

Numerical integration and differentiation project

OVERVIEW

Numerical integration and differentiation are frequently performed on experimental data. In this project these techniques will be used on several sample data sets, using both MATLAB and Excel.

THEORY

The central difference method for performing numerical integration is preferred. The equation for this is

$$\frac{dy}{dx} = \frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}} \quad (1)$$

The trapezoidal method is most commonly used for numerical integration. The equation is

$$I_i = I_{i-1} + \frac{y_{i+1} - y_i}{2} (x_{i+1} - x_i) \quad (2)$$

Numerical integration and differentiation can be easily performed in MATLAB or in Excel. The procedural details for doing this can be found in Appendix A.

PART A: FREE-FALLING OBJECT

Table 1 lists time, displacement, and acceleration data for an object in free-fall.

Table 1. Data for a falling object.

| Time (sec) | Displacement (ft) | Acceleration (ft/sec ²) |
|------------|-------------------|-------------------------------------|
| 0 | 0 | 32.2 |
| 1 | 21.1 | 32.2 |
| 2 | 74.4 | 32.2 |
| 3 | 159.9 | 32.2 |
| 4 | 277.6 | 32.2 |
| 5 | 427.5 | 32.2 |
| 6 | 609.6 | 32.2 |
| 7 | 823.9 | 32.2 |
| 8 | 1070.4 | 32.2 |
| 9 | 1349.1 | 32.2 |
| 10 | 1660 | 32.2 |

Using either Microsoft Excel or MATLAB and the data in Table 1:

1. Differentiate the displacement data once to obtain velocity.
2. Differentiate the displacement data a second time to obtain acceleration.
3. Integrate the acceleration data once to obtain velocity. Remember that you need an initial velocity for the integration—where can you get this?

4. Integrate the acceleration data a second time to obtain displacement. What is the initial displacement of the object?

Produce the following plots and discuss the results:

- a) Velocity results from (1) and (3) plotted together. Do they match exactly? What would happen if you didn't include an initial velocity? Does this explain why displacement and acceleration measurements are often taken together?
- b) Acceleration result from (2) plotted with original given acceleration. How well do they match?
- c) Displacement data from (4) plotted with original given displacement. How well do they match?

PART B: SINE WAVE 1

Either Excel or MATLAB may be used for this part.

Create data for a single cycle of a sine wave, with points in 10° increments. For procedural details of how to do this in Excel, see Appendix A. For MATLAB, see the MATLAB code in Appendix B.

1. Differentiate the data once.
2. Differentiate the data a second time.
3. Integrate the data once. Remember an initial condition.
4. Integrate the data a second time. Remember an initial condition.

Produce the following plots and discuss the reasons for any discrepancies:

- a) It is possible to find the analytical, "correct" result for each of these integrations and differentiations. Therefore, plot the result from (1) with the exact answer.
- b) The result from (2) with the exact answer.
- c) The result from (4) with the exact answer.

PART C: SINE WAVE 2

Either Excel or MATLAB may be used for this part.

Create data for a single cycle of a sine wave, with points in 30° increments.

1. Find the second derivative of the sine wave.
2. Find the second integral of the sine wave.

Produce the following plots and discuss the reasons for any discrepancies. Why are these results worse than in Part B?

- a) The result from (1) with the exact answer.
- b) The result from (2) with the exact answer.

PART D: SINE WAVE 3

Either Excel or MATLAB may be used for this part.

The data points in Table 2 describe a sinusoidal signal.

Table 2. Data for a sinusoidal signal.

| x | y |
|------|--------|
| 0.00 | 5.00 |
| 0.08 | 24.74 |
| 0.17 | 39.19 |
| 0.25 | 44.48 |
| 0.33 | 39.19 |
| 0.42 | 24.74 |
| 0.50 | 5.00 |
| 0.58 | -14.73 |
| 0.67 | -29.19 |
| 0.75 | -34.48 |
| 0.83 | -29.19 |
| 0.92 | -14.75 |
| 1.00 | 4.99 |
| 1.08 | 24.73 |
| 1.17 | 39.18 |
| 1.25 | 44.48 |
| 1.33 | 39.20 |
| 1.42 | 24.75 |
| 1.50 | 5.01 |
| 1.58 | -14.73 |
| 1.67 | -29.18 |
| 1.75 | -34.48 |
| 1.83 | -29.20 |
| 1.92 | -14.76 |
| 2.00 | 4.98 |

1. Integrate the data once, using an initial condition of 1.88.
2. Integrate the data a second time, using an initial condition of 0.35.
3. Differentiate the data once.
4. Differentiate the data a second time.

Produce the following plots and explain the results:

- a) The result from (1). Does this produce the expected result? If not, why not? Re-examine the original data and see if you can determine the reason.
- b) The result from (2). Again, explain the results.

- c) The result from (3). Did this produce the expected cosine wave?
- d) The result from (4). Did this produce the expected $-\sin$ wave?

What is going on with the original data which is causing these results in the numerical processing?

APPENDIX A MATLAB and Excel techniques

EXCEL

In this project, it is necessary to generate a sine wave using Excel, and then numerically integrate and differentiate this signal.

Generating a sine wave

To generate a sine wave signal in Excel, first the “x” values must be produced. If, for example, a sine wave in 5° increments is desired, first create a column of values in degrees, as shown in column A of Fig. A-1. The sine calculation in Excel, however, requires a value in radians as the input. Therefore convert all of the degree values to radian values, using an equation as shown in column B of Fig. A-1.

| | A | B | C |
|---|---------|-------------------|---|
| 1 | | | |
| 2 | degrees | radians | |
| 3 | 0 | =A3*2*3.14159/180 | |
| 4 | 5 | | |
| 5 | 10 | | |
| 6 | 15 | | |
| 7 | 20 | | |

Fig. A-1. Creating x-values for plot.

Now simply find the sine of the column B values, as shown in Fig. A-2.

| | A | B | C |
|---|---------|----------|----------|
| 1 | | | |
| 2 | degrees | radians | sin(x) |
| 3 | 0 | 0 | =sin(B3) |
| 4 | 5 | 0.174533 | |
| 5 | 10 | 0.349066 | |
| 6 | 15 | 0.523598 | |
| 7 | 20 | 0.698131 | |

Fig. A-2. Calculating sin(x).

The result of this is shown in Fig. A-3.

| | A | B | C |
|---|---------|----------|----------|
| 1 | | | |
| 2 | degrees | radians | sin(x) |
| 3 | 0 | 0 | 0 |
| 4 | 5 | 0.174533 | 0.173648 |
| 5 | 10 | 0.349066 | 0.34202 |
| 6 | 15 | 0.523598 | 0.5 |
| 7 | 20 | 0.698131 | 0.642787 |

Fig. A-3. Result of calculations.

When performed over a full cycle and plot, the result is a sine wave like that shown in Fig. A-4.

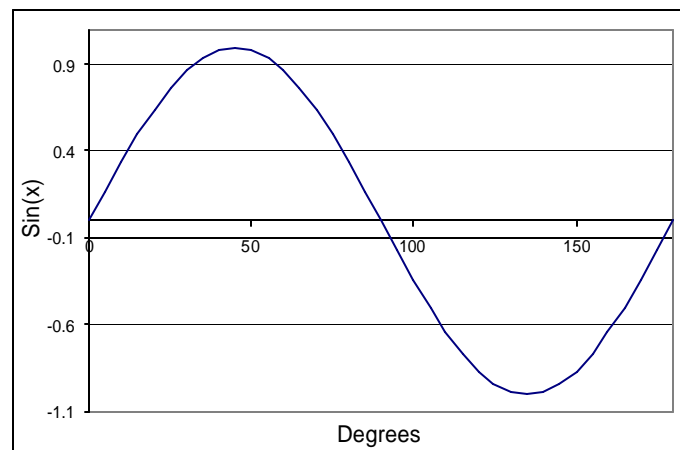


Fig. A-4. Resulting sine wave.

Numerical integration and differentiation

To perform numerical integration and differentiation using Excel, simply use the equations as given in the theory section.

MATLAB

The `cumtrapz` command computes the cumulative integral of y with respect to x using trapezoidal integration. The syntax for the command is:

$$z = \text{cumtrapz}(x,y)$$

Numerical differentiation can be performed in MATLAB by simply using the basic equations as part of a `for` loop. To numerically differentiate a data set where the x -values are in a vector named `x`, and the y -values are in a vector named `disp`, the syntax would be:

```
for i = 2:length(x)-1;
    vel(i) = (disp(i+1) - disp(i-1))/(x(i+1) - x(i-1));
end
```

The attached MATLAB file provides an example of the use of these commands.

APPENDIX B

MATLAB code to perform numerical integration and differentiation

```
% Numerical integration and differentiation
% Using built-in Matlab functions

% Clear the command window.
clc
% Clear all variables.
clear all

% Start with simple sine function, representing displacement
% Recall that MATLAB assumes that the input to the sine function is
% in radians, so a full cycle goes from 1 to 2 pi.
x = 0:0.1:2*pi;
disp = sin(x);

%-----
% Numerical differentiation:
% Differentiate the displacement once to obtain velocity.
% The central difference method is used, therefore the first and
% last points are "lost"
for i = 2:length(x)-1;
    vel(i) = (disp(i+1) - disp(i-1))/(x(i+1) - x(i-1));
end

% Plot the displacement and the calculated velocity:
figure(1)
% Because those two points were "lost" in the differentiation, the
% plot command must consider that
plot(x,disp,'-',x(2:length(x)-1),vel(2:length(vel)),'--');
legend('Displacement','Velocity obtained by differentiation')

%-----
% Numerical integration:
% The acceleration of this object would be a -sin function.
% We will therefore start with this acceleration, and integrate
% once to obtain velocity.
acc = -sin(x);

% Z = cumtrapz(X,Y) computes the cumulative integral of Y with
% respect to X using trapezoidal integration. We will therefore
% use this to calculate velocity:
vel2 = cumtrapz(x,acc);

% However, this has not yet included an initial condition.
% We know that the first integral of -sin is cos, and cos(0) = 1,
% so this is the initial condition for our integration. We will add
% this to each element of vel2, to obtain our final result.
vel2 = vel2 + 1;

% Plot the acceleration and the calculated velocity:
figure(2)
plot(x,acc,'-',x,vel2,'--');
legend('Acceleration','Velocity obtained by integration')

% Compare the two different calculated velocities.
figure(3)
plot(x(2:length(x)-1),vel(2:length(vel)),'-',x,vel2,'--');
legend('Velocity obtained by differentiation','Velocity obtained by integration');
```

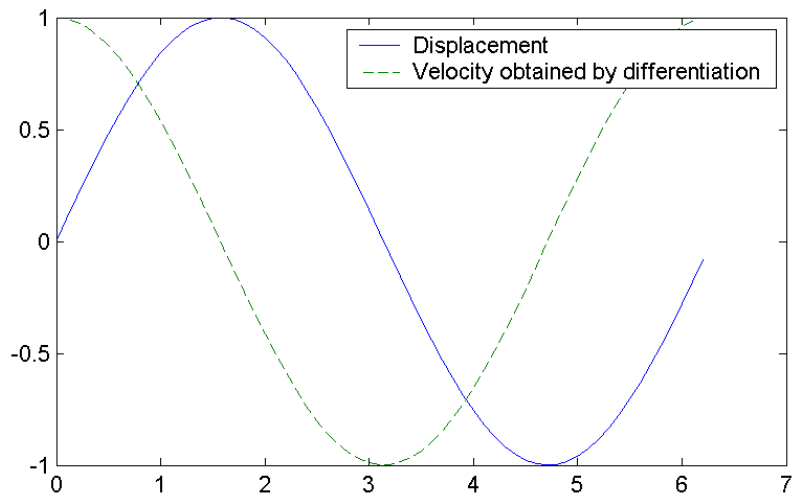


Fig. B-1. Figure 1 from MATLAB code.

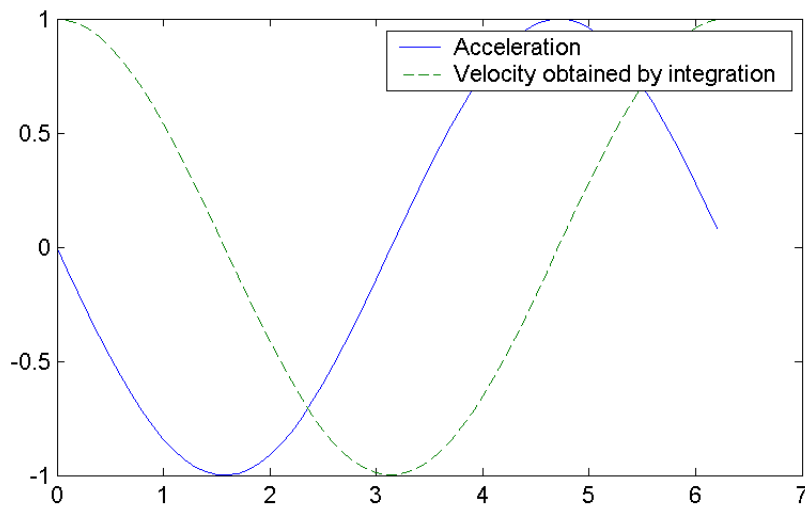


Fig. B-2. Figure 2 from MATLAB code.

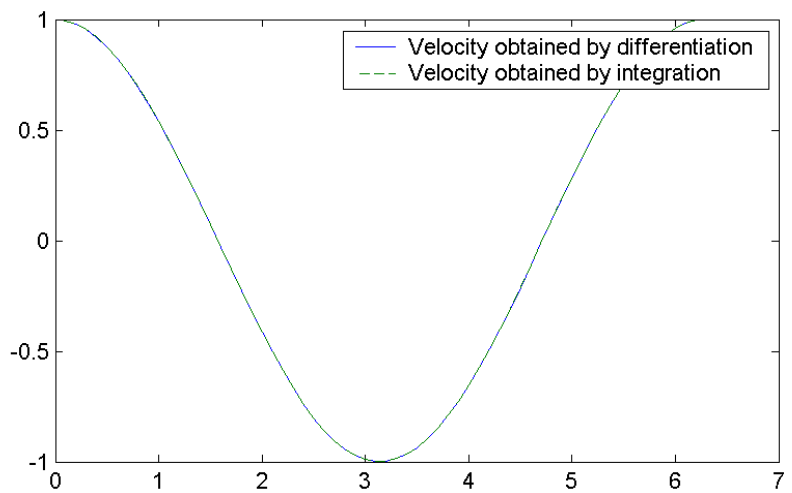


Fig. B-3. Figure 3 from MATLAB code.