experiences is also lacking. Applying these theories to study of teachers' learning in data teams will provide researchers with information they can use to frame further research into teacher learning in collaborative situations in schools. Much of the professional development literature is focused on measuring what teachers learned through pre and post-tests sandwiched around workshops. While measuring teacher learning in this way is relevant to any professional development experience, utilizing a lens that closely studies group learning as it happens will produce more of an understanding of how teachers learn from their engagement in collaborative situations. Understanding about how learning happens in collaborative situations requires a focused study of the dynamic nature of a groups' collaborative activity. Because schools have chosen collaborative settings for the professional growth of teachers, there should be ways of learning more about how their collaboration impacted what teachers learned from the experience.

METHODS

This study will be an exploratory qualitative study that seeks to build on a slowly emerging research base on high school data teams and what teachers learn from participation in them. Observations of data team meetings, interviews of participants, and collection of artifacts from data team meetings will provide data to be analyzed from the perspective of situated learning. The dynamic interaction between theories of social learning and situated learning serve as the theoretical framework for understanding more about how and what teachers learn through their participation in data teams. It provides an avenue to understand changes in teachers' participation, discourse, and self-efficacy as it relates to their learning in social situations.

Approaching this study from the social constructivist perspective requires an emphasis on the fact that learning is not an activity that occurs solely within a person's mind, but involves the whole person, their environment, and the activities they engage in during the learning process (Sawyer, 2006). Consistent with the theories of social learning and situated learning, this study will be conducted using an integrated theoretical approach, meaning that the unit of analysis in this study will be the processes, practices, and perspectives of data team members, combined with everything that encompasses the entire situation, including the situation's physical environment (Bandura, 2006;

Greeno, 2006). The research questions were designed to illicit data about the processes, practices, and perspectives of team members. In this study I will observe and audio record data team meetings, audio record interviews with data team members, survey responses of data team members, and collect documents used during data team meetings. Analysis of these data will answer each of my research questions, with each question relying on different aspects of the collected data.

Data Collection

Three things will be simultaneously unfolding during my observations, relations between participants, practices they engage in, and their individual learning. To understand what teachers learn from being engaged in a data team, all three of these areas must be studied collectively (Sawyer, 2006). Focusing only on an individual's thought and not on the discursive context leaves the collaborative activity incompletely understood and is why interaction analysts do not use experimental designs in their studies. Without closely examining the collaboration as it happens, its power to generate learning and change within participants cannot be fully understood (Barab, 2006).

Conversational interaction is a critical component to learning within collaborative contexts, therefore it must be studied closely. Interaction at the conversational level "… results in the emergence of new insights and representations …[that] both constrain and enable the ongoing collaboration (Barab, 2006). To identify how individual knowledge emerges, interaction analysis and cognitive methodologies need to be employed to find out more about the individual learning that results from collaboration (Barab, 2006). Interviews will be used to gauge the individual learning gains relative to concepts discovered within these broader analyses of the meeting discourse (Barab, 2006). Interview questions will be based on self-efficacy surveys to illicit information about teachers' self-efficacy (Bandura, 2006). Both the observational and interview data will be collected from participants to pursue emerging themes and patterns from the discourse of group members. This data will identify shifts in teachers' self-efficacy over the course of their engagement as members of a data team, while also providing rich data relative to the level of participation of data team members, and the content of data team meeting conversations. Design, questions, and data collection are consistent with this theoretical framework within the social constructivist perspective.

Analysis of Collected Data

At least five data team meetings will be observed for a duration of over forty-five minutes for the purpose of being able code data team conversations to compare conversational patterns and to group of threads of conversation within each meeting into organizing units. Such a breadth of conversational data is required to isolate relevant conversational threads and investigate them further within the theoretical framework. The dynamic nature of both the social and situated learning of the participants engaged in collaborative activity requires a larger sample size from which to base the analysis of the organizing units drawn from the theoretical framework. An example of such a unit might be "legitimate peripheral participation", where the researcher anticipates that over time members of the community of practice would change their level of engagement in the work of the group. Others might be changes seen in conversational participation, feelings of self-efficacy to make instructional changes, and in discursive patterns that open or close down learning opportunities.

To understand what teachers are learning from their situated engagement as members of a high school data team, collected data will be triangulated. The observations and transcripts of data team meetings will provide a record of teachers' situated learning. Topics of discussion, actions, attitudes, and changes in teachers learning can be discovered through a close analysis of this record. Changes in behavior and speech can also be outlined and investigated. Analysis of verbal data can isolate specific areas of teacher learning to be further investigated in interviews. It can also be cross-referenced with documents from meetings to corroborate what and how teachers learned from others and from the documents in the data team meeting situations. The theoretical framework guides this analysis by looking at changes in the amount and quality of participation that can be catalogued over time and explained. Looking at all of these areas allows for the analysis of the processes, practices, and perspectives of data team members, all of which play a critical role in determining what teachers learn from their situated engagement in a high school data team.

Closer analysis of the observation and audio recording of data team meetings can provide insight into how opportunities for teacher learning were opened up or closed down by teacher discourse. Analysis of this data can look closely at avenues of inquiry that were pursued and ignored in the course of meetings over the year, and can potentially identify behavioral patterns that might have impacted learning opportunities for the group. Analysis of verbal data can highlight patterns of conversation, concepts, and ideas ordinarily hidden within the discourse of a group. These patterns may prove helpful in guiding analysis of the other questions, can prompt interview and survey questions for further investigation.

Analysis of this data will also help to answer the question of the extent to which the situated engagement of

teachers on a high school data team impacts their self-efficacy to make instructional changes. This is one perspective of the individual participants that can be investigated at varied points during the course of the school year. Changes over time in participants' self-efficacy can also be cross-referenced with other forms of data collected to offer possible reasons why the change occurred, and what, if anything, resulted from a change in self-efficacy. The theoretical framework guides this analysis by helping to frame any changes in the participants' self-efficacy from the social constructivist perspective.

IMPLICATIONS

This study will likely contribute the discovery of important and relevant variables that operate within data teams. It will also provide commentary relevant to on-the-job professional development from the perspective of situated learning of teachers. As schools attempt to stimulate collaboration amongst teachers, this study can also provide insight into the value of this type of data teams as an organizational routine that could be adapted by other schools (Spillane, 2011). Historically, attempts to incorporate collaborative elements within the daily practice of high schools have proved difficult (Nehring, 2009; Sizer, 2004; Tyack & Cuban, 1997). Therefore, most empirical studies on this topic chose to investigate elementary or middle school settings. Because high schools offer a unique and challenging context with many different variables such as the function of departmental cultures, my study can offer new information to researchers interested in data teams at the high school level (Datnow, 2011; Datnow et al., 2012). My study will deepen the understanding regarding the notion of teachers collaborating with data as a method of teacher learning that contributes to the educational success of students.

FUTURE DIRECTIONS FOR RESEARCH

This study may stimulate further scholarly investigation. Possible future studies may include looking more closely at teachers' instructional practices within the classroom, and how those might be affected by participation in a data team. It may also be possible to examine school-wide outcomes as a result of data teams related to a school's culture. More knowledge of outcomes of data teams can also lead to comparative studies of other types of collaborative efforts in schools. As many have come to realize the importance of professional learning within the teaching profession, it is hoped that the perspective of situated learning might be employed in other studies of collaborative on-the-job professional development experiences to learn more about how teachers learn in collaborative settings.

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The Lewis Model

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ABSTRACT

The ability to competently represent a molecule's structure and predict its properties is the essence of understanding chemistry. This qualifying paper is focused on Lewis's (1916) theory of bonding and the model he used to visualize this abstract concept. The Lewis model helps to explain bonding through diagrammatic representations that would permit a knowledgeable user to not just mentally visualize the organization of a molecule showing how atoms are combined, but also to predict the chemical and physical properties of the molecule. These diagrammatic representations are emphasized as being critically important to learning chemistry. Yet research shows that graphical representations of molecules pose a challenge to undergraduate students enrolled in general and organic chemistry courses. Furthermore, many do not even understand the purposes and uses of these representations. This paper presents a rationale for a future study to examine how proficient undergraduate chemistry students use the Lewis model and develop conceptual understanding in bonding and molecule structure.

It can be said that chemistry is about making and breaking of bonds between atoms. Chemical bonding theories, though ill-defined (Ponec & Uhlik, 1997), are central to chemistry because of their immense impact in the progress of this science. These theories explain how atoms combine together to form molecules, and why some combinations of atoms are stable while others are not. Since a molecule's structure determines many of its chemical and physical properties, the investigation of the relationship between structure and properties is the foundation of chemistry.

Currently three theories on chemical bonding are accepted and illustrated in chemistry text books – Lewis theory, Valence Bond theory, and the Molecular Orbital theory. All three theories explain the nature of the covalent bond, which in simple language is a shared pair of electrons. While Valence Bond theory and Molecular Orbital theory have roots in quantum mechanics, Lewis's theory, introduced in 1916 before the formulation of quantum mechanics, provided a crucial step in rationalizing and understanding the structure and properties of molecules, without relying on the mechanical behavior of the atom (Bader, Johnson, Trang & Popelier, 1996; Ponec, & Uhlik, 1997).

The key principle of Lewis's (1916) theory is that the covalent bond is a result of sharing a pair of electrons. To represent such an abstract concept, and help others to visualize the essential features of his theory, Lewis suggested graphical representations of the concept in a clever, resourceful, and elegant way. These representations have been widely used in the literature and textbooks, and they still maintain their status as an essential to comprehend the key chemistry concepts. For the sake of clarity and coherence, I will refer to Lewis model as a tool for the graphical representation of the major principle of Lewis's theory of the covalent bond.

The pictorial representations created by Lewis (1916) are referred to in textbooks (e.g., Tro, 2008) and in most of the literature as *Lewis structures* (e.g., Cooper, Grove, Underwood, & Klymkowsky, 2010; Furio & Calatayud, 1996; Nicoll, 2003), *Lewis octet structures (LOS)* (Zandler & Talaty, 1984) or *Lewis electron dot structures (LEDS)* (DeKock, 1987; Lever, 1972; McGoran, 1991). Again for clarity and coherence, I will refer to them here as two-dimensional (i.e., plane) representations of a molecule's structure.

By *structure* we will understand arrangement of atoms in a molecule. For example, consider the formula of an organic molecule $C_3H_7NO_2$. This formula, commonly referred to as the molecular formula by chemists, is the symbolic representation of a molecule. It only provides information about the type and the number of each atom present. To explain the molecule's structure, one needs to describe the exact positions of all the atoms, in relation to each other, making up the molecule. It is important to note that different molecules, known as isomers, can have exactly the same type and number of atoms. However, the position or the exact arrangement of these atoms can be different. The precise location, i.e., the spatial arrangement, of each atom of a molecule is known as its structure or the *geometrical shape*. I will refer the spatial arrangement of atoms of a molecule to the three-dimensional or just simply as the spatial representation of a molecule's structure.

From this it can be inferred that knowing the type and the number of each atom in a molecule, the position and the exact arrangement of these atoms, and the precise location of each atom allow one to predict the geometrical shape of the molecules, and other important properties and connections. The Lewis model incorporates all these parameters and provides the basis for learning other chemistry concepts such as acids and bases, resonance, structural isomerism, polarity, electronegativity, intermolecular forces, and reaction mechanisms (Carroll, 1986; DeKock, 1987; McGoran, 1991; Miburo, 1998; Packer & Woodgate, 1991;

Pardo, 1989; Purser, 1999; Wan-Yaacob & Siraj1992; Zandler & Talaty, 1984). Given the importance and utility of the Lewis model, research shows that generating the basic two-dimensional representation of a molecule's structure appear to be an unattainable objective for many chemistry students (Cooper, Grove, Underwood, & Klymkowsky, 2010; Nicoll, 2003; Wang & Barrow, 2011).

STATEMENT OF THE PROBLEM

This paper addresses the issues related to difficulties that many students from high school to graduate school level face in using the Lewis model to create the two-dimensional representations of a molecule's structure and using this to build its spatial representation.

Studies on students' difficulties regarding these representations have mostly focused on what students do not know or could not do (Cooper *et al.* 2010; Nicoll, 2003), or on students' understanding or perceptions of bonding in molecules with the goal of identifying and isolating misconceptions (Birk & Kurtz, 1999; Boo, 1998; Butts & Smith, 1987; Dogan & Demirci, 2011; Goh, Khoo, & Chia, 1993; Peterson & Treagust, 1989; Peterson, Treagust, & Garnett, 1986; Unal, Costu, & Ayas, 2010). These studies may assist instructors to learn the kinds of difficulties students may experience while learning chemistry. However, they obviously lack information regarding how the proficient chemistry students ***use*** the Lewis model to create the two-dimensional and three-dimensional representations of the structure of the molecule and predict its properties.

The two-dimensional representations of a molecule's structure are probably the first pictorial representations of molecules that chemistry students encounter. According to American Chemical Society (2013), in the United States, the students may be even introduced to the two-dimensional representation of a molecule as early as middle school. It is difficult to ascertain how instructors in the middle school and high school actually introduce the Lewis model and the length of time they dedicate to studying this concept. Cooper, Grove, Underwood, and Klymkowsky (2010) indicated that at the undergraduate level, instructors often assume that students are familiar with the Lewis model and area bale to generate the two-dimensional representations of a molecule's structure from its symbolic molecular formula. The students are then expected to represent the three-dimensional structure of the molecule and connect it with the myriad properties of the molecule. This poses a real challenge to the novice students. Panasuk (2010) affirmed that the "abilities to recognize, create, interpret, make connections and translate among representations are powerful communication tools for mathematical thinking" (p. 239). Likewise in chemistry, students are expected to demonstrate similar abilities by translating fluently between macroscopic (e.g., observations), submicroscopic (e.g., atoms, molecules, and bonds) and symbolic (e.g., formulae, equations, representations, and models) levels of reasoning (Johnstone, 1991, 2000).

Building on the analogy with the Paanasuk's Taxonomy of Conceptual Understanding in Algebra (2011), I will assume that those students who would be able to use the Lewis model and illustrate the two-dimensional structure of the molecule, associate this structure with the symbolic representation of the molecular formula and connect the properties of the molecules to the molecule's structure would be demonstrating conceptual understanding of the theory of covalent bonding**.** In other words, they should be able to fluently translate among multiple representations of structurally the same concept (Panasuk, 2011). Kozma and Russell (1997) contended that this ability to "transform the expression of a chemical concept or situation in one form to a different form" demonstrates "representational competence" (p.963), which is an attribute demonstrated by a practicing chemist.

Mastering the Lewis model to illustrate molecules' structures is a prerequisite to comprehending many of the sophisticated chemistry concepts. To explore this assumption a logical place to begin with is by reviewing studies on students' representations of molecules, and how they use the Lewis model. This paper intends to develop a foundation for a future study to explore characteristics of those undergraduate chemistry students identified as being proficient in the discipline to determine whether they demonstrate representational competence.

PERTINENT AREAS OF RESEARCH

The first strand discusses the Lewis model and representations of molecules' structure. It begins by discussing Lewis's (1916) pictorial representations of molecules, and explores the modifications to Lewis's (1916) original illustrations. It also examines the spatial representations used to illustrate a molecule's structure and discusses some of the limitations. This is followed by the use of Lewis's model along with the Valence Shell Electron Pair Repulsion (VSEPR) theory to determine the geometric shape of molecules. It is worth mentioning here that I have only used the term model to refer to the key principles of Lewis's theory, and have refrained from using this term to describe pictorial, physical or virtual representations. However, these two-dimensional and spatial representations are referred to in the literature as either two-dimensional or three-dimensional models respectively. Gabel (1999) warned that the use of language is a barrier to understanding essential concepts in chemistry, and this may not necessarily be related to the

complexity of the discipline. Therefore for this reason, I will continue to refer to these entities as representations.

The second strand explores studies related to students' use of the Lewis model and their representations of molecules' structure. In this strand the most appropriate studies investigating students' difficulties in representing a molecule's two-dimensional and spatial structures are reviewed. Additional studies investigating experts and novices, and high-ability and low-ability students' approach to creating two-dimensional and/or spatial representations of a molecule's structure are also considered. However, studies identifying and isolating misconceptions are ignored as they are beyond the scope of this paper.

THE LEWIS MODEL OF AND VARIATIONS

The development of the Lewis's (1916) shared electron-pair theory impinges on so many intellectual developments and "conceptual transformations" (Kuhn, 1970, p.8). Prior to Lewis (1916) proposing the shared-electron pair theory of the covalent bond, there were several competing theories based on the electrical attraction of atoms to explain bonding (Ihde, 1984; Mackle, 1954; Stranges, 1984). Although these theories were successful in explaining majority of the inorganic reaction of polar molecules, they failed to explain the type of bonding in organic compounds (and other polar molecules). Lewis's theory, which preceded the development of quantum mechanics, convincingly explained the formation and existence of non-polar molecules, as well as polar molecules. Lewis's pictorial representations were based on the arrangement of valence electrons in the formation of chemical bonds. He used these illustrations to explain the difference in bonding between polar and non-polar molecules, and related the structure of the molecule to its property (Lewis, 1916).

Lewis (1933) revealed the importance of visualization in creating representations of molecules when he wrote that he "visualized a sort of articulated skeletal structure" (p.17) to explain how he arrived at his model. Lewis (1916) had suggested using the chemical symbol of the element to be the "kernel" (p.768). This was to symbolize the nucleus of the atom, and all of the electrons, except the outermost ones (the valence shell electrons). He then suggested the use of the "colon or two dots arranged in some other manner, to represent the two electrons, which act as the connecting links between the two atoms" (p.777). So for example, Cl_2 would be represented as Cl **:** Cl, which explains that the two chlorine atoms are linked by one covalent bond. He also illustrated bond polarity (un-equal sharing of electrons in the covalent bond) by bringing the "colon nearer to the negative element … [and] write Na **:**I and I **:**Cl" (p.777). Further, Lewis (1916, 1933) used this model to create the two-dimensional structure of molecules as illustrated in Figure 1.

Lewis used the shared electron-pair theory to solve the structures of many molecules, which today appears to be simple, yet "proved extremely embarrassing" (Lewis, 1916 p. 777) to the early twentieth century chemist. The Lewis model is still employed in the teaching and learning of chemistry from high school to graduate school level, and is used widely by practicing chemists. However, the details in the pictorial representations of molecules have changed over the course of time.

Changes to Representations of Molecules

The most common changes to Lewis's (1916) original representations used in chemistry text books are solid lines as the bond between atoms. This automatically implies a shared-pair of electrons. However, the dots are still used to indicate un-bonded or lone-pairs of electron, as shown in Figure 2. These representations are now referred to as structural formulae because they imply the molecule's structure.

Harrison and Treagust (1996) described these pictorial representations as having the advantage of being easy to use because they show all bonding and non-bonding electrons, and allow students to be able to deduce the geometrical shape of the molecule. However, they commented that these representations are flawed because they are "only two-dimensional and do not show bond type and fail to work in more complex cases" (p. 514).

The Lewis model, as it preceded quantum mechanical theory of the atom, is based on static electron theories. As a result some have argued that this may mislead students to believing that electrons are stationary (Clark, 2002). From suggestions made to address this fact, the widely accepted alterations to the pictorial representation of the two-dimensional structure of molecules was to replace the

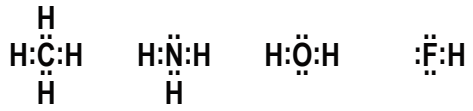


Figure 1. Lewis's (1916) representations of the structures for methane (CH_4), ammonia (NH_3), water (H_2O), and hydrogen flouride (HF)

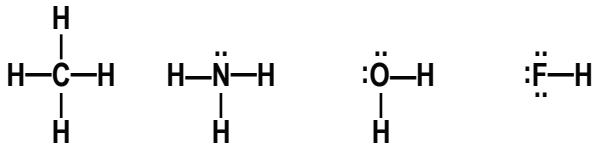


Figure 2. The most common representations of the structures for methane (CH_4), ammonia (NH_3), water (H_2O), and hydrogen flouride (HF).

electron dots with a line or a bar as shown in Figure 3 (Wan-Yaacob & Siraj, 1992; Imkampe, 1975; Pardo, 1989).

Spatial Representations of Molecules

Lewis's pictorial representations illustrated the two-dimensional structure of molecules. However, not all molecules will exhibit this two-dimensional arrangement. The Valence-Shell Electron-Pair Repulsion (VSEPR) theory proposes that the stereochemistry (spatial arrangement) of an atom is determined primarily by the repulsive interactions between the negative electron pairs in the valence shell. Thus to graphically represent the three-dimensional structure of the molecule on the plane of a paper, the Lewis's (1916) representations are further modified using dotted or dashed lines and wedges. The dashes depict the behind-the-plane, and the wedges show in-front-of-the-plane orientations as shown in Figure 4. These illustrations depict the geometrical shape of molecules, which will be addressed later.

These splayed representations, though allowing one to illustrate the three-dimensional structures of molecules on the plane of a paper, Harrison and Treagust (1996) commented that their limitation is that they do not accurately show the bond angles. The authors further added that students need time to develop the "visualization skills needed to read these diagrams" (p. 514).

With the advancement of technology, it is now possible to physically build the three-dimensional arrangement of atoms in molecules or virtually create them using sophisticated computer software. In either case there are two representations used to depict the three-dimensional structure. These are known as the space-filling and ball-and-stick representations, which are shown in Figure 5.

The space-filling representations of molecules show atoms as spheres whose radii are proportional to the radii of the actual atoms. In addition the center-to-center distances are proportional to the distances between the atomic nuclei. These representations depict atoms and bond angles all in the same scale with adequate accuracy. The limitation of these representations, as noted by Harrison and Treagust (1996) is that they do not show bond number. In other words, it is not possible to determine if the atoms are sharing one, two, or three pairs of electrons therefore making it impossible for students to decide whether the bonds are single, double or triple.

The ball-and-stick representations display both the spatial location of the atoms and the bonds between them. As in the space-filling representations, spheres are used as atoms, which are connected by rods to indicate the bonds. Double and triple bonds are usually represented by two or three curved rods, respectively, and they maintain their non-rotatable nature in contrast to the fully rotatable single bond. In more advanced ball-and-stick representations, the angles between the rods are the same as the angles between the bonds, and the distances between the centers of the spheres are proportional to the distances between the corresponding atomic nuclei. The limitation of this representation, according to Harrison and Treagust (1996) is that these representations give an impression of an openness of the molecule. Furthermore they also warned that as the spheres are generally the same size, they indicate that atoms are all the same size, which of course they are not. Moreover, as the double and triple bonds are represented as bent by using a plastic or spring, these models give the impression that single and multiple bonds are the same length, which again is incorrect.

Geometrical Shape of Molecules

Gillespie and Nyholm (1957) developed the VSEPR theory to explain the spatial arrangement of atoms of a molecule. They used this theory to qualitatively determine the location of each atom with respect to each other, and described the shape using geometrical terms. The key principle of this theory, Gillespie (1963) asserted, is based on Lewis's (1916) theory of shared electron-pairs. Crudely put, the shape of a molecule is a result of repulsion between electron pairs. This can be understood by considering the methane (CH_4) molecule shown in Figure 4(a). The central atom, which is carbon, has four electrons in its valence

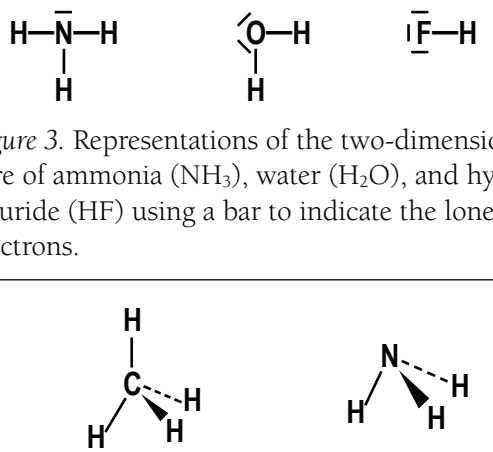


Figure 3. Representations of the two-dimensional structure of ammonia (NH_3), water (H_2O), and hydrogen flouride (HF) using a bar to indicate the lone pair of electrons.

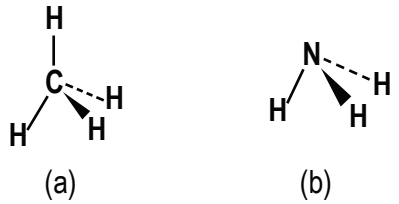


Figure 4. Splayed representations of (a) methane (CH_4) and (b) ammonia (NH_3).

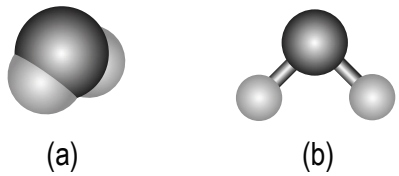


Figure 5. Spatial representation of water (a) space-filling and (b) ball-and-stick

shell. The molecular formula for methane implies that there are four hydrogen atoms. This means that four C-H bonds are formed. According to Lewis's theory, each bond is a shared pair of electrons. Therefore the four electrons from the valence shell of the carbon atom combine with one electron from each hydrogen atom. Overall then, there are eight electrons around the central atom, which means that there are four pairs of electrons. As all four pairs of electrons are involved in bonding, there are no lone-pairs. Subsequently, the four bond-pairs experience equal repulsion and each hydrogen atom bonding to the C is located approximately 109 apart in the molecule giving the molecule the shape of a tetrahedron.

It is now clearly established that shapes of molecule of many organic and inorganic compounds can be predicted if one knows the number of pairs of electrons around the central atom of a molecule. Table 1 shows the geometric shapes proposed for molecules where the central atom has 2, 3, 4, 5, and 6 pairs of electrons. However, if a molecule has one or more lone pair on the central atom then there is a deviation from the predicted geometric shape. So for the ammonia molecule NH_3 in Figure 4(b), the central atom, nitrogen has five valence electrons and there are three N-H bonds. This means out of the eight electrons around the central atom, only three pairs are involved in bonding and the fourth pair is un-bonded, i.e., it is a lone pair. So although there are four electron pairs around the central nitrogen atom, the shape is not a tetrahedron because the VSEPR theory states that the repulsion experienced by bonding electron pairs is greater when there is a lone pair present. So the shape is a said to be a deviation from the tetrahedron shape, and in this case it is a pyramid shape. Due to the increased repulsion of the lone-pair of electrons with the bonding-pairs of electrons, the bond angel is now much smaller than the tetrahedral angle.

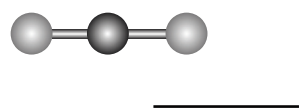
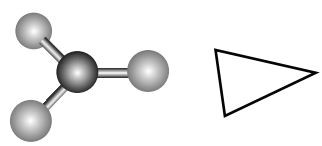
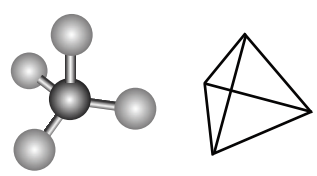
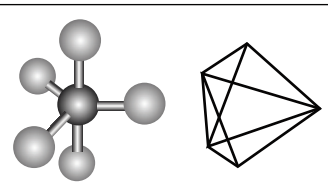
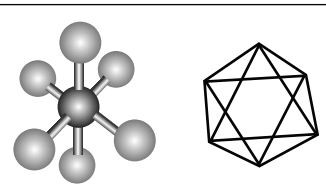
When students are asked to predict the spatial structure of a molecule they need to be able to identify the central atom, determine how many electrons in the valence shell of the central atom, use the symbolic molecular formula to deduce the number and type of bonding between the central atom and the other atoms of the molecule, and determine the number of bonding and non-bonding electron pairs.

Discussion

Currently there exist several deviations of Lewis's (1916) pictorial representations for the structure of a molecule, with each representation displaying some modifications. These may have been instigated for the ease of use in communication and/or for closely resembling the actual concept. It is possible that different instructors may have a preference to a particular way of representing a molecule's two-dimensional structure and the three-dimensional structure. Furthermore, it is also possible that the same instructor may wittingly or unwittingly use different formats of the same representation (e.g., H – Cl or H:Cl). If this occurs, then it is possible that this may add to the confusion of students understanding the representational syntax. In other words students may not comprehend how a representation encodes and present information if different formats are used (Ainsworth, 2008).

In addition, it appears that different forms of representations, whether they depict the two-dimensional arrangement or the three-dimensional arrangement of a molecule's structure, they will always be met with many limitations.

Table 1
Geometric shapes of molecules

Number of electron pairs	Example	Geometrical Shape	Bond angle
2	$BeCl_2$	 Linear	180°
3	BCl_3	 Equilateral Triangle	120°
4	CH_4	 Tetrahedron	109.5°
5	PCl_5	 Trigonal Bipyramid	120° and 90°
6	SF_6	 Octahedron	90°

However, the point that must be made here is that a representation will never correspond exactly to the more complicated abstract entity it represents. It brings certain features into focus while minimizing or obscuring other features. The Lewis's (1916) pictorial representations bring into focus the fact that a covalent bond shares a pair of electrons, and enable one to show whether the electron pair is shared equally or not. It does not show that electrons are in motion, as this is not something that can be represented accurately in a two-dimensional illustration.

Furthermore, Lewis's (1916) two-dimensional representations are often used as the most effective way to predict the geometrical shape of molecules using the VSEPR model (Carroll, 1986; DeKock, 1987; Gomba, 1998; Miburo, 1998; Purser, 1999). However, it must be emphasized that these two-dimensional representations only show how atoms are bound together in a molecule. They represent how the valence electrons are placed in the molecule so that each atom has eight electrons (except in the case of hydrogen which has two). A two-dimensional representation does not illustrate molecular shape. It can only be used to deduce molecular geometry.

Finally, the process of predicting the spatial arrangement of atoms of a molecule and generating a three-dimensional representation from the symbolic molecular formula is quite a complicated task. Bodner and Domin (2000) asserted that what the instructor may write as symbols represents a physical reality, but to the novice students these are just letters and numbers devoid of physical meaning. It is only when students connect these letters and numbers as representations of a physical reality will they be able to recognize absurdities and flaws in their responses. These difficulties are discussed in the next strand.

STUDENTS' REPRESENTATIONS OF MOLECULES' STRUCTURE

Problems in chemistry can be either mathematical or non-mathematical in nature. Being asked to generate the two-dimensional or three-dimensional representation of a molecule's structure from its symbolic molecular formula and then describe its properties is an example a non-mathematical problem. Bodner and Domin (2000) asserted that when solving such a problem in chemistry, the first step is to build an internal representation of the problem. Panasuk (2010) associated internal representations to mental images that an individual may generate in his or her mind. Bodner and Domin (2000) describe an internal representation as "information that has been encoded, modified, and stored in the brain" (p. 26). They note that each internal representation is unique to each student, and that each internal representation may highlight different features of a situation or problem. In contrast, the external representation is the means by which the student attempts to convey their internal representation. Although two students may possess different internal representations, they may both provide the same external representation.

Studies that focused on internal representations or mental images often refer to these as 'mental models' (Coll & Treagust, 2002; Coll & Taylor, 2002; Coll & Treagust, 2003; Harrison & Tregust, 1996). These studies mainly concentrate on determining students' preference for one particular representation of a molecule or type of bonding. Although these are interesting studies, it is often difficult to capture an individual's internal representations. Therefore in this strand, studies pertaining to describing students' external representations of a molecule's two- or three-dimensional structure are discussed.

Two- and Three-Dimensional Representations

There are many discussions spanning over four decades implying that students struggle with generating the two-dimensional representation of a molecule's structure (Wan-Yacob & Siraj1992; Bell, Adkins, Gamble, & Schultz, 2009; Carroll, 1986; Clark, 1984; Clark, 2002; DeKock, 1987; Gomba, 1998; Imkampe, 1975; Lever, 1972; Mcgoran, 1991; Miburo, 1998; Packer & Woodgate, 1991; Pardo, 1989; Purser, 1999; Zandler & Telaty, 1984; Wan-Yacob & Zakaria, 2000). Each one of these discussions provides step-by-step guidelines, or algorithms to support instructors in teaching students to graphically illustrate a molecule's two-dimensional representation. There are only a few studies that do actually explore what difficulties students may have using the Lewis model to generate a molecule's two- or three-dimensional representations. The studies reviewed in this section of the strand, Cooper et al., (2010), Furio and Calatayud (1996), and Nicoll (2003) focuses on students' perceptions and difficulties with plane representations of a molecule's structure.

Furio and Calatayud (1996) used a survey-type instrument to analyze students' difficulties with geometries and polarities of molecules. They were interested in students' declarative (what they should know) and their procedural (what they should know how to do) knowledge regarding geometries and polarities of molecules. Their instrument consisted of 10 items concerning geometry of molecules, and six items about the polarity of molecules. The participants in their sample included Grade 12 students (*N=85*), first year undergraduates (*N=151*), and third year undergraduates (*N=100*). The researchers had made several assumptions. They assumed that students were familiar with the VSEPR theory and be able to use it to predict the spatial structure of molecules. They assumed that students would

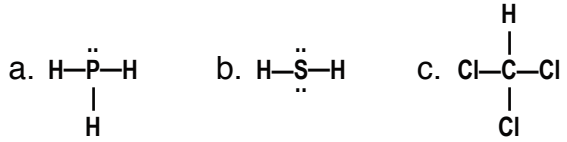
be able to pictorially represent the plane two-dimensional structure of molecules and decide how many lone pairs of electrons are on the central atom to predict the shape of the molecule. Only part of Furio and Calatayud's (1996) study is discussed here.

To test the hypothesis that students' have difficulties choosing the central atom, completing its valence shell, and pictorially representing the molecule's two-dimensional plane structure, Furio and Calatayud (1996) used the first two items of their instrument (Figure 6).

They found that over 80 % of the Grade 12, first year and third year undergraduate students were able to pick out the correct answer for item 1 – i.e., when two-dimensional representation of the molecule was presented to them. However, percentage of Grade 12 students giving the correct answer dropped in the responses to item 2 – i.e., when the symbolic molecular formula of the molecule was presented.

Difficulties in using the symbolic formula to create the two-dimensional representation of a molecule's structure is further evident in Nicoll (2003) and Cooper *et al.*'s (2010) studies. Nicoll (2003) was interested in how students conceived the submicroscopic representations of molecules. She interviewed participants (*N=56*) from five university chemistry courses. The students in her sample ranged from freshman to seniors. The interview protocol was designed to probe how students transformed from symbolic level to submicroscopic level described by Johnstone (1991). She provided students with the symbolic molecular formula of formaldehyde and asked them to first create a molecule's two-dimensional representation using dots as electrons, similar to the pictorial representations created by Lewis (1916). Then the students were presented with different colored modeling clay and two different lengths of stick to build the three-dimensional spatial representation of the molecule.

Item 1. Which of the following molecules has tetrahedral geometry?



Item 2. Which of the following molecules has tetrahedral geometry?

(a) CF_4 (b) SCl_4 (c) $SeCl_4$

Figure 6. First two items from Furio and Calatayud's (1996) instrument.

In Nicoll's (2003) pilot study, she presented the symbolic molecular formula of formaldehyde as CH_2O. This, she commented " resulted in all of the freshmen believing that the structure was simply water, H_2O, with a carbon bonded off of the oxygen" (p. 206). Based on this information, she represented the symbolic molecular formula to formula COH_2.

The most interesting finding from Nicoll's (2003) study was that all the participants in her sample had built their three-dimensional representations identical in form to their pictorial two-dimensional representations and had failed to take into account the stereochemistry of the molecule. Other interesting finding was that although over half the participants had identified carbon as the central atom, 39 % had built their models with oxygen in the center. She had also found that students do not take into consideration the type of bonding because they had omitted the double bond in the three-dimensional representation with modeling clay in many cases.

Finally, in Cooper *et al.*'s study (2010) a tablet-PC program was used for students to create the plane two-dimensional structures of molecules and ions. Cooper's group conducted two studies and collected both quantitative and qualitative data. The quantitative data were collected from students enrolled in Organic Chemistry I (*N=70*) on their ability to create two-dimensional plane representations of nine molecules. The participants were provided with the symbolic molecular formula of the molecules as: CH_4O, CH_3COOH, CH_2O, HCN, CH_3OH, CH_6N^+, $C_2H_5O^-$, CH_5O^+, and $C_2H_3O_2^-$. The qualitative data were collected first through interviewing students (*N=21*), selected from a range of chemistry courses including graduate students, and faculty (*N=6*). During the interview, the participants were asked to create the two-dimensional representations of five different molecules and ions. Again only their symbolic molecular formulae were provided as NH_2, NO, CH_4S, C_2H_6O, and C_3H_7NO. They were also asked a series of questions regarding their views of what these representations were, essential features contained in structures of these representations, and information that could be gathered from them. These same questions were then asked of a larger sample size (*N=170*) through an online data collection program.

There were two finding in Cooper *et al.*'s (2010) study, which is pertinent to this paper. First, the percentage of students constructing correct two-dimensional representations of the molecules' structures dropped as the number of atoms in the molecular formula increased from six to seven and above. Second students had difficulty in creating the two-dimensional representation if the symbolic formula was presented to them "without structural cues" (p. 871). The example in this study was using CH_4O and CH_3OH, which represents the same molecule. More students (>90%)

correctly illustrated the two-dimensional representation when the molecular formula was presented as the latter. Only 60 % of students correctly did when the molecular formula was presented as the former. This is very similar to Nicoll's (2003) finding from her pilot study when she presented the molecular formula of formaldehyde as CH_2O (without structural cues) and as COH_2 (with structural cues).

Other remarkable revelations from Cooper *et al.*'s (2010) interviews were that most of the organic chemistry instructors assume that students were capable of creating the two-dimensional representations of molecules, that little time was spent on this activity. Furthermore, a few of the faculty were also confused about producing valid two-dimensional representations of the molecules' structures.

The results from these studies clearly indicate that there are students who struggle to transform the symbolic molecular formula of a molecule to creating a valid two-dimensional representation of a molecule's structure. In addition, one of the studies (Nicoll, 2003) also showed that students struggle with building an appropriate three-dimensional representation of a molecule's structure from the pictorial representation they had just created. Two studies, Cooper et al., (2010) and Nicoll (2003), further showed that the symbolic representation format of the molecular formula influences students' ability to produce valid plane two-dimensional representations of the molecule's structure.

There may be several reasons to explain the observations from these studies. However, what was interesting in these last two studies is that the researchers assumed that students were familiar with the use of Lewis model and the theory form which this model was derived. To be able to create valid two-dimensional representations of a molecule's structure, students need to have conceptualized that a bond is a shared pair of electrons. It is possible to memorize this concept. However, conceptual understanding of this concept can only be assessed through its application such as in the case of illustrating the two-dimensional representations of a molecule's structure and then correctly assembling its spatial form and using this to predict some of the molecule's properties.

Representational Competence

Kozma and Russell (1997) examined how expert and novices responded to different symbol systems that can be used to represent information in different ways of a particular chemistry concept. The experts in their study were postgraduates (*N=11*) and the novices were undergraduates (*N=10*). Although the chemical concept they investigated was chemical equilibria, the findings from their investigations have produced fascinating insight into how experts and novices use various representations to solve problems.

In their investigation, Kozma and Russell (1997) showed the subjects 14 dynamic and still images of several chemical reactions varying from animations to chemical equations and symbolic formulae. The first task the participants were asked to complete was to group the cards with these images, label each group, and explain their reasoning behind the grouping. In the second task, the subjects were asked to describe the dynamic and still images, and transform them into various other forms, e.g., to transform a mathematical relationship into a graph.

From the first part of the investigation, it Kozma and Russell (1997) observed that the novices created more groupings with fewer cards. Each group usually incorporated the same type of representation e.g., all graphs or all equations. The experts on the other hand used a range of representations and therefore had fewer groupings, and more cards per group. In the second part, it was observed that the novices struggled with transferring chemical expressions in one medium to that of another such as creating a graph from a verbal description. In many occasions the novices were constrained by specific surface-features of the representations and struggled to move beyond this. Experts, on the other hand, were able to see different surface features as representing the same principle or concept and transform the concept from one form to another. In other words, experts in this sample demonstrated "representational competence" (Kozma & Russell, 1997, p. 963).

Bearing this in mind it is now possible to analyze results obtained by Furio and Calatayud's (1996), Nicoll (2003), and Cooper *et al.,* (2010) in terms of students' representational competence. In these studies it may be inferred that when students were presented with different symbolic representations of the same molecule (Cooper et al., 2010; Nicoll, 2003) or when given different representations of the same shape of different molecules (Furio and Calatayud, 1996), only those students who had qualities of expert chemists were able to correctly solve the problem. This means that those students who were able to successfully illustrate the two-dimensional representation of a molecule's structure would have had a single mental image of the molecule's structure regardless of the number of different ways it is presented to them. For example when the symbolic molecular formula for methanol is presented as CH_4O and CH_3OH (Cooper et *al.,* 2010), those students proficient in chemistry would easily match these two external representations with a single internal representation. Therefore it is possible to develop representational competence in students, so that they approach and perform tasks in their chemistry classes as expert chemists would.

Representational competence, though not referred to as such, was also discussed by Bodner and Domin (2000).

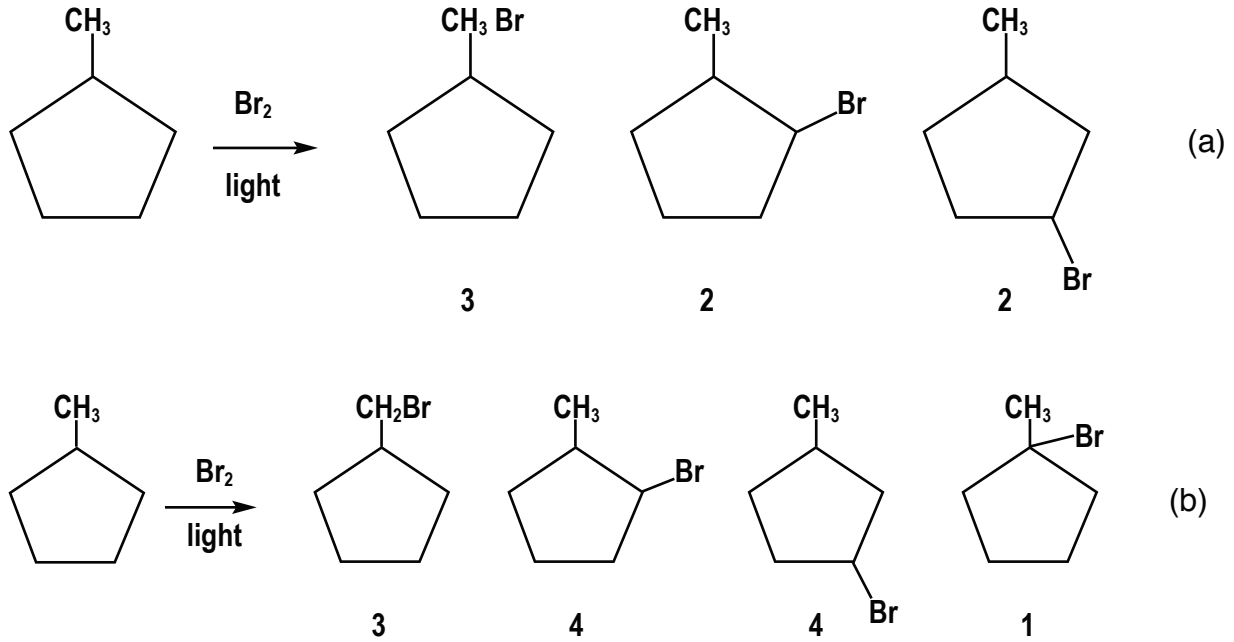


Figure 7. Organic chemistry problem from Bodner and Domin (2000).

In an organic chemistry course, students were asked to predict the ratio of the products formed when bromine reacted with methycyclopentane. The students (*N*>*200*) were provided with two options, (a) and (b), as shown in Figure 7.

Most of the students in the course had indicated that three products were formed in the ratio 3:2:3 as shown in Figure 7(a). However, a few had chosen the correct answer Figure 7(b) and predicted that there would be four products in the ratio 3:4:4:1. Bodner and Domin (2000) observed that every one of those students who had successfully solved the problem did exactly the same thing. They had "translated the line drawing for the starting material into a drawing that showed the positions of the hydrogen atoms in this compound" (p. 25) (as shown in Figure 8) and none of the students who gave the incorrect answer had done this.

Bodner and Domin (2000) thus concluded that successful problem solving of this sort include creating appropriate representations. However, the authors also noted that such a suggestion requires more rigorous research and analysis.

Students of Varying Abilities

Wang and Barrow (2011) were interested in undergraduate students' internal representations, which they had referred to as 'mental models' of molecules. Although it was decided not to review studies pertaining to students' internal representations, this study was reviewed for the reason that the researchers were interested in how students' of varying abilities understood geometry and polarity of molecules. Wang and Barrow (2011) had used various diagnostic instruments from previous studies to categorize the participants as high-scorers (*N*=*3*), moderate-scorers (*N*=*3*), and low-scorers (*N*=*3*) according to their performance on the diagnostic instruments.

Each group was then to physically create three-dimensional representations of three molecules using play-dough and straws. They were given the symbolic molecular formula of the three molecules as H_2S, SCl_2, and BF_3. This was very similar to what Nicoll (2003) had asked of her participants in her study. Wang and Barrow (2011) however, wanted the students to describe the features of their physical representations of the molecule's structure and explain the molecule's polarity.

On analyzing the students' approach to the problem situation, and their ability to create the three-dimensional structures, Wang and Barrow (2011) identified five characteristics, which they assert distinguished the high-scoring individuals from the low-scorers. From these five characteristics, the first two were the most interesting as they describe the students' approach to generating the three-dimensional

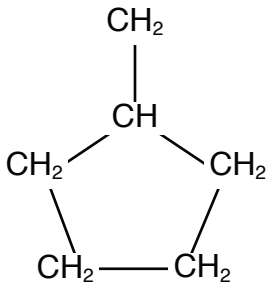


Figure 8. Two dimensional representation of methylcyclopentane.

representations of a molecules' structure, whereas characteristics three, four and five described the metacognitive skills the students had employed.

The first characteristic, according to Wang and Barrow (2011) was to do with the ability to visualize. They asserted that high-scoring students were able to perform visualization skills because these students were able to generate the physical spatial representations of the molecules' structure, without illustrating its two-dimensional representation on paper. The researchers commented that this ability may have been a result either from their familiarity of the molecule or it was a result of creating a mental image of the molecule's two-dimensional structure in their head. The low-scoring students however, relied on creating the two-dimensional pictorial representation of the molecule's structure on paper. According to Wang and Barrow (2011), this reliance on the two-dimensional representation often resulted in the low-scoring students making incorrect inferences about the molecule's spatial structure. However, the researchers did not disclose too much detail as to what those incorrect inferences were.

Second characteristic identified by Wang and Barrow (2011) was based on how students used two-dimensional representations. The researchers found from their interviews that the high-scorers only created pictorial two-dimensional representation of unfamiliar molecules using dots as electrons in much the same ways as Lewis (1916) had done. These illustrations were performed either prior to generating the spatial representation of the molecule's structure, or at the end as a means of verification (Wang & Barrow, 2011). The low-scoring students, on the other hand, relied completely on creating a pictorial two-dimensional representation of the molecule's structure before creating its spatial representation.

From these observations Wang and Barrow (2011) contended that for the high-scorers the pictorial representations "provided cues about numbers of bonds and the lone pairs in the molecule" (p. 580). However, the researchers implied that the low-scorers in their sample used the cues from their pictorial representation of the molecule's structure to recall from memory an algorithm to create the spatial representation. If the molecule was unfamiliar to them, the low-scorers used the pictorial two-dimensional representation as the "basis for inferring and reasoning" (p.586).

In Wang and Barrow's (2011) study high- and low-scoring students from diagnostic instruments were compared in attempting to solve the same problems. As several diagnostic instruments were used by the researchers to categorize the students, it may be assumed that the high-scorers in this sample are in fact proficient chemistry students. However, the analysis was based on a sample size of three students. Furthermore these students were asked to create spatial representations of molecules that were much less complex than those used by Cooper *et al.* (2010) in their study. In addition, Wang and Barrow (2011) used the term 'mental models' very broadly to refer to the mental image students may have had and to refer to the three-dimensional spatial representation the students created. Gaining access to students' mental images is a difficult task. Coll and Treagust (2003) warned that because mental representations are personal in nature, research findings "inevitably represents the researcher(s)' interpretations of participants' expressed models" (p. 467) and this is mediated by the researchers' beliefs and the research tools used. Nevertheless, Wang and Barrow's (2011) study provided some important information regarding the fact that there is a difference in the manner in which different abilities of students generate two-dimensional representations and/or spatial representations of a molecule's structure given only the symbolic molecular formula.

Discussion

The ability to use multiple representations and translate between multiple representations is a unique quality of an expert chemist (Kozma & Russell, 1997). Bodner and Domin (2000) had observed that in organic chemistry classes, students who were successful in solving problems involving reaction mechanisms, created appropriate two-dimensional representations of the symbolic molecular formulae in the chemical equations. Furio and Calatayud's (1996), Nicoll (2003), and Cooper *et al.,* (2010) all found that even though a majority of the students were not able correctly determine or generate the two- and/or three-dimensional structures of molecules, there were still a minority who were able to do so. This therefore appears to indicate that there are undergraduate students who may have reached the level of competence of an expert chemist in their conceptual understanding of the Lewis model. However the studies reviewed in this strand had all assumed that students were familiar with the Lewis model and the theory of the covalent bond. The students in these studies were not questioned as to how they used the Lewis model to either create the two-dimensional representations and/or the three-dimensional representation of the molecule's structure.

CONCLUSION

Lewis's theory of the chemical bond is one of the most important contributions to chemistry. Literature emphasizes that the Lewis model is an essential part of chemistry because an understanding of it and using it to create the plane or spatial representation of a molecule's structure is crucial for predicting many of the important properties of the molecule. Moreover the Lewis model is said to subsume many

essential chemistry concepts such as acids and bases, resonance, molecular geometry, structural isomerism, polarity, electronegativity, intermolecular forces, and reaction mechanisms. There are however, four decades of anecdotal testimonials of various instructors, and a few studies suggesting that students from high school to graduate school level experience difficulties in creating valid plane and spatial representations of a molecule's structure.

Much of the literature focuses on what students are unable to do, their incorrect interpretations, and limited understanding of issues regarding important chemical concepts. However exploring how proficient undergraduate chemistry students use the Lewis model and develop conceptual understanding in bonding and molecule structure is yet an uncharted research area.

Representational competence is a skill and a practice whereby expert chemists are able to use a variety of representations and transform between them to express and communicate a particular chemical concept (Kozma & Russell, 1997). Although many students may struggle with creating two-dimensional or spatial representations of a molecule's structure from its symbolic formula, there are a few who are able to successfully perform this task. Perhaps by exploring the characteristics of representational competence in these proficient chemistry students, with respect to representing a molecule's two-dimensional and spatial structures, it may be possible to develop this quality in other students.

POTENTIAL AREA OF RESEARCH

Based on the above, an argument can be made that there is a need for additional and perhaps novel research to explore the ways the competent and proficient chemistry students use the Lewis model. It seems important to investigate how these students use the model and how they build conceptual understanding of bonding "using multiple representations of the structurally the same concept" (Panasuk (2011, p. 228). In turn, if these students demonstrate conceptual understanding of the theory of covalent bonding, one can hypothesize that they should be able to use the Lewis model and illustrate the two-dimensional structure of a molecule and associate this to the symbolic representation of its molecular formula. Then they are likely to be able to generate the spatial representation of the molecule, and finally connect the properties of the molecule to the molecule's structure. Bearing this in mind the potential research question is proposed.

- How do the proficient undergraduate chemistry students use the Lewis model and develop conceptual understanding in bonding and molecule structure.

In order to address this main question the following sub-questions will be considered to design an appropriate instrument to measure conceptual understanding of covalent bonding and representational competence.

- What are the ways in which the successful students approach the problem of creating the plane two-dimensional and spatial three-dimensional structures of molecules when two representations of the molecular formulae are presented, first as the symbolic molecular formula with no structural cues and second with structural cues?
- What are the ways in which the successful students approach the problem of creating the plane two-dimensional and spatial three-dimensional structures of molecules with more than five atoms?
- How do the successful students use the plane two-dimensional and spatial three-dimensional structures of molecules to predict some of the properties of the molecules?
- How can representational competence be developed in low-achieving chemistry students?

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The Accumulation Function:

What Does Research Tell Us?

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Editor's Note: Because of the significant amount of inline mathematical copy, this qualifying paper is presented in its original format.

Abstract

Research shows that students do not have a strong conceptual understanding of the relationship between the derivative and the integral in calculus. It is suggested that studying the accumulation function may strengthen students' conceptual understanding of that relationship. This paper examines components in understanding the accumulation function, the presentation of the accumulation function in three typical high school textbooks and the Advanced Placement (AP) course description for Calculus, as well as an analysis of the literature.

The Accumulation Function: What Does the Research Tell Us?

Statement of the Problem

The study of calculus is required in many domains of mathematics, science, and business. There are three major concepts in calculus: the limit, the derivative and the integral. Researchers have shown that students have little conceptual understanding of the relationship between limit, derivative, and integral and, therefore, struggle to apply calculus to solve real problems (Orton, 1983; Bagni, 1999; Nguyen & Rebello, 2011; Judson & Nishimori, 2005). There is one more concept that is very comprehensive in calculus called *accumulation*. Kouropatov and Dreyfus (2012) state that the concept of accumulation can be thought of in two different ways: getting more of a known quantity or accumulating small parts of a whole when the whole is unknown. These two facets of the notion of accumulation are directly linked to the concept of the accumulation function in calculus (p.3). It has been proposed that understanding the accumulation function may strengthen students' conception of the relationship between the derivative and the integral (Thompson, 1994, Kouropatov & Dreyfus, 2012). Anecdotal evidence suggests that students struggle to understand the accumulation function (Kaput 1997; Tall, 2010).

This paper analyzes the literature related to learning the accumulation function as a major concept in calculus. In the next sections the accumulation function will be defined. I will address some ideas why the accumulation function is critical in mathematics. In the first strand, *the Nature of the Accumulation Function*, I will discuss the mathematics of the accumulation function, and sub-concepts of the accumulation function. The second strand of this paper is *Learning the Accumulation Function*. How is this concept presented in textbooks and in the

Advanced Placement guides? What have research studies shown about learning the accumulation function? Finally, I will propose two questions for further research.

Defining the Accumulation Function

In the study of calculus the definite integral is the area under a curve on a given interval. The accumulation function is a function that describes the area under a curve on a given interval, where the right bound of the interval is changing. Alternatively, the accumulation function can describe the net change of a quantity for a given rate during a certain time interval where the amount of time elapsed is changing. Thus, the accumulation function is represented by the integral. For example, if the velocity of a particle is represented by $\mathrm{v(t)} = 0.125\mathrm{t}^2 - 1$, then the distance the particle travels can be described as function of *t*. If the velocity of the particle is measured between 3 and 5.75 seconds, then the total distance traveled over the 2.75 seconds can be calculated as a sum of the parts, i.e., $\int_3^{5.75}(0.125\mathrm{t}\hat{}2 - 1)\mathrm{dt}$. This is a definite integral. Figure 1 (see below) shows a representation of this distance as the area between the curve, *v(t),* and the *x* axis from *x* =3 to values of *x*= 5.75. This area is the distance traveled by the particle during the 3 second and 5.75 second interval:

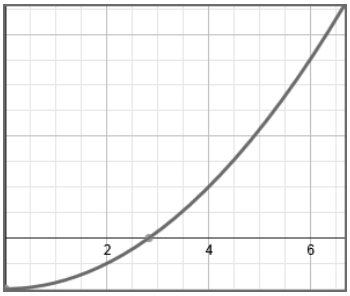

Figure 1. Example of the accumulation function.

If the time elapsed was between 3 and 6.75 seconds, the distance traveled would be equal to the distance traveled between 3 and 5.75 seconds plus the distance traveled between 5.75 and 6.75 seconds. In other words, if we want to calculate the distance over the time period from 0 to *t*, then the distance will be calculated as $\int_0^t (0.125t\text{^}2 - 1)dt$. This is an accumulation function.

The Place and Significance of the Accumulation Function in K-12 Curriculum

There are certain reasons that support the importance of learning the accumulation function. Kaput (1997) states that the study of calculus is part of a venerable institution that provides the foundation for much of our studies in science and technology (p.731). Students interested in these fields of study are often required to learn calculus. Kaput and Roschelle (1997) consider the underlying concepts in calculus to be variable rates of changing quantities, the accumulation of those quantities, the connections between rates and accumulations, and approximations (p. 2). While Kaput (1997) contends that only 10% of the school population will use calculus, he advocates that the mathematics of variation and change should be taught to all students to deepen their understanding of k-12 mathematics curriculum (p. 2). For example, Nemirovsky, Kaput & Roschelle (1998) suggest that quadratic functions can be taught as the accumulation of linearly changing quantities, and the linear function as the rate of change of quadratic quantities (p. 3). Roschelle and Kaput (1996) argue that "By supporting conceptual growth of powerful ideas, we will prepare all citizens to describe, discuss, design, and analyze processes of change" (p. 1).

Historically, two prominent scientists, Newton and Leibnitz, independently recognized that the concepts of rate of change and accumulation are closely related (Thompson, 1994, p.8). Newton observed that the rate of change of the accumulated quantity is equal to the immediate accrual (Carlson & Thompson, 2005, p.9). In other words, the value of the function is an

accumulation of the rate. The connection between the derivative (rate of change of a quantity) and the integral (change in a quantity given a rate) is the foundation for the Fundamental Theorem of Calculus.

Various researchers discuss the importance of the relationship between the rate of change and the concept of accumulation. Schnepp and Nemirovsky (2001) state that "accumulation of speed or the speed of accumulation both get us back to the quantity we started with, with the added property that in the process we lose information about a constant value" (p.91). According to Schnepp and Nemirovsky (2001) the relationship between the concept of accumulation and rate of change should be clear, so that the concept of accumulation is understood to always occur at a certain rate. In relation to the integration concept one should realize that the rate at any given point is equal to the value of the function being integrated. In relation to the differentiation concept, the value of the function being differentiated is equal to the amount accumulated up to a specific point. In essence, the rate of change is cumulative (p. 91). In summary, the notion of accumulation embraces the three major concepts in calculus and it is critical in mathematics education to build conceptual understanding in calculus.

When high school calculus students analyze the accumulation function the students are expected to reason about the behavior of the accumulation function. Students are expected to make the connection between functions, their derivatives and integrals. They are expected to recognize that analyzing the functional values and other properties of the functions (e.g., slope) assist in analysis of the behavior of the accumulation function. Thus the accumulation function can provides the basis to relate several essential concepts of calculus, providing students with the opportunity to develop conceptual understanding in calculus.

Since the accumulation function is a critical component of learning calculus, it is important that teachers and researchers clearly determine the components of a conceptual understanding of the accumulation function. If a student can apply Riemann sums, does it mean the student understands the accumulation function? Must the student be able to visualize the impact of changing the upper limit of the integral on the area under the curve? Are all of these necessary for conceptual understanding of the accumulation function? In this paper I will examine how various textbook authors and researchers have defined understanding of the accumulation function and how this function is presented in a sample of textbooks used in high schools, as well as *in the Advanced Placement Course Description for Calculus* (2012). Additionally, I will analyze specific research studies that have been done in the areas teaching and learning of integration and accumulation.

Strand I: The Nature of the Accumulation Function and Its Components

According to Thompson and Silverman (2008) "the major source of students' problems with the idea of the accumulation function is that it is rarely taught with the intention that students actually understand it" (pp. 4-5). Instructors tend to teach for conceptual understanding of the definite integral, a specific instance of an accumulation function (p. 5). Since students have little experience with the accumulation function, despite students' ability to use algorithms to solve problems involving the definite integral, there is little evidence of students' conceptual understanding of the accumulation function (Tall, 1990).

In the next sections I will look at the definition and its components that may be critical for a student to understand the accumulation function.

Definition of the Accumulation Function

The formal mathematical definition, given by Kouropatov and Dreyfus (2012), of the accumulation function is:

Given a function *f(x)*, defined and bounded on $[a,b]$, the function $F_a(x) \approx F_{\Delta x,a}(x) = \sum_{i=0}^{\frac{x-a}{\Delta x}} f(a + i\Delta x)\Delta x\,, a \leq x \leq b,$ also defined on $[a,b]$, is called the accumulation function or integral of *f(x)* (equality is obtained in the limit $\Delta x \to 0$), or

$$F_a(x) = \int_a^x f(t)dt = lim_{\Delta x \to 0} \sum_{i=0}^{\frac{x-a}{\Delta x}} f(a + i\Delta x)\Delta x\,, a \leq x \leq b$$

(p.3). As previously stated, the accumulation function can be informally defined as the area under a curve over a changing interval.

The accumulation function is not always represented as a graph with the area under the curve. Figure 2 is an example from the *2013 AP Calculus AB* exam where the given function is a rate at which unprocessed gravel arrives at a plant. The total amount of unprocessed gravel that has arrived at a specific time, x, in an 8-hour period will be represented by the accumulation function $\int_0^x G(t)dt$.

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \leq t \leq 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 8$, the plant processes gravel at a constant rate of 100 tons per hour.

Figure 2. AP Calculus AB 2013 Free Response #1.

What Does It Mean to Understand the Accumulation Function?

To discuss "understanding" in mathematics one must have a theoretical framework to which to refer. There are several frameworks that may provide relatively valid explanations to the process of building and assessing conceptual understanding in mathematics.

APOS theory. Dubinsky and McDonald (2001) have developed a theory, Action-Process-Object-Schema (APOS), which helps to understand the student learning-process. This theory is an extension of Piaget's work on reflective abstraction, which is based on a general coordination of actions. The abstraction from these actions synthesizes into a new object with its own properties (Piaget, 1980). Dubinsky and McDonald hypothesized that "mathematical knowledge consists in an individual's tendency to deal with perceived mathematical problem situations by constructing mental actions, processes, and objects and organizing them in schemas to make sense of the situations and solve the problems" (p. 2).

According to Dubinsky and McDonald (2001), an *action* stage is perceived by the individual as an external process that transforms an object through step-by-step instructions either from memory or from a set of instructions (p. 2). For example, an individual with an action conception of the accumulation function might only be able to evaluate an accumulation function for a specific value of x. Suppose an individual is given $F(x) = \int_3^x (t+1)\, dt$and asked to evaluate *F(5)*. A student with an action conception would likely rewrite the integral as ${}_3^5\, t + 1\, dt$, take the anti-derivative of $t+1$, *F(x)*, and apply the Fundamental Theorem of Calculus ($F(5) - F(3)$).

Dubinsky and McDonald (2001) contend that if the individual repeats an action enough times and reflects upon the process, the student is likely to make an internal mental construction called a *process*, the next stage of learning.The student with a process conception should be able to execute an algorithmwithout the need for the external stimulus. At this level of understanding a student can imagine performing the process without actually doing it, and may be able to reverse the process or combine it with other processes (p. 3). For example, suppose we are given that$F(x) = {}_3^x \frac{t}{t^3+1}\, dt$and *F(3) = 2*, and asked to find *F(5)*. A student with an action conception

would struggle with a problem such as this, since there is no elementary anti-derivative of $\frac{t}{t^3+1}$.When he is asked to find *F(5)* he will need to recognize he cannot simply substitute in 5 for *x* and take the anti-derivative of the integrand. The individual with a process conception of the accumulation function would be able to re-organize the Fundamental Theorem of Calculus into the form $F(5) = 2 + \int_3^5 \frac{t}{t^3+1} dt$and recognize that the value of the integral can be numerically approximated using a graphing utility, even if there is no elementary anti-derivative. Being able to combine these concepts would be a demonstration of *processunderstanding*.

An individual is said to have *object understanding* when the student realizes that the process is an object in itself, and "is aware of the transformations that can act on it" (Dubinsky & McDonald, 2001, p. 3). In regards to the accumulation function, an individual with object understanding would be able to recognize that$F(x) = \int_3^x t + 1\ dtis$ a function and be able to comment on features of *F(x)* including intervals over which *F(x)* is increasing or decreasing, and finding extrema.

The final level of understanding in APOS Theory is *schema*. Dubinsky and McDonald (2001) state that the schema is the individual's framework relating the actions, processes, objects and other schemas by general principles (p.3). According to Dubinsky and McDonald schema is very similar to the concept image of Tall and Vinner (1981). Tall and Vinner define a concept image as the set of all mental images a student has about a given concept, including its properties (p. 356). However, Dubinsky and McDonald (2001) claim that the requirement of coherence differentiates schema and concept image, "the framework must be coherent in the sense that it gives, explicitly or implicitly, means of determining which phenomena are in the scope of the schema and which are not" (p. 3).

While Dubinsky and McDonald (2001) contend that there is a hierarchy to the constructions of APOS, they state that when an individual is developing his understanding of a concept the order is not necessarily linear. Instead, a student might think about one type of problem, reflect on these, start to think of a process, then return to action conception for more complex problems of the same concept, reflect on these new examples and further develop the process understanding (p.4).

Possible genetic decomposition for accumulation function. In order to use APOS theory to describe particular mental constructions researchers develop a *genetic decomposition*. Cooley, Trigueros, and Baker (2007) define genetic decomposition as "a description of specific mental construction one may make in understanding mathematical concepts and their relationships" (p.372). In this section I discuss components that might be considered in developing a genetic decomposition for the accumulation function.

Thompson and Silverman. In discussing a well-structured understanding of accumulation functions, Thompson and Silverman (2008) develop four different components: conception of function, covariational understanding between x and f, imagining accumulation and its quantification, and understanding Riemann sum as a function. Thompson and Silverman contend that students must have a *process conception* of the integrand, $f(t)$, as a function's defining formula. Thompson and Silverman describe process conception of the formula $f(t)$ as the student understanding, without having to actually do the calculations, that calculated values for $f(t)$ will be numbers and that these numbers depend only on t (p.2). In APOS theory, Dubinsky and McDonald (2001) use a similar term, *process understanding*, meaning the learner can imagine the result of a process, without actually doing the process (p. 3).

Thompson and Silverman (2008) state that to understand the accumulation function students must have "covariational understanding of the relationship between *x* and *f*" (p. 2), where *f(x)* is the integrand. Students must not only know that as *x* changes there is a corresponding change in *f(x),* but also have a mental image of how the change in *x* effects the change in *f(x)* (p.2).

The third component in Thompson and Silverman's (2008) description of understanding the accumulation function requires students to "imagine an accumulated area for each *x*" (p. 3). Assuming the accumulation function is represented by $F(x) = \int_a^x f(t)dt$, students must be able to coordinate three different quantities: *x*, *f(x)* and *F(x).* Students must simultaneously coordinate changes in *x* with changes in *f(x)* and imagine how the bounded area under the graph is accumulating (p.3). Fundamentally this image is a space curve, with the general point $(x, f(x), \int_a^x f(t)dt)$. While it is not necessary for students to conceptualize a space curve, Thompson and Silverman contend that understanding accumulation as a function is the same as expecting understanding of a space curve (p. 3).

Riemann sums are often used to estimate area under a curve. Thompson and Silverman (2008) state that for students to understand the accumulation function they must be able to have a mental image of the Riemann sum as the sum of products, where one factor is the change from one point to the next (Δx) and the other factor is a function value (*f(c)*) (p.4). Recall the problem from before where unprocessed gravel is arriving at a plant is modeled by the function *G(t),* where *t* is measured in hours and *G(t)* is measured in tons per hour. The total number of tons of unprocessed gravel that arrives at the plant during the first two hours of operation, based on 30 minute intervals, can be approximated by the following Riemann sum: *G(0) * (0.5) + G(0.5) *(0.5) + G(1)*(0.5) + G(1.5)*(0.5) + G(2) *(0.5).* Each product represents the approximate

number of tons of unprocessed gravel that arrived in that half hour interval. According to Thompson and Silverman, if students understand the accumulation function, they would be able to visualize the sum of the products or the graph with the rectangles approximating the area under the curve(p.4).

Thus, a genetic decomposition for the accumulation function according to Thompson and Silverman (2008) would likely include the following components: function, covariation, accumulation, and Riemann sums.

Kouropatov and Dreyfus. To describe students' understanding of the accumulation function, Kouropatov and Dreyfus used the Abstraction in Context theory developed by Hershkowitz, Schwartz and Dreyfus (2001, p.202) as a framework. In Abstraction in Context, learners reorganize previous concepts that are vertically aligned to construct new concepts. These concepts are specific to a context, in this case, the accumulation function. Kouropatov and Dreyfus (2012) state the accumulation function is *proceptual* in nature, i.e., symbolic or numeric representations that includes both concept and process. The accumulation function is a process of change, as the right border of the area moves. This process can be described by a graph,which is the objectification of the process of change (p. 13)

In an analysis of the content domain for the accumulation function, Kouropatov and Dreyfus determine three "knowledge elements" (p.13): complex co-variation, accumulation function meaning, and accumulation function properties. In complex co-variation, the value of the accumulation function depends on the point where the accumulating ends as well as on the integrand function (p. 13). Kouropatov and Dreyfus define *accumulation function meaning* to be if a function is defined and continuous on a closed interval [a,b], when a sub-interval is chosen,

and only the end point for the accumulation changes, the value of the accumulation will have a corresponding change in value (p.14). The third knowledge element of the accumulation function is that the properties of the function of the curve will affect the properties of accumulation (p. 14). As stated previously, the accumulation function is a function, and as such its graph has certain properties. For example, when the function of a curve is increasing the value of the accumulation function will increase as well.

In addition, Kouropatov and Dreyfus (2012) categorize constructs of sub-concepts that are likely to be relevant in developing the accumulation function concept. Kouropatov and Dreyfus list four such components: simple covariation of a function, understanding the symbolic meaning of definite integrals, understanding the symbolic meaning of a definite integral with a variable upper limit, and approximation of area using geometric formulas of rectangles, trapezoids, etc.

Thus a genetic decomposition of the accumulation function using Kouropatov and Dreyfus'(2012) elements of knowledge might consist of the following components: simple covariation, complex covariation, accumulation function meaning, accumulation function property, understanding symbolic meaning of definite integral, and approximation of area.

Comparison of two frameworks.The three knowledge elements of Kouropatov and Dreyfus (2012) are somewhat encapsulated in Thompson and Silverman's (2008) component of imagining and quantifying the accumulation function. Kouropatov and Dreyfus' list of sub-concepts of the accumulation function is similar to the decomposition of Thompson and Silverman. Both decompositions include an element of covariation of the integrand's function. Kouropatov and Dreyfus (2012) define the symbolic meaning of definite integrals in a similar manner to Thompson and Silverman's Riemann sum component, the integral as a sum of the

partial products. Kouropatov and Dreyfus' fourth sub-concept includes recognizing that if the upper limit is a variable, the meaning of the parts of the definite integral hold (i.e., sum of the partial products, values of the endpoints, etc.) with one difference: the endpoint of the accumulation is changing (p.14). Finally, Kouropatov and Dreyfus (2012) state students must have prior knowledge of approximation of area using rectangles, etc. In other words, students must understand that the value for any accumulation function of a continuous function can be estimated by using the area of known objects: rectangles, trapezoids, triangles, etc. (p.14). Thompson and Silverman do not explicitly state the need for students to be able to find the area under a curve using geometric objects in their discussion of either the Riemann sum component or the imagining accumulation and quantification component. However, traditionally students are taught to estimate the area under a curve geometrically at the start of learning integration. It is reasonable to assume that Thompson and Silverman would expect students to be able estimate accumulation based on geometric objects.

Thus the accumulation function is composed of sub-concepts such as the Riemann sum, covariation, function and accumulation. The accumulation function can be viewed as a process, such as summing the products of a rate and time, and as an object as an area describing this process.

Strand II: Learning the Accumulation Function

Learning about the accumulation function is a valuable activity in itself because it is a sophisticated and complex mathematical object that requires the process of abstract reasoning (Thompson & Silverman, 2008; Kouropatov & Dreyfus, 2012). Hershkowitz, Schwartz, and Dreyfus (2001) define abstraction as "an activity of vertically reorganizing previously

constructed mathematics into a new mathematical structure"(p.203). Their characterization of abstraction is that of a process that takes place in a complex setting. The setting affects the process of abstraction and consists of not only the task, but the tools, the teaching methods, and the personal histories of the students (p.204). According to Hershkowitz, et al., abstraction is often viewed as a linear process from concrete to abstract. Abstraction has been viewed as a de-contextualization of a task, recognizing similar properties in concrete tasks and being able to view the commonalities as an object in itself (p. 1971). However, Hershkowitz, et al., contend that abstraction is not a linear process. Instead, the process of abstraction begins with an undeveloped form of the abstract entity and progresses to a more developed form of the entity, one which emphasizes new aspects of the concrete representation (p.200). In studying the accumulation function, students find the value of definite integrals with a changing upper limit. This is followed by the interpretation of the changing area as a function of the right-hand boundary of the area, or the upper limit of the definite integral.

Formal Representation of Accumulation Function in Textbooks

Three different high school calculus text books will be compared, as well as the *Advanced Placement Calculus Course Description (2012).* First it is necessary to differentiate between two calculus curricula: traditional and reform.

Traditional calculus and reform calculus. As previously stated, calculus has a venerable tradition of hundreds of years. Kaput (1997) contends that calculus textbooks written in the 1970's are remarkably similar to the original textbooks written by L'Hopital, the Bernoullis and Euler in the fifteenth and sixteenth centuries (p.731). In traditional calculus, according to Tall, Smith & Piez (2008), there is an emphasis on symbolic manipulation for

differentiation, integration and solving differential equations. Graphs are used to illustrate or complement the symbolic representations. Traditional calculus is typified by a formal approach based on the epsilon-delta definition of a limit (p.6).

In the 1980's mathematicians became concerned that students in calculus were not learning mathematics in a meaningful way and, in addition, many students were choosing not to study mathematics (Tall, Smith & Piez, 2008; Kaput, 1997; White & Mitchelmore, 1996). Tall, Smith & Piez report that a committee was formed in the United States called the Calculus Reform on the First Two Years (CRAFTY), a sub-committee of the Mathematical Association of America (p.2). The sub-committee made several recommendations for teaching calculus. Some of the recommendations included historical and cultural impact, engaging realistic applications, and methodology to build conceptual understanding. In addition, the committee recommended that all students should initially read and write mathematical arguments intuitively, and students who will continue on in higher mathematics should be required to read and write more rigorous mathematical arguments (p. 2).

There has always been a need for calculus students to learn rate of change and integration in multi-modal presentations: verbal, numerical, graphical and symbolic. New technologies allow for students to use these different representations (Tall, Smith & Piez, 2008; Kaput, 1997). The findings of committees like CRAFTY, coupled with newer technologies led to calculus reform. According to Tall (2010), the objectives of calculus reform are to focus on the inter-relationships of different representations of the limit, rate of change and integration (p.3). Tall, Smith and Piez (2008) note that concepts are introduced to students through experiences with verbal, numeric, symbolic and graphic representations, and is often referred to as *rule of four*.

Teachers introduce a new concept in a real-life context, requiring some sort of technology, active student exploration and discussion(p. 11).

Reform calculus has had many variations, but for the most part Tall (2010) believes that the current curricula of calculus reform reflects "largely a retention of traditional calculus ideas now supported by dynamic graphics for illustration and symbolic manipulation for computation" (p. 3). Dubinsky (1994) states that while the original goals of calculus reform were to change the content of calculus, to make the concepts more modern, and the applications more realistic, the current state of reform calculus is very similar to traditional calculus. While it is difficult to examine every text that is used in high school calculus, many textbooks support both Tall's and Dubinsky's claims. In the next section I will examine the presentation of the accumulation function in a sample of three calculus texts often used in high schools: Larson, Hostetler & Edwards (2006), Hughes-Hallett et al. (2002) and Stewart (2003).

Accumulation function in textbooks. In examining the indices of the three textbooks mentioned above, only Larson et al. (2006) have a listing for the accumulation function. However, the entry in the text occurs as an interpretation of a specific example, rather than part of the main body of the text. While neither Hughes-Hallett et al. (2002) nor Stewart (2003) specifically use the term *accumulation function*, the components of understanding the accumulation function as defined in the earlier part of this paper are present in all three texts.

Although Larson et al. (2006) refers explicitly to the accumulation function, the structure of the presentation of the accumulation function in the text is very traditional, emphasizing symbolic manipulation and formal mathematics. First, anti-derivatives are introduced as a solution to a differential equation and then the properties of indefinite integrals are presented.

Area under the curve is introduced as an approximation of a plane region, followed by the definition of a Riemann sum, definition of a definite integral, and the properties of definite integrals. These are considered to be sub-concepts of the accumulation function, according to Kouropatov and Dreyfus (2012). Next Larson et al. present the Fundamental Theorem of Calculus, followed by the concept of average value of a function over a closed interval. The accumulation function is mentioned in discussion of the second part of the Fundamental Theorem of Calculus. Larson et al. (2006) make the connection of the sum of the partial products in notes of this section,

Although Stewart (2003) and Hughes-Hallet et al. (2002) do not specifically use the term accumulation function, their presentations focus less on symbolic manipulation and more on the connections between the concepts of derivative and integral than Larson et al. (2006). Stewart begins his discussion of integration with anti-derivatives, but does not introduce integral notation. He then considers the relationship between areas and distance, "in trying to find the area under a curve or the distance traveled by a car, we end up with the same special type of limit" (Stewart, 2003, p. 315). In this manner Stewart introduces the notion of Riemann sums, and follows this with the definition of the integral notation as the limit of a Riemann sum and properties. Again, the components of understanding the accumulation function are clearly presented. Stewart introduces the Fundamental Theorem of Calculus as an accumulation function without identifying that function as accumulation. Thus, Stewart uses a more conceptual approach to the presentation of the Fundamental Theorem and directly discusses the accumulation function without using the specific term, *accumulation function*.

The third text that was examined was Hughes-Hallet et al. (2002), one of the texts citedby Tall, Smith and Piez (2008) as an example of a reform calculus text. In the introduction of the

integral, Hughes-Hallet et al. discuss how to find distance traveled given velocity and define the integral as computing the total change in a function from its rate of change (p.221).This is the same method advocated by Schnepp and Nemirovsky (2001), previously cited in this paper. Hughes-Hallet et al. (2002) use tables and graphs of velocity functions to find distance, and upper and lower estimates of the distance. Sums of partial products and the limit are used to further refine the estimates. Summation notation is introduced, followed by the definite integral. When the definite integral is presented, the relationship is explicitly stated, "Change in $F \approx$ rate of change of $F(x)$ *time elapsed = $F(b) - F(a) = \int_a^b F'(t)dt$" (Hughes-Hallet, 2002, p.237). Anti-derivatives are then represented graphically, numerically and analytically. Differential equations are introduced, followed by the Second Fundamental Theorem of Calculus, which is defined as a function of the upper limit of the definite integral. Again, the term accumulation function is never used in the text of Hughes-Hallet (2002), but the components for understanding, specifically the relationships between rate of change and total change, are emphasized in this text. This may be contrasted with Larson et al. (2006) where the term accumulation function is used, but the focus is on building a formal mathematical understanding rather than a strong conceptual understanding.

Advanced Placement (AP) curriculum. While textbooks may be a good indicator of curricula in the high school classroom, many schools offer Advanced Placement Calculus and are constrained by the College Board, its governing body, to use a curriculum consistent with the philosophy and goals of the College Board. According to the *AP College Board Calculus Course Description, Calculus AB, Calculus BC* (2012), the students should focus on learning the broad concepts of calculus, as well as methods that are widely applicable (p. 5). Students' experiences in calculus should consist of a multi-modal approach: numerical, graphical, verbal

and analytical. The relationships between the different representations should be emphasized. The College Board recommends that technology is regularly used by both teachers and students, and unifying themes utilized to present the course as a cohesive body of knowledge rather than a collection of topics (p.5). This philosophy and these objectives are reminiscent of the findings of the CRAFTY committee.

While the accumulation function is not specifically mentioned in the goals of the course, the pre-requisite knowledge as articulated by Kouropatov and Dreyfus (2012) is clearly stated in the College Board *Course Description (*2012*)*, "Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and should be able to use integrals to solve a variety of problems."(p. 6). In addition, the course description for AP Calculus specifically states that students should understand both parts of the Fundamental Theorem of Calculus and the relationships between the derivative and the definite integral (p.6). In the *Topic Outline* of the course description, there is mention of accumulation as an application of the integral. The *Topic Outline* explicitly states students should be able to find accumulated change from rate of change.

While only three textbooks were analyzed in this paper, Lin McMullen (2011), an educational consultant and past mathematics teacher, has analyzed a variety of high school calculus texts used in AP courses. He found that none of the texts he examined used the accumulation function$f(x) = f(a) + \int_a^x f'(t)dt$,a form he states is extremely useful in problem-solving (p.2).

Research Studies on Student Learning of the Accumulation Function

Researcher studies completed prior to reform calculus have concluded that students' conception of the integral tends to be limited to a process understanding (Orton, 1983; Bagni,

1999). In recent years research has focused on student's conceptual understanding of the integral, however, there is much less research on learning the accumulation function and the development of students' understanding of the connection between rate of change and area under the curve. Nonetheless, several researchers include the components of understanding of the accumulation function in their studies. The first studies summarized below focus on the components of understanding the accumulation function as part of a broader study in conception of the integral. The latter research studies focus specifically on learning the accumulation function.

Understanding and applying area under the curve to physics. Nguyen and Rebello (2011) investigated how students who have already taken calculus use graphical methods to evaluate definite integrals in introductory physics problems. The purpose of the study was to examine whether students recognized when to use the area under the curve to solve a physics problem, understood what the quantity represented, and whether students could match a definite integral with the area under the curve (p. 1). Nguyen and Rebello hypothesized that students who have successfully completed calculus do not have a strong enough conceptual understanding of the integral and the integral-area relationship to solve problems, despite being able to successfully calculate the value of an integral or find the area under a curve (p. 2). In their review of literature, Nguyen and Rebello discuss Thompson and Silverman's (2008) model of accumulation as the foundation for understanding the definite integral. Nguyen and Rebello also refer to Sealey's (as cited in Nguyen & Rebello, 2011, p.2) study where students struggled to relate the area under a curve to a corresponding Riemann sum, despite being able to find the area under the curve.

Nguyen and Rebello (2011) conducted their study to answer three questions: How did students recognize the use of area under the curve in physics problems? How did students in the study understand what quantity was being accumulated when calculating the area under a curve? How did students understand the relationship between the area under the curve and the definite integral (p.2)? Nguyen and Rebello interviewed first-year university students in first and second semesters of a calculus-based physics course. Twenty students were involved in the research study during the first semester, and fifteen of the twenty were again interviewed in the second semester.

Nguyen and Rebello (2011) interviewed students four times, using physics problems that involved the concept of the area under the curve. The graphical form of the problem required the student to calculate the integral using the area under the curve in the context of a work problem. Nguyen and Rebello state that the students tried to use a formula for work and struggled to recognize that the integral or area under the curve was a tool to solve the problem (p.5). By the fourth interview, Nguyen and Rebello found that students tried to use the area under the curve to find work, but since the graph was of velocity vs. angular displacement, rather than linear displacement, the area under the curve did not represent work (p. 6). For the question on accumulation, Nguyen and Rebello concluded that students did not necessarily understand which quantity was being accumulated, and thus could not solve problems in novel situations (p. 8). Nguyen and Rebello state that the results of their study are limited by their small sample size, a consequence of the interview method (p.13).

Comparing Japanese and American students' comprehension. In most of the research studies pertaining to the learning of concepts in calculus the targeted population has been first year university students. However, Judson and Nishimori (2005) compared high school calculus

students' conceptual understanding and ability to use algebra in problem solving. Judson and Nishimori were interested in reports that over the past few decades the skills of calculus students in both Japan and America had declined (pp.24-25). Despite the fact that AP Calculus students scored highly on the Third International Mathematics and Science Study (TIMSS) and calculus reform, Judson and Nishimori claim that mathematicians and mathematics educators point to a decline in mathematical skills (p.25). In Japan, university professors claim students do not calculate well, reject logical reasoning, abstract thinking, and proof in preference to following patterns and using templates (p. 25). Judson and Nishimori's study was developed to determine if there was a difference in the conceptual understanding of students between the two countries.

Judson and Nishimori (2005) involved students from Portland, Oregon and Sapporo, Japan. The 18 American students had completed BC Calculus, with a previous year of AB Calculus, while the 26 Japanese students were Suugaku 3 Calculus, third-year high school students. Judson and Nishimori (2005) wanted to find the answers to the following research questions: "Is there a difference in the conceptual understanding of the best Japanese and U.S. high school calculus students? Is there a difference in algebraic skills between the best Japanese and U.S. students and what are some of the issues surrounding calculator use?" (p. 27).

The study by Judson and Nishimori (2005) consisted of two parts of a written examination followed up by interviews. The first part of the written exam included problems from two American reform calculus textbooks, where little or no algebra was required to correctly solve the problem. The problems on the second part of the written exam were more traditional, and similar to Japanese Center Examinations and AP Calculus tests from previous years. Following the examinations, Judson and Nishimori conducted student interviews to

determine background and student attitude, as well as conceptual understanding of function, derivative, integral, and the Fundamental Theorem of Calculus (p. 34).

In general, the Judson and Nishimori (2005) found little differences in the two populations on reform-type calculus questions on the written examination. Japanese students appear to be significantly more fluent in algebraic manipulation (p.30). One of the most interesting findings pertinent to understanding the accumulation function was the weakness of both populations in relating the derivative and the definite integral, although all students knew about the Fundamental Theorem of Calculus (p.36).

Judson and Nishimori (2005) report similar issues in understanding the elements of the accumulation function (as described by Kouropatov and Dreyfus (2010)) as did Nguyen and Rebello (2011). Both studies found that successful calculus students have difficulties relating the Riemann sum to the integral, as well as the relationship between the derivative and the integral. Judson and Nishimori state that since they did not use random sampling in determining their study's population, and since their population was small, their study is not widely applicable to populations of calculus students. However, Judson and Nishimori claim their findings are probably representative of other high achieving student populations (p.39).

Comparing effects of different types of instruction. Huang (2007) states that results of the 2003 TIMSS indicate Taiwanese students are high-achieving, but have low interest and low self-confidence in mathematics (p. 31). According to Huang, students have traditionally been taught by demonstration by the teacher, followed by student imitation and practice of similar exercises. In combination with a low birth-rate and an expansion of the universities in Taiwan, Huang contends that the mathematical abilities' of university students has declined (p. 31).

For his study, Huang (2007) uses a theoretical framework based on the works of Hiebert and Lefevre, as well as Thompson (as cited in Huang, 20007, p. 31-32). Hiebert and Lefevre define two different categories of mathematical knowledge: conceptual and procedural. Conceptual knowledge is defined as "knowledge rich in connections, which may serve as an interconnected network" (as cited in Huang, 2007, p.31), while procedural knowledge is further divided into two components. One component is "comprehension of mathematical symbols and awareness of symbol syntaxes. The second type consists of the algorithms or rules for solving mathematical tasks" (Huang, 2007, p.31). Huang contends that true understanding in mathematics requires connections between the two types of knowledge, procedural and conceptual. He states that the definite integral is a sub-concept of the concept of the integral, and implies that this is the connection between the procedural and conceptual knowledge for integration (p. 31).

Huang (2007) briefly discusses Thompson's (1994) work, particularly the point that students have a limited understanding of the integral due to a weak understanding of covariation and "multiplicatively-constructed quantities" (as cited in Huang, 2007, p.32). Huang states that the idea of the integral is based on the concepts of the Riemann sum, accumulation, and variation.Furthermore, for the purpose of his study, Huang assumes that the accumulation function is the foundation for teaching integration (p. 32). Huang(2007) contends that "the concept of the accumulation function used in this study was the Riemann sum and this concept was employed to derive a relationship between the definite integrals, indefinite integrals and the Fundamental Theorem of Calculus" (p. 32).

Huang (2007) states the purpose of his study was to examine the impact of concept-based instruction of definite integrals on students' understanding of the integral (p.31). He chose 203

first-year university engineering students in Taiwan who had just completed a course in differentiation to participate in his study. The students were divided into two groups. The instruction for the students in one group, the *procedural group,* focused on procedural techniques for integration. The second group, the *conceptual group,* was instructed with an emphasis on developing students' conceptual understanding of integration (p.32). Huang (2007) states that after instruction students were simultaneously given 50-minute tests. For example, for the first question students were asked to find the values of two definite integrals. Other questions asked students to calculate the shaded area of a graph, find the area under the curve of a specific function bounded on a specific interval without a graph, and find the specific value of a function given a graph of its derivative and the area under the curve. For each of these problems students' responses were categorized and analyzed (p.34).

Huang (2007) concludes that students who received conceptual-based instruction demonstrated a higher level of comprehension of integrals and the ability to apply concepts they had learned than those students in the procedural-based instruction (p. 31). If Huang's research is valid, then the next step in better understanding student learning in calculus may be researching whether a student who exhibits comprehension of the components of understanding the accumulation function also demonstrates a strong conceptual understanding of the accumulation function.

Focus on accumulation as a function. Estrada-Medina and Arenas-Sanchez (2006), Kouropatov and Dreyfus (2009), and Yerushalmy and Swidan (2013) conducted studies that specifically focused on students' conception of the accumulation function.

Estrada-Medina and Arenas-Sanchez (2006) state their goal is to use technology to promote student understanding of calculus. Their research is based on Thompson's (1994) work

that conceptual understanding of the Fundamental Theorem of Calculus is dependent on the relationship between the process of the accumulation of change and its derivative. Estrada-Medina and Arenas-Sanchez (2006) cite the tasks of a research study by Carlson et al. (p.850), where students were presented with a graph representing the rate of change of water in a tank. Students were asked questions about the amount of water accumulated in the tank (p. 850). Estrada-Medina and Arenas-Sanchez state this task was static, using paper and pencil to complete the task. They decided to design a computer simulation of water filling and leaving a tank. Students could explore the relationship between the process of accumulation and its rate of change. This idea was based on the work of Kaput (as cited in Estrada-Medina & Arenas-Sanchez, 2006, p. 850). Estrada-Medina and Arenas-Sanchez refer to this computer simulation as a *dynamic setting* (p.851).

Estrada-Medina and Arenas-Sanchez (2006) created nine different activities to use with the simulator that ranged in increasing conceptual difficulty (p. 851). Tasks either involve constant rate of net consumption over intervals, or combinations of constant and non-constant rates of net consumption (p.852).Two university engineering students were videotaped while working on the program. These students had already completed differential calculus (p.851). Three questions from this research study relate to student understanding of the accumulation function: To what degree does the dynamic setting support student understanding of accumulation of a quantity and its relation to rate of change? What connections do students establish between the ordinate of the accumulation function and its rate of change? To what degree do the students identify that the ordinate at each point represents the area below the curve of its rate of change? (p.850)

Estrada-Medina and Arena-Sanchez (2006) conclude that dynamic activities have *potential* to increase student understanding of the relationship between the accumulation of a quantity and its rate of change (p. 854). However, while students were able to articulate that the net consumption was increasing with speed in a given activity, they were not able to correctly graph the water quantity if the consumption was not constant (p. 854).While this research focused on developing student comprehension of net rate of change as a variable, it did not examine students' conception of accumulation assuming a well-developed understanding of net rate of change as a variable.

Yerushalmy and Swidan (2013) conducted a similar study using a dynamic environment to study stages of development in student understanding of the accumulation concept. The researchers were interested in how students make sense of the graph of area under the curve which they term "integral graph" (p. 287) when it is first introduced in calculus. The framework used for this study is Tall's (2003) three modes of mental representation: embodied, symbolic-proceptual, and formal world (p. 3). Yerushalmy and Swidan were concerned with how students develop their understanding of the accumulation function in the embodied world (p. 287). Tall (2003) defines the embodied world as "based on human perceptions and actions in a real-world context including but not limited to enactive and visual aspects" (p.3). The researchers expect that when the students manipulate various parts of the graph, they will be able to see the effects. These experiences will develop the students' conceptual understanding of the integral (p. 287).

Yerushalmy and Swidan(2013) state the students in the study had been taught functions and derivatives, but not integrals (p. 293). The two seventeen year-old students used Calculus UnLimited (CUL),a software interface that would allow students to interact and build experiences with the accumulation function. CUL utilizes graphing tools that include two

graphs, one of the function, and one of accumulation of the area under the curve. Students were asked to use the CUL tools to hypothesize a relationship between the two graphs and explain their reasoning (p. 293). The researchers conclude that when students interact with CUL, the students were able to focus on the lower boundary of the accumulation function as being the pivot between positive and negative areas of accumulation. Yerushalmy and Swidan contend objectifying of the zero increased students' understanding of the balance between the non-zero areas.

Yerushalmy and Swidan (2013) note the narrowness of the task in their case study, as well as the lack of mediation by an instructor, limit the ability to generalize the findings of their study. However, the study does reveal how some students conceptualize area under the curve and accumulation as a function, one of the components to understanding the accumulation function.

The third study specifically focusing on learning the accumulation function was designed by Kouropatov and Dreyfus (2009). The researchers state that many students in high school are taught that anti-derivatives "undo" derivatives and to calculate value of the definite integral. However, it is not common for high school students to learn the reason anti-derivatives can be used to calculate the area under the curve or the relationship between definite and indefinite integrals (p. 1). Kouropatov and Dreyfus(2009) cite research by Orton, Thomas and Hong, and Belova that provide evidence that while students have elementary integration skills, the students have a weak conceptual understanding (p. 1). Kouropatov and Dreyfus were interested in determining if students were given a series of tasks to build their conceptual understanding of the accumulation function would it strengthen their understanding of the integral?

Kouropatov and Dreyfus (2009) examined 117 twelfth-grade students from five different high schools in Israel, who had recently finished studying integrals using the "common approach" (p. 3), presumably an approach based on symbolic manipulation. The students were given eight questions that assessed conceptual understanding of the integral. The results of their questionnaire provided evidence that students' conceptual understanding of the integral was low (p.3).

Kouropatov and Dreyfus (2009) provided three eleventh-grade students with an opportunity to experience the integral as an accumulation of a function, dependent only on the function's properties rather than finding the anti-derivative (p. 5). For the purposes of their instructional unit, Kouropatov and Dreyfus (2009) defined accumulation as "the accumulating sum of very small parts.The basic idea is that of Riemann integration, leading in parallel to the definite and the indefinite integral, as well as their connection by the FTC" (p.4). The instructor guided the students as they worked on the following tasks: approximate the area under a parabola or the volume of a frustum using known geometric formulas or directly measuring, estimate the accumulation value of a positive increasing continuous function and a sign-changing decreasing function, calculate the value of the accumulation of a linear function algebraically, calculate the definite integral of a linear function geometrically, and calculate the definite integral of $\int_0^6 (x^2+1)\,dx$symbolically to use the results in evaluating $_0^6(x^2)dx$ (pp.5-7). In a series of four sixty-minute periods the students were able to complete the tasks while guided by the instructor.

Kouropatov and Dreyfus (2009) conclude that a strong understanding of the integral is critical to understanding calculus. Further, they state that since calculus developed from attempts to understand our world, the integral cannot be understood without understanding the

strong connection between the concept and its applications (p. 7). According to Kouropatov and Dreyfus, the accumulation function is central to the integral and thus, teaching should be a central focus in teaching integration (p.7).

Thus, researchers have found some evidence that students who explore the accumulation function through specific learning activities may help students develop conceptual understanding of the integral, as well as the relationship between the derivative and the integral. However, since the textbooks sampled tend to present a traditional calculus with graphs and tables, rather than a focus on conceptual understanding, it is unlikely that most students have the opportunity to build a conceptual understanding in calculus.

Conclusion and Future Research

Many students in high school are studying calculus, a foundational course for fields of study in mathematics and science. The core concepts of calculus are the limit, the derivative, and the integral, with the Fundamental Theorem of Calculus relating these concepts. Students today need to have a strong understanding of these inter-relationships to solve real-world problems, which may be strengthened by a strong understanding of the accumulation function. The possible components to understanding the accumulation function have been outlined in this paper.

If textbooks do not present a cohesive presentation of calculus, teachers will need to provide opportunities for students to integrate the concepts of calculus.Understanding the accumulation function may be a significant component of building a conception of the relationship between the derivative and the integral (Thompson and Silverman, 2008). If the accumulation function is divided into smaller knowledge-elements (Kouropatov & Dreyfus,

2012; Thompson & Silverman, 2008) and a genetic decomposition for the accumulation function is written both teachers and students may be able to better articulate students' understanding of the accumulation function.

The research studies described in this paper have suggested that students' understanding of the accumulation function may influence their conception of the integral. The three studies that focused specifically on teaching the accumulation function examine the relationship between a quantity being accumulated and its rate of change and the area under the curve and accumulation.

Questions for further research are (a) if high school students demonstrate understanding of the individual components of the accumulation function do they demonstrate a strong understanding of the accumulation function; (b) is there a hierarchal relationship between the components of understanding?

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