Block Diagram Modeling of Second-Order Systems

INTRODUCTION

Block diagrams are a method of describing the behavior of a dynamic system. In a block diagram, each discrete component, or block, represents part of the system. These blocks are connected together, representing how the "signal" flows between components. This can aid in understanding the interaction between various components, particularly for complex systems. Simulink allows block-diagram modeling of systems, and will be used for the examples in this tutorial. The concepts described here, however, are applicable to block diagrams in general.

In this document, the basics of modeling second-order differential equations using block diagrams will be discussed. A single-degree-of-freedom mass-spring-dashpot system will be used as an example in the construction of the model. Other tutorials are available which contain more general information on second-order dynamic systems.

CONSTRUCTING THE MODEL

The response of the mass-spring-dashpot system is described by a second-order differential equation known as the equation of motion,

$$m\ddot{x} + c\dot{x} + kx = f(t), \tag{1}$$

where
m = the mass,
x = the displacement,
$\dot{x}$ = the velocity,
$\ddot{x}$ = the acceleration,
f(t) = the forcing function (input force),
c = the damping, and
k = the spring constant.

For the purposes of this tutorial, (1) will be rearranged to give

$$\ddot{x} = \frac{1}{m}\left[f(t) - c\dot{x} - kx\right]. \tag{2}$$

To model this system, begin by observing the terms in the brackets. There are three input signal lines: f(t), $\dot{x}$, and x. Both $\dot{x}$ and x are multiplied by constants, which is accomplished by using gain blocks. After each is appropriately modified, the three input signals are then summed together as shown in Fig.1.

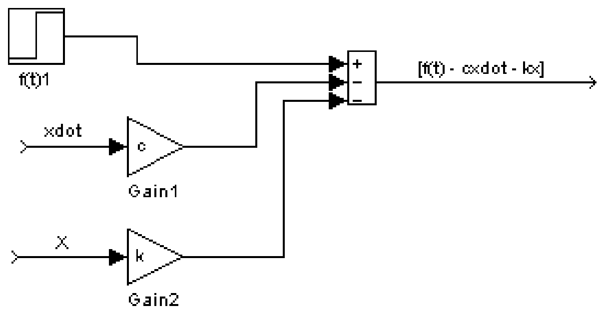

Fig.1. Summing the input signals.

The block in the center of the diagram is referred to as a summing junction. Note that in the Simulink model labels, “xdot” and “xddot” refer to $\dot{x}$ and $\ddot{x}$.

Next, it can be seen that the bracketed term $\left[f(t) - c\dot{x} - kx\right]$ is multiplied by the constant 1/m. Therefore, the output signal is passed through a gain block as shown in Fig.2.

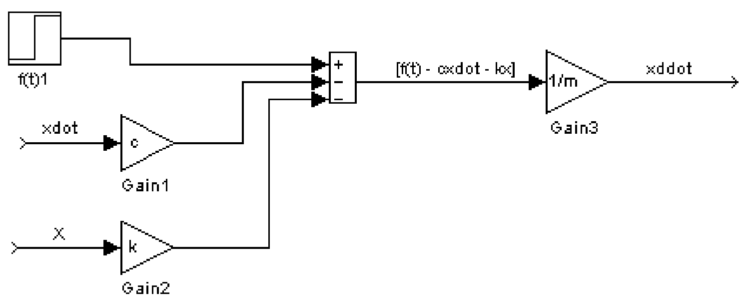

Fig.2. Applying a gain to the output.

The output of this final gain block is $\ddot{x}$. The output of interest, however, is the displacement x. To obtain the x output signal, the $\ddot{x}$ output signal is integrated twice, as shown in Fig.3.

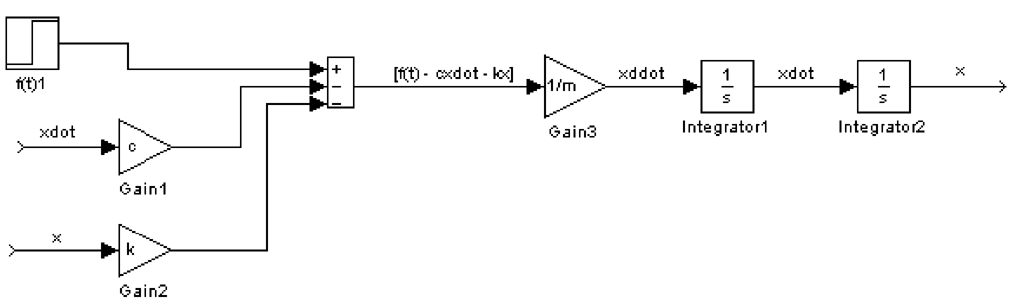

Fig.3. Integrating the output signal twice to obtain x.

As can be seen in Fig.3, $\dot{x}$ and x are both outputs from and inputs to the model. Therefore, two feedback loops are needed. A line is tapped off after the first integrator block and fed back in to the open $\dot{x}$ port, and a line is tapped off after the second integrator and fed back in as x. After adding a scope block (“Scope1”) to allow the output to be viewed in Simulink, and with some rearranging of blocks, the resulting model should look like Fig. 4. The block diagram is now complete.

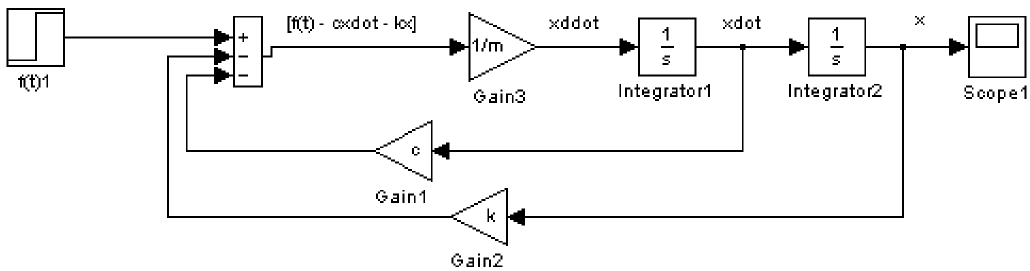

Fig. 4. The finished Simulink model for a second-order mass-spring-dashpot system.

EXAMPLE

Numerical values will now be entered into the Simulink model, shown in Fig. 5 below, so that it can be used to determine the step response of a second-order system.

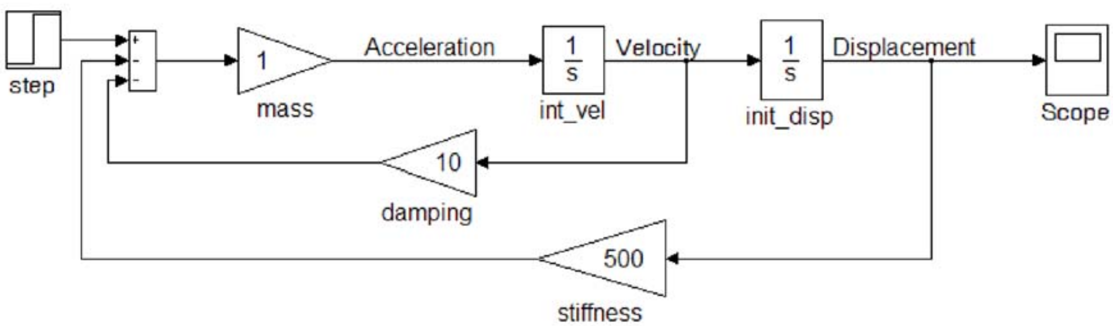

Fig. 5. Simulink model of second-order system.

Construct the model as shown, and enter the values given in Table 2.

Table 2. Values used in Simulink model.

Step block	Step time	0
	Initial value	0
	Final value	1
Mass gain block	Gain	1
Damping gain block	Gain	10
Stiffness gain block	Gain	500
Integration blocks (both)	Initial condition	0

When the result is viewed using the scope block, the result will appear as shown in Fig. 6.

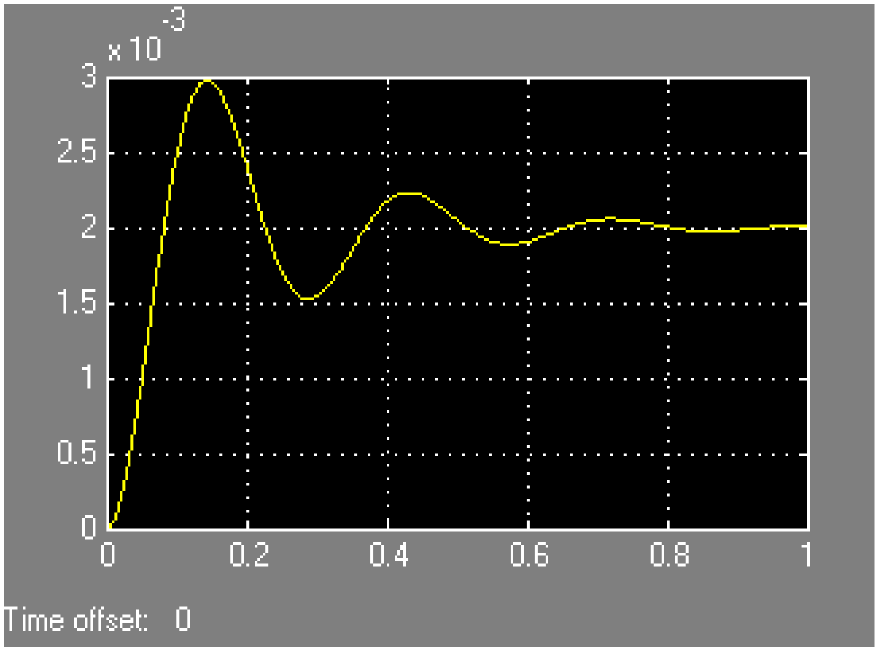

Fig. 6. Step response of second-order system, found using Simulink.