Magneto-optical study of donor-level crossing in tipped GaAs/(Ga,Al)As quantum wells

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We present evidence for an anticrossing between the $2p_{\pm 1}$ and $2p_0$ levels of Si donors situated near the center of a GaAs quantum well. This anticrossing, which does not appear for magnetic fields perpendicular to the plane of the sample, is induced by tipping the sample normal away from the direction of the applied magnetic field. All donor transitions were detected with photoconductivity, a method that is standard for GaAs bulk donors but has only recently been employed successfully for donors in quantum wells.

I. INTRODUCTION

In 1985, Jarosik and co-workers\(^1\) reported the observation of far-infrared (FIR) spectra from donors located in GaAs/(Ga,Al)As quantum-well structures. These authors were able to study the magnetic-field dependence of the energies of the observed spectral lines, mainly by measuring the small decrease in sample transmission that occurs when the energy of the incident photons matches the energy of an allowed donor transition. The spectroscopy of donors in quantum wells is more difficult than that of bulk donors. In general, quantum-well samples contain fewer donors per unit area, and the absorption energies are smeared out over a wider energy range, primarily because of the variation of donor transition energy with the position of the donor in the well.\(^2\) Typically, only a few percent of the incident photons are absorbed to produce the transmission minima. However, recently a photoconductivity detection method analogous to the technique proven valuable in bulk film studies has been applied to quantum wells.\(^3\)–\(^5\)

In the present work, we investigate donors in quantum wells using the photoconductivity method in conjunction with FIR laser sources. To achieve photoconductive detection in GaAs films, an electric field is applied in the plane of the sample, which is held at liquid-He temperatures. Optical excitation of donors is detected as a change in sample conductivity. Employing this method, which is known to be capable of extreme sensitivity in the detection of donor absorption in bulk epitaxial GaAs, we have obtained relatively sharp spectral lines with high signal-to-noise ratios in quantum-well samples. This has allowed us to explore, with high precision, differences between the spectra of shallow donors in GaAs quantum wells and in bulk GaAs. In particular, we have made a critical study of an effect not previously reported—the anticrossing of two $p$-like levels of donors in the center of a quantum well.

In the limit of an infinitely wide well, the energy levels of the central donor must become indistinguishable from those of a bulk donor. We identify the energy levels of the donor in the center of the well by the label we attach to bulk states into which the well state adiabatically evolves as the quantum-well width is slowly increased to infinity, with the donor always remaining in the center of the well. For example, we label the ground state of the well donor 1$s$ (even though that well state is not spherically symmetric). This labeling scheme does not lead to ambiguities for any of the states of interest in the present paper. Since the growth direction of quantum-well samples is the axis of rotational symmetry for the donors in the absence of external fields, we take the axis of quantization, our $z$ axis, along this direction, which is normal to the plane of the sample.

Many interesting differences exist between the spectra of hydrogenic donors confined in the center of a one-dimensional quantum well and those of hydrogenic donors in bulk GaAs (the latter behave very nearly like weakly bound hydrogen atoms\(^6\)–\(^7\)), and these persist even for wells as wide as several donor Bohr radii. For example, a difference between bulk and well donors is that the bulk donor is isotropic, so its spectrum in a magnetic field
depends only on field strength and not on field direction. The well donor spectrum, on the other hand, depends upon both the magnitude of the field and the angle $\theta$ that it makes with the normal to the planes of the wells.

Another difference between bulk and well donor spectra is that at zero magnetic field the well potential lifts certain degeneracies that exist for a spinless electron in the Coulomb field of a point positive charge in vacuum. States having the same principal hydrogenic quantum number $n$, but labeled by different values of orbital angular momentum $l$, are not degenerate for the donor in the well. As an example, the $2p$ levels of the well are not degenerate with the $2s$ level. In addition, the quantum-well barriers also break the degeneracy of states labeled with the same $n$ and $l$ but having different magnitudes of the quantum number $M$, where $M$ is the projection of the orbital angular momentum on the $z$ axis. Thus at zero magnetic field the $2p_0$ level always lies above the degenerate $2p_{+1}$ and $2p_{-1}$ levels in a quantum well, whereas all three levels are degenerate in the simple hydrogenic model.

The splitting of these levels in the well results in a wealth of level crossings that occur at various magnetic-field strengths for $\theta = 0^\circ$. Of special interest to us is the crossing between the $2p_{+1}$ and $2p_0$ levels, shown by the dashed lines (calculated) in Fig. 7. Since $M$ remains a good quantum number in magnetic fields along the $z$ axis ($\theta = 0^\circ$), and since these levels have different $M$ values, they may cross. However, if the sample is tilted ($\theta \neq 0^\circ$), $M$ is no longer a good quantum number and the levels can no longer cross. Instead, an anticrossing occurs due to the presence of a nonzero component of the field in the GaAs well planes. This component couples the $2p_{+1}$ and $2p_0$ levels causing those levels to "repel" each other.

A final difference between bulk and well states, which is much less well known, is that the basic tuning behavior of energy levels in a magnetic field at $\theta = 0^\circ$ is different for a well donor than for a bulk donor. Although all bulk donor states with $M \leq 0$ always remain below the lowest $(N=0)$ Landau level in energy, the same is not necessarily true in a quantum well. The rule forbidding level crossings of states with the same good quantum numbers leads to the conclusion that, for quantum wells that are not too wide, the vast majority of excited states with $M \leq 0$ penetrate through at least one Landau level with increasing magnetic field. For example, while in sufficiently low magnetic fields the $3p_0$ level of a donor at the center of the well lies below the lowest Landau level associated with the first excited subband, in high fields we expect it to lie below but near to the lowest Landau level associated with the third excited subband. By contrast, the $3p_0$ state in bulk remains, irrespective of field strength, below the single $N=0$ Landau level present in the bulk. Generally, all but the lowest donor state of each symmetry can be expected to tune upward in energy with increasing field more rapidly than their bulk counterparts, at least for a range of fields and well widths comparable to those employed in the present work.

Anticrossings of electronic levels in a magnetic field in systems with one-dimensional confinement of electrons have been reported previously. Schlesinger, Hwang, and Allen observed the splitting of the cyclotron resonance frequency of free electrons confined near a GaAs/(Ga,Al)As interface. This splitting was induced by tipping the sample so as to couple the $N=1$ Landau level associated with the ground subband level to the $N=0$ Landau level associated with the first excited subband level. Jarosik and co-workers have reported anticrossings associated with levels for donors in a quantum well located near the GaAs/(Ga,Al)As interface ("edge-doped" donors). Although their data clearly show a tilt-induced anticrossing, the semiquantitative theoretical curves purporting to account for that effect exhibit sizable discrepancies with the data. A significant uncertainty remains as to the position of the donors that contribute to the anticrossing spectra reported. Moreover, the data do not appear to support directly and clearly the theoretical prediction that two anticrossings (involving three levels) occur in the region studied.

The present paper describes results of measurements of anticrossings of levels belonging to donors in the center of the quantum well. Only two levels participate significantly in the level crossing process observed, allowing a relatively simple theoretical treatment. A description of the samples used and their preparation is presented in Sec. II. In Sec. III, the measurements are described, and the experimental and theoretical results are presented. The method of calculation of the energy levels is discussed in Sec. IV. The results of this paper are summarized in Sec. V.

II. SAMPLES

GaAs/(Ga,Al)As quantum-well samples were grown using elemental-source molecular-beam epitaxy. Growth parameters included a substrate temperature of 690°C and ion gauge beam flux ratios of 12 to 1 and 18 to 1 for As/(Ga+Al) and As/Ga, respectively. Each sample consisted of a (100)-oriented semi-insulating liquid-encapsulated Czochralski substrate with the following epitaxial layers: an unintentionally doped GaAs buffer layer, an intentionally doped Ga$_{0.8}$Al$_{0.2}$As buffer layer, a 25-period quantum-well structure with 35-nm-thick Ga$_{0.8}$Al$_{0.2}$As barriers and center-doped ~50-nm-wide GaAs quantum wells, an unintentionally doped 100-nm-thick Ga$_{0.8}$Al$_{0.2}$As isolation layer, and a 10-nm-thick unintentionally doped GaAs cap layer. The center third of the quantum wells was lightly doped $(5 \times 10^{15}$ donors/cm$^3$) with silicon donors. Growth pauses with the sample under an As flux were used at each heterointerface and at each termination of doping. Photoluminescence measurements indicated that the samples were of high quality and lightly doped. The GaAs well widths as determined by scanning electron microscopy (SEM) were 50±5 nm, and the (Ga,Al)As barrier widths were 35±5 nm.

The fabrication process for forming electrical contacts to the samples consisted of the following steps: beveling a pair of opposite edges of a 6×6 mm sample to expose the quantum wells, metallizing the edges with layers of Ni, Ge, and Au with a lift-off process to form contacts, alloying the contacts, and attaching 50-μm-thick gold
wires to the contacts with conducting epoxy. Three methods for beveling the edges of the sample were successfully demonstrated. In the first method, angled chlorine ion-beam-assisted etching is used to etch a sidewall into the wells at an angle of 70° with respect to the normal of the sample surface. The second method consists of quickly dipping the edge in a slow isotropic etch and slowly withdrawing it to form the bevel. In the third method, three lithographic/wet-etch steps are used to form a staircase-shaped sidewall. The second and third methods have thus far produced samples with better signal-to-noise ratio than the first method. The layer thicknesses of the Ni, Ge, and Au layers were 30, 40, and 340 nm, respectively, and the layers were alloyed in nitrogen for 30 s at 450 °C. Figure 1 shows a sketch of one of our contacted samples along with a diagram of the epitaxial layers employed.

III. EXPERIMENT

The measurements were carried out using CO₂-laser-pumped FIR gas lasers at about 20 wavelengths from 890 to 70.5 μm. Cavity tuning of the FIR laser along with external wire grid polarizers were used to select the desired FIR line when more than one was present. The magnetic fields were supplied by a 6.8-T superconducting solenoid placed in a liquid-He immersion Dewar. The samples also were immersed in liquid He, thereby maintaining them at a constant temperature of 4.2 K. A light-pipe system (0.5-in.-diam brass) propagated the radiation into the Dewar. From ~1 to 500 μW of laser radiation, depending on the laser line employed, emerged onto the sample from the end of the section of pipe running parallel to the axis of the solenoind. In this arrangement, experiments on a bulk GaAs FIR detector showed that the excitation of the 1s → 2p₀ transition was very weak compared to the 1s → 2p₁⁺ or 1s → 2p₋₁ transitions.

The magnetic field was measured by an in situ Hall probe, which was calibrated in situ against another Hall probe, which was in turn calibrated at the National Magnet Laboratory. Great care was taken to minimize stresses on the probe during the measurements. The calibration of the probe was checked every ten thermal cycles, a precaution suggested by the results in Ref. 13. Thermal cycling occurred between 4.2 K and room temperature. Further, the calibration was checked against cyclotron resonance in bulk GaAs, a phenomenon that has been thoroughly studied. The calibration remained stable within 0.25%, which is probably due to the care taken to reduce stresses on the probe. Values of magnetic field quoted in this paper are accurate to within better than 0.5%.

Samples immersed in liquid He were illuminated by white light from an external 20-W quartz halogen lamp in order to free carriers from traps in the (Ga,Al)As prior to the spectroscopy runs. The current-voltage (I-V) characteristics of the samples were observed to be unstable until the sample was illuminated. After illumination the I-V characteristics remained stable, exhibiting a sharp breakdown point as long as the sample remained in liquid He. (Illumination-induced persistent effects have been observed previously.) Upon surveying a number of quantum-well samples we found that only those which showed a sharp breakdown point produced useful donor spectra in photoconductivity. All samples exhibited an asymmetry in I-V characteristic upon reversing polarity of the applied voltage.

In the runs made with the sample tilted, the tilt angle was determined by measuring the length of the sample and the height of the raised side. The angles quoted are accurate to within approximately ±2°. To minimize the possibility of the sample orientation changing during the experiment all data for a given angle θ were taken without removing the light-pipe assembly between measurements.

Figure 2 is a schematic diagram of the circuitry associated with the monitoring of the photoconductive response of the samples. The constant voltage supply is battery powered and was operated at 0.8 V. This operating value is well below the observed breakdown voltage of ~3 V. At voltages below breakdown, sample resistances in the dark were in excess of 20 MΩ. At the bias voltage employed, the dark bias current was observed to be less than 50 nA. A liquid-He-cooled Ge bolometer was used as a reference detector to monitor the stability of the FIR laser. The reference signal was also fed into a lock-in amplifier whose reference frequency was the chopping frequency of the CO₂ laser.

Photoconductivity spectra obtained by sweeping the
magnetic field at fixed laser frequency are displayed in Fig. 3. These measurements were taken in the Faraday configuration (θ=0°, light incident along the sample normal). A typical field sweep took approximately 3 min. The spectra displayed are not averaged; each spectrum is taken in a single sweep. Fluctuations in laser intensity during a run were monitored and found to be of the order of 1% of the total laser power. The data presented were not corrected for such fluctuations. The arrows indicate the peak of the $1s \rightarrow 2p_{+1}$ transition. Other sharp, prominent features appearing in the spectra (e.g., lines A, B, and C in Fig. 3) are relatively sensitive to the magnitude and polarity of the bias voltage applied and, to a lesser extent, the intensity of the laser. Under certain conditions of bias these features disappear leaving only the $1s \rightarrow 2p_{+1}$ line. The peak position of this line for each laser frequency is, however, independent of bias and laser intensity over the range of these parameters employed. We do not at present have a clear understanding of the origin of the extra structure in Fig. 3. The peaks A and B bear at least a superficial resemblance to peaks 1 and 2, respectively, reported in Ref. 16.

Figure 4 is a plot of the peak positions versus the magnetic field for the $1s \rightarrow 2p_{+1}$ transition and the A and B transitions. Error bars are given only for the four lowest field points of the $1s \rightarrow 2p_{+1}$ transition. The width of the error bars represents the uncertainty in locating the peak position of the lines in the magnetic field. There is no vertical error bar since the laser-line frequencies are known to better than one part per million. At higher magnetic fields, where the transitions are sharper, uncertainties in the resonant field values are dominated by noise and calibration errors inherent in the Hall probe, and are too small (≤0.5%) to be shown in Figs. 4–7. It should be noted that the data points without error bars in these figures are, in general, larger than the experimental error. The upper line in Fig. 4 is the result of theoretical calculations described in Sec. IV for the $1s \rightarrow 2p_{+1}$ transition. The deviations of the data points from this line are outside the bounds of measurement error and represent a failure of the theory to account for the experimental results. We believe that these data are the most accurate yet published for any shallow donor transition in quantum wells.

The spectral peak positions shown in Fig. 5 are ob-

![Figure 3](image-url)  
**Fig. 3.** Photoconductivity spectra obtained by sweeping the magnetic field at fixed laser frequency. Some of the more prominent features discussed in the text are labeled A, B, and C; the arrows indicate the $1s \rightarrow 2p_{+1}$ transitions.

![Figure 4](image-url)  
**Fig. 4.** Plots of peak positions of the transition energy vs magnetic field in Faraday geometry (θ=0°) for three of the transitions studied. The upper line is the theoretical curve for the $1s \rightarrow 2p_{+1}$ transition, the lower line is the theoretical curve for bulk cyclotron resonance in the absence of nonparabolicity with an effective mass ratio of 0.0665, and the points represent the experimental results for θ=0°.
tained when the sample is rotated into the Voigt configuration ($\theta=90^\circ$ and the radiation is deflected by a mirror so that the light remains incident along the sample normal). Experiments in this configuration have been reported recently. The calculations of Sec. IV indicate that the positions of the peaks observed in the Voigt configuration are much more sensitive to the well width $L$ than are those in the Faraday configuration. We have used the Voigt data to establish a working value for $L$. The theoretical results in Fig. 5 are for $L=51$ nm. The agreement shown with our data is markedly better than that obtained for either $L=50$ or 52 nm. Thus in our theoretical calculations for all geometries we have taken $L=51$ nm, a value consistent with our SEM value of $50\pm5$ nm quoted earlier.

Figures 6 and 7 show the experimental data for $\theta=16^\circ$ and $31^\circ$, respectively, and also compare these results with our coupled-state theory presented in Sec. IV. (For these tilt angles the radiation propagates along the direction of the magnetic field.) We judge the agreement very satisfactory considering that there are no adjustable parameters in the calculations for these figures. (Bulk values of $46.1\ \text{cm}^{-1}$ for the donor Rydberg energy and $0.0665$ for the ratio of the conduction-band mass to the vacuum electron mass are used throughout.) We take this agreement as strong evidence for the $2p_0-2p_{+1}$ anticrossing.

The level anticrossing just described may be responsible for an interesting effect in the Faraday data of Fig. 4. It seems quite possible that the deviation of the $1s-2p_{+1}$ transition peak at $74.63\ \text{cm}^{-1}$ energy from the theoretical curve for $\theta=0^\circ$ is the result of a small misalignment of the sample normal with respect to the magnetic field. In contrast to all the other peaks, this peak would be expected to be very sensitive to such misalignment since it lies very close to the energy at which the level crossing occurs at $\theta=0^\circ$. In fact, upon restoring the sample to the Faraday configuration after tilting it, we observed a slight change in the field at which the $74.63\ \text{cm}^{-1}$ peak occurred. In contrast, the positions of the other $1s-2p_{+1}$ points remained fixed to within the precision of our measurements.

Transitions labeled $A$ and $B$ in Figs. 3 and 4 both display anticrossing behavior upon tilting the sample, indicating that they too are associated with electrons confined in the quantum wells. The $A$ transitions are cyclotron-resonance-like as indicated by the close proximity of the $A$-peak data to the straight solid line in Fig. 4, representing bulk GaAs cyclotron resonance behavior.

![Figure 5](image-url)  
**FIG. 5.** Plot of peak position of the transition energy vs magnetic field in Voigt geometry ($\theta=90^\circ$). The solid line is the theoretical curve, and the points represent the experimental results for $\theta=90^\circ$.

![Figure 6](image-url)  
**FIG. 6.** Plots of peak positions of the transition energy vs magnetic field. The dashed lines are the theoretical curves for the Faraday geometry ($\theta=0^\circ$), the solid lines represent the theoretical results for $\theta=16^\circ$, and the points represent the experimental results for $\theta=16^\circ$.

![Figure 7](image-url)  
**FIG. 7.** Plots of peak positions of the transition energy vs magnetic field. The dashed lines are the theoretical curves for the Faraday geometry ($\theta=0^\circ$), the solid lines represent the theoretical results for $\theta=31^\circ$, and the points represent the experimental results for $\theta=31^\circ$. 
in the absence of nonparabolicity. These transitions behave much as expected for cyclotron resonance when the sample is tilted. They may be associated with bound states of donor ions located deep within the barriers. (The barriers are, however, not intentionally doped in our samples.)

The location of the energy region in which strong anticrossing is observed in the B transitions is relatively close to that of 1s → 2p_{+1}. We note that the experimental points labeled B in Fig. 4 can be fairly well fit by assuming a spike in the density of shallow donors at a position in the well that is considerably outside the region intentionally doped. We do not, however, at the present time regard this explanation of the B transitions with great confidence, especially in view of the fact that we have seen only a single transition, corresponding to donors located in the center of the wells, in our Voigt data. \[18\]

We tentatively ascribe the C peaks in Fig. 3 to 1s → 2s donor transitions which are turned on by the combination of the close proximity of the 2s and 2p_{+1} levels and the presence of random electric fields in the sample. Such fields arise from compensating acceptors, which are certainly present in our samples. Where the C peaks are resolved, their separation from the 1s → 2p_{+1} peaks is in fairly good agreement with values deduced from our 2s variational calculations. \[8\] The 1s → 2s donor transition has been observed in bulk GaAs. \[19\]

IV. CALCULATION OF ENERGY LEVELS

Theoretical transition energies reported in this paper are determined by taking energy differences between the appropriate levels calculated for isolated donors located in the center of a quantum well. The quantum wells, in turn, are themselves assumed situated in the sense that the barriers separating adjacent wells are considered to be infinitely thick (although of finite height).

Energy levels are calculated variationally from an effective-mass Hamiltonian in which the quantum well is represented by a square-well potential given by

\[ V(z) = \begin{cases} 0 & \text{for } |z| < L/2 \\ V_0 & \text{for } |z| > L/2 \end{cases} \]

where \( z \) is measured from the center of the well and, as discussed in Sec. III, the well width \( L \) is taken to be 51 nm. For the 20 at. % Al concentration in (Ga,Al)As barriers of our samples we use a barrier height \( V_0 \) of 25R, \[20,21\] where \( R \) is the bulk donor Rydberg in GaAs (\( R = 46.1 \) cm \(^{-1} \)). For convenience we introduce donor atomic units, wherein energies are in units of \( R \) and lengths in units of the bulk GaAs donor Bohr radius \( a_0 \) (\( a_0 = 10 \) nm). Without loss of generality we take the magnetic field \( B \) in the form

\[ B = B_x i + B_y k = B (\sin \theta, 0, \cos \theta) \]

We write our vector potential \( A \) as

\[ A = A^x + A^y \]

where

\[ A^x = B \sin \theta (1, -\delta z, 1 - \delta y) \]

\[ A^y = \frac{1}{2} B \cos \theta (-y, x, 0) \]

The parameter \( \delta \) is a variational constant as discussed in Ref. 17. It is trivial to verify that, independent of \( \delta \),

\[ \nabla \times A^x = B_x i \text{ and } \nabla \times A^y = B_x k \]

and therefore \( \nabla \times A = B \).

With choice of gauge given by (4), we can write the effective-mass Hamiltonian in our donor atomic units for the central donor as

\[ H = H_F + H' \]

\[ H_F = -\frac{\nabla^2}{r} + \frac{\gamma_x}{r} \left( \frac{x}{\partial y} - y \frac{x}{\partial x} \right) + \frac{1}{2} \gamma_x \rho^2 + V(z) \]

\[ H' = H'_1 + H'_2 \]

\[ H'_1 = \frac{2 \gamma_x}{i} \left( \frac{1}{2} - \xi \right) \frac{\partial}{\partial z} + \frac{\gamma_x}{\partial y} \frac{\partial}{\partial x} - \gamma_x \gamma_x \xi \]

\[ H'_2 = \gamma_x^2 \left[ \left( \frac{1}{2} - \xi \right)^2 + \xi^2 \right] \]

Here, \( r = (\rho^2 + z^2)^{1/2}, \rho^2 = x^2 + y^2, \gamma_x = \gamma \sin \theta, \text{ and } \gamma_x = \gamma \cos \theta \) with \( \gamma = \hbar \omega_c / 2 R \), where \( \omega_c = eB / m^* c \) and \( R = h/2m^* a_0^2 = e^2/2 \varepsilon_0 a_0^2 \), with \( a_0 = h/2m^* e^2 \) and \( m^* \) equal to the conduction-band mass in GaAs. It is to be understood that in the above Hamiltonian all coordinates are in units of \( a_0 \) and that \( V(z) \) is the potential given by (1) but in donor atomic units.

In choosing variational trial functions we have followed Ref. 17, assuming that the wave functions and their first derivatives are continuous across the barriers and employing trial functions similar to those described there. As in that reference, we introduce a set of wave functions \( f_n(U_0, z) \), depending on the variational barrier height \( U_0 \), and satisfying

\[ \left\{ -\frac{\partial^2}{\partial z^2} + U(z) \right\} f_n(U_0, z) = E_n f_n(U_0, z) \]

where \( U(z) = 0 \) for \( |z| < L/2 \) and \( U(z) = U_0 \) for \( |z| > L/2 \) (\( L \) here is assumed in units of \( a_0 \)).

In the Faraday configuration \( H' = 0 \), and we take trial functions of the form

\[ \rho^{IM} e^{-iM \phi} e^{i\phi \rho \zeta} f_n(U_0, z) \]

where

\[ \phi(\rho, z) = \exp \left[ -H \rho^2 - k (\rho^2 + \alpha z^2)^{1/2} \right] \]

and \( H, \kappa, \alpha, \text{ and } U_0 \) are variational parameters. If \( f_1 \) and \( f_2 \) represent the ground state and first excited state of (7), respectively, then we make the following correspondences between the quantum states and their trial functions from (8):

\[ 1s\leftrightarrow M = 0, \quad n = 1 \]

\[ 2p_{+1}\leftrightarrow M = 0, \quad n = 2 \]

\[ 2p_{+1}\leftrightarrow M = \pm 1, \quad n = 1 \]

\[ 4f_{-2}\leftrightarrow M = -2, \quad n = 2 \]
For each state we choose variational parameters that minimize the expectation value of $H_F$ in the appropriate normalized wave function of the form (8). Calculations for the fit to the Voigt data indicated in Fig. 5 were made using the two-orbital trial functions of Ref. 17.

In order to understand the level crossing observed when the sample is tilted with respect to the magnetic field it is useful to consider the limit of very small tilt angle ($\theta \ll 1$). In that limit we have $\gamma_x \ll \gamma_z$, and we can treat $H'$ as a perturbation on $H_F$. By inspection, $H'_1$ dominates $H'_2$ and, acting on an eigenfunction of $H_F$, has the effect of raising or lowering $M$ by one and reversing the $z$ parity. Thus, for example, $H'_1$ couples $2p_{+1}$ (a state with even $z$ parity and $M=+1$) to $2p_0$ (odd $z$ parity and $M=0$). Those two unperturbed levels cross in a magnetic field accessible to us (as indicated by the dashed lines in Fig. 7); for $\theta \neq 0$ they will repel each other near the unperturbed crossing point. From degenerate perturbation theory we expect that the splitting near the crossing point will be proportional to $\gamma_x$ and therefore to $\sin \theta$.

Since the above argument applies not just to $2p_0$ but also to $3p_0$ and to all higher-lying states with odd $z$ parity and $M=0$, one might ask whether any of those states might also produce a level crossing accessible to our experiment. Variational calculations show that, as suggested in our introductory comments, these higher excited states tune upward rapidly with increasing magnetic field. As a result they cross the $2p_{+1}$ level at very much higher fields than our 6.8-T maximum. Odd $z$-parity states with $M=+2$, which are also coupled to $2p_{+1}$ by $H'_2$, all lie higher than $2p_{+1}$ at zero magnetic field and tune upward so rapidly with increasing magnetic field that they cannot be expected to cross the $2p_{+1}$ level at all.

Although the discussion above seems to suggest that the $2p_0-2p_{+1}$ anticrossing exhausts the anticrossings induced by $H'$ up to fields as high as 6.8 T, it does not preclude the possibility of other low-field anticrossings. Additional anticrossings, which arise from treating $H'_1$ in higher order or which involve $H'_2$ are predicted; these are, however, weak in the sense that their level splittings at the crossing points of the relevant unperturbed levels are of higher order than $\gamma_x$ as $\gamma_x \rightarrow 0$. For example, the $4f_{-2}$ state is coupled in third-order perturbation theory to the $2p_{+1}$ level. The two unperturbed levels cross at a field near 4.2 T (see the upper crossing of the dashed lines in Fig. 6). The minimum splitting at the anticrossing is of order $\gamma_x^2$ and, although visible in Fig. 7 ($\theta = 31^\circ$), is too small to be clearly discerned in Fig. 6 ($\theta = 16^\circ$).

Excited-state levels participating in the anticrossings indicated by solid curves in Figs. 6 and 7 were calculated by minimizing the lowest eigenvalue of the $4 \times 4$ Hamiltonian matrix of $H$ formed from the following orbitals:

\[ \begin{align*}
2p_x \rightarrow x \exp[-Qy^2 - H \rho^2 - \kappa(\rho^2 + az^2)^{1/2}] f_1(U_0, z), \\
2p_y \rightarrow iy \exp[-Qy^2 - H \rho^2 - \kappa(\rho^2 + az^2)^{1/2}] f_1(U_0, z), \\
2p_0 \rightarrow \exp[-Hz \rho^2 - \kappa(\rho^2 + az^2)^{1/2}] f_2(U_0, z), \\
4f_{-2} \rightarrow \rho^2 e^{-2i\xi} \exp[-H \rho^2 - \kappa_2(\rho^2 + az^2)^{1/2}] f_2(U_0, z).
\end{align*} \] (11a) (11b) (11c) (11d)

In this minimization the variational parameters were $Q$, $H$, $H_z$, $\kappa$, $\alpha$, $U_0$, and $\xi$. Parameters $H_z$, $\kappa_2$, and $\alpha_2$ for the $4f_{-2}$ orbital were determined in a separate variational calculation by minimizing the $4f_{-2}$ energy in the Hamiltonian for the Faraday configuration, $H_F$. The ground-state energy was minimized independently using the trial function

\[ 1s \rightarrow \exp[-Qy^2 - H \rho^2 - \kappa(\rho^2 + az^2)^{1/2}] f_1(U_0, z), \] (12)

where $Q$, $H$, $\kappa$, $\alpha$, $\xi$, and $U_0$ were employed as variational parameters.

V. SUMMARY

We have observed that in a center-doped quantum-well sample the sign of the shift of the position of the $1s \rightarrow 2p_{+1}$ donor transition induced by tilting the sample in a magnetic field depends upon the photon energy absorbed in the transition. For photon energies above the (calculated) transition energy at which the $1s \rightarrow 2p_0$ and $1s \rightarrow 2p_{+1}$ transitions cross in the untilted sample (B parallel to the sample growth direction), the peak shifts to lower magnetic fields; at photon energies lower than this crossing point, the observed peak shifts to higher magnetic fields. Comparison of calculations to the experimental data at the tilt angles studied ($\theta = 16^\circ$ and $31^\circ$) lends quantitative support to the hypothesis that our experimental results reflect the anticrossing of the $2p_{+1}$ and $2p_0$ donor levels in the quantum well. The experimental technique, which combines far laser sources and photoconductive detection with lightly doped samples, gives the most accurate donor data yet reported for quantum wells.

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5. A related technique in which the photocurrents are not measured directly, but are instead detected by a change in capaci-


15L. G. Rubin (private communication).


18We consider very doubtful the tentative assignment of line 2 of Ref. 16, which appears to be the analog of our line B, to a so-called 1s→3p+1 transition associated with donors in the barrier.

