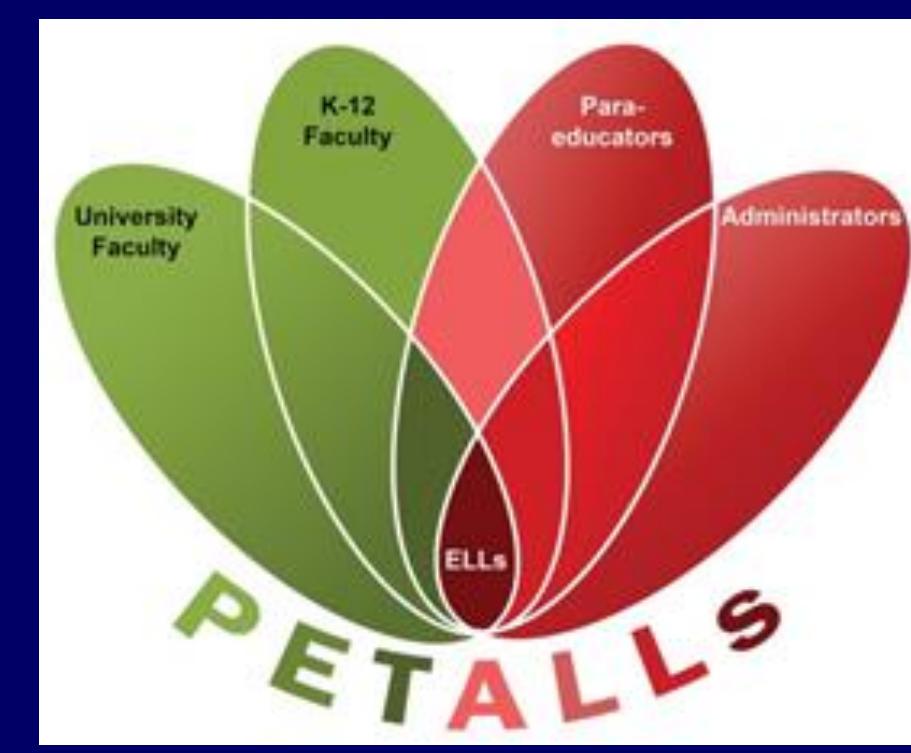




Learning with Purpose

# Developing Mathematical Literacy through Solution Structures

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## INTRODUCTION

As written languages evolved, rules were invented to disambiguate writing. This included inserting spaces between words, punctuation, and more recently the paragraph.

Counting the number of paragraphs on any page is second nature for readers whose primary language is a modern descendant of the Latin language family. But consider some of the rules that the brains of fluent readers handle automatically:

- The first sentence of a paragraph is indented and each indentation is the same width and aligned with adjacent paragraphs.
- Sentences begin with a capital letter and end with a punctuation mark.
- We read from left-to-right and then top-to-bottom.
- Sentences do not overlap or cut through each other.

**Example:** The layout and continuity of text on a page serves to increase its readability and demarcate transitions.

This is made possible by the use of text spacing and blank space.

Standardized tests have become ubiquitous in U.S. elementary and secondary education over the last few decades. About ten years ago, states across the country expanded their assessment systems in response to the federal No Child Left Behind Act's requirement that students take annual math and reading tests in grades 3-8 and once in high school. In the coming years, states will need to make the most significant changes to their assessment systems since the passage of NCLB as they implement the Common Core State Standards, a common framework for what students are expected to know that will replace existing standards in 45 states and the District of Columbia.

The Common Core effort has prompted concerns about the cost of implementing the new standards and assessments, especially in states that have historically spent very little on their tests. Unfortunately, there is little comprehensive up-to-date information on the costs of assessment systems currently in place throughout the country. This report seeks to fill this void by providing the most current, comprehensive evidence on state-level costs of assessment systems, based on new data gathered from state contracts with testing vendors.

### INFORMATION!

We find that the 45 states from which we obtained data spend a combined \$669 million per year on their primary assessment contracts, or \$27 per pupil in grades 3-9, with six testing vendors accounting for 89 percent of this total. Per-pupil spending varies significantly across states, with Oregon (\$13 per student), Georgia (\$14), and California (\$16) among the lowest-spending states, and Massachusetts (\$64), Delaware (\$73), and Hawaii (\$105) among the highest spending. We find that larger states tend to spend substantially less, per student, than smaller states, which is not surprising given that larger states save on fixed costs like test development by spreading them over more students and may have more bargaining power.

### INFORMATION!

We estimate that states nationwide spend upwards of roughly \$1.7 billion on assessments each year, after adjusting the \$669 million figure to (1) account for the fact that six percent of students are located in states for which we were unable to obtain data, (2) reflect spending on assessments not included in states' primary assessment contracts, and (3) include state-level spending on

## DESCRIPTION

Writing has order and structure. We actively teach and correct errors in language when written or spoken. This is especially true in the English Language Arts; however, it remains a problem in written mathematical problem-solving when order and structure are often overlooked and uncorrected because the importance of such feedback to students is secondary to a correct numerical solution.

Consider the following solutions to the same problem, submitted by students in the same class.

Example 1	Example 2
$KE = \frac{1}{2}mv^2$ $KE = \frac{1}{2} \cdot 0.5\text{kg} \cdot 50^2\text{m/s}$ $KE = \frac{1}{2} \cdot 0.5\text{kg} \cdot 2500\text{m/s}$ $KE = 625\text{J}$	$\frac{1}{2} (0.5\text{kg}) (50\text{ m/s})^2$ $\frac{1}{2} (0.5) (2500)$ $\frac{1}{2} (1250)$ $(625)$ $625$

Both answers arrive at the same final solution, however example 1 looks "cleaner." This is because the solution is more logically structured and organized.

## Analysis of Example 1

Clear beginning - as if the "formula" the student is using functions as the first sentence in a paragraph.

$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2} \cdot 0.5\text{kg} \cdot 50^2\text{m/s}$$

$$KE = \frac{1}{2} \cdot 0.5\text{kg} \cdot 2500\text{m/s}$$

$$KE = 625\text{J}$$

Clear solution - this functions as if it was the last sentence in a paragraph.

$$KE = 625\text{J}$$

Each individual line of work is aligned to the equal sign above and below it - in the same way paragraphs are aligned by an indentation.

$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2} \cdot 0.5\text{kg} \cdot 50^2\text{m/s}$$

$$KE = \frac{1}{2} \cdot 0.5\text{kg} \cdot 2500\text{m/s}$$

$$KE = 625\text{J}$$

The final answer ends with the proper unit of measurement, akin to how the proper punctuation mark ends a sentence.

## Analysis of Example 2

When a calculator is provided, this work is not required - and it obscures the logical coherency of the solution structure. Moreover, our students generally believe that this constitutes the required part of "calculations" and "work" for open responses - it only consumes time and adds a potential source of error.

Correct use of "formula" but forgetting to square one term may lead students to the wrong solution.

The multiple circle around the final solution obscures the units that the solution needs for complete credit.

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