Practical Lessons Learned from Applying the Reliability Methods to LRFD for the Analysis of Deep Foundations

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ABSTRACT: NCHRP Project 24-17 “LRFD for Deep Foundations” was recently completed (Paikowsky, 2002). The large databases compiled during the research, enabled to assess the uncertainty of different deep foundation design methods, hence, to develop resistance factors based on monitored performance. Some practical lessons learned are presented, including the direct use of databases to estimate the probability of failure (in contrast of the calculated PDF), the influence of the resistance factors calculation method (FOSM vs. FORM) on the outcome parameters, and sensitivity analysis inspecting outliers and adjusting the number of cases while examining its influence over the resulting resistance factors.

1 BACKGROUND – NCHRP PROJECT 24-17

National Cooperative Highway Research Program, project 24-17 (NCHRP 24-17) “LRFD for Deep Foundations” was initiated to: (i) Provide recommended revisions to the driven pile and drilled shaft portions of section 10 of AASHTO Specifications and (ii) Provide a detailed procedure for calibrating deep foundation resistance factors. The current AASHTO specifications as well as other existing codes based on Load and Resistance Factor Design (LRFD) principles were calibrated using a combination of reliability theory, fitting to ASD (Allowable Stress Design) and engineering judgment. The main challenges of the project were therefore: (a) Compilation of large, high quality databases and (b) Framework for a procedure and data management to enable LRFD parameter evaluation, and future updates.

The AASHTO specifications are traditionally observed on all federally aided projects and generally viewed as a National code of the US Highway practice, hence influences the construction of all the deep foundations of highway bridges throughout the USA.

Since 1994 (1st LRFD edition) the existing AASHTO code, similarly to others worldwide, is based on Load and Resistance Factor Design (LRFD) principles. The current attempt is the first, however, to use reliability based calibration utilizing databases. In the present phase, the developed databases relate to axial capacity of single driven piles and drilled shafts only. Databases containing 527 and 136 case histories were compiled at the university of Florida for static evaluation of driven piles and drilled shafts respectively. For dynamic evaluation of driven piles, a database (PD/LT2000) containing information related to 210 piles and 403 dynamic measurements was compiled at the University of Massachusetts at Lowell.

2 UNCERTAINTY OF PILE DESIGN METHODS

A procedure to determine the nominal geotechnical pile strength out of static load-test results was defined, and its uncertainty was evaluated. As such, a unique and consistent ultimate pile/drilled shaft static capacity could be determined for all the analyzed cases, and compared to the various prediction methods. Paikowsky (2002a, 2002b) and Paikowsky and Stenerson (2000) detailed the various pile design methods that were examined during the project. Table 1 summarizes the different design methods for which uncertainty was evaluated. The number of cases under each category varied according to the specificity. Several histograms and frequency distributions for
the uncertainty evaluations are presented in Figures 1 through 6. The figures are presented in the form of the bias, i.e. the ratio of the measured resistance over the calculated (predicted) resistance, vs. the number of cases and relative frequency. Histograms and frequency distributions are presented using normal and lognormal distribution functions, suggesting in general a reasonable match between the histogram and the fitted lognormal distribution function.

Table 1. Summary of Uncertainty Evaluations for the Major Pile Capacity Design Methods

<table>
<thead>
<tr>
<th>Type of Analysis</th>
<th>No. of Design Methods</th>
<th>Soil Type or Driving Conditions</th>
<th>Pile Type or Construction Type</th>
<th>Total No. of Evaluations¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Analysis of Driven Piles</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>45 / 39</td>
</tr>
<tr>
<td>Dynamic Analysis of Driven Piles</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>30 / 14</td>
</tr>
<tr>
<td>Static Analysis of Drilled Shafts</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>36 / 30</td>
</tr>
</tbody>
</table>

¹Total number may differ from the possible combinations. Lower values represent the actual final evaluations used for the recommended resistance factors

Figure 1. Histogram and Frequency Distributions of the bias $K_{sx}$ for 52 Cases of All Pile Types (concrete, pipe, H) in Clay

Figure 2. Histogram and Frequency Distributions of the bias $K_{sx}$ for 74 Cases of All Pile Types (Concrete, Pipe, H) in Sand.
Figure 3. Histogram and Frequency Distributions of the bias $K_{sx}$ for 146 Cases of All Pile Types (Concrete, Pipe, H) in Mixed Soil.

Figure 5. Histogram and Frequency Distributions of the bias for All EOD (128) Energy Approach Pile-Cases in PD/LT2000.

Figure 6. Histogram and Frequency Distributions of the bias $K_{sx}$ for 53 Cases of Drilled Shafts in Clay.

Practical Lessons Learned from Applying the Reliability Methods to LRFD (S. G. Paikowsky)
3 CALIBRATION METHODS

The process by which the load and the resistance factors are assigned to match the prescribed probability of failure is recognized as calibration.

The 2001 AASHTO LRFD Bridge Design Specifications for Foundations provides that the ultimate resistance \( R_u \) multiplied by a resistance factor \( \phi \), which thus becomes the factored resistance \( R_f \), must be greater than or equal to the summation of loads \( Q_i \) multiplied by corresponding load factors \( \gamma_i \), and a modifier \( \eta_i \). For strength limits states:

\[
R_f = \phi R_u \geq \sum \eta_i \gamma_i Q_i
\]

where:

\[
\eta_i = \eta_D \eta_R \eta_I > 0.95
\]

\( \eta_i \) = factors to account for effects of ductility \( (\eta_D) \), redundancy \( (\eta_R) \), and operational importance \( (\eta_I) \).

The existing AASHTO LRFD specifications are based on first-order, second-moment (FOSM) analysis originally proposed by Cornell (1969). Using \( \eta=1 \) in equation 1, and assuming lognormal distribution for the resistance, leads to the relation (Barker et al., 1991):

\[
\phi = \frac{\lambda_R \left( \sum \gamma_i Q_i \right)}{Q \exp \left\{ \beta \sqrt{\ln \left( \frac{1 + COV_R^2}{1 + COV_Q^2} \right)} \right\}}
\]

where:

\[
\lambda_R = \text{resistance bias factor}
\]
\[
COV_Q = \text{coefficient of variation (standard deviation divided by mean) of the load}
\]
\[
COV_R = \text{coefficient of variation of the resistance}
\]
\[
\beta = \text{target reliability index}
\]

When just dead and live loads are considered equation 3 can be rewritten as:

\[
\phi = \frac{\lambda_Q D \left( \frac{\gamma_D Q_D}{Q_L} \right) + \gamma_L}{\left( \frac{\gamma_D Q_D}{Q_L} + \lambda_{QL} \right) \exp \left\{ \beta \sqrt{\ln \left( \frac{1 + COV_{Q_D}^2 + COV_{Q_L}^2}{1 + COV_R^2} \right)} \right\}}
\]

where:

\( \gamma_D, \gamma_L \) = dead and live load factors
\( Q_D/Q_L \) = dead to live load ratio
\( \lambda_{QD}, \lambda_{QL} \) = dead and live load bias factors

The probabilistic characteristics of the random variables DL and LL are assumed to be those used by AASHTO (Nowak, 1999) with the following load factors and lognormal distributions (bias and COV) for live and dead loads, respectively:

\[
\gamma_L = 1.75 \quad \lambda_{QL} = 1.15 \quad COV_{QL} = 0.2
\]

\[
\gamma_D = 1.25 \quad \lambda_{QD} = 1.05 \quad COV_{QD} = 0.1
\]

Following Ayyub and Assakaf (1999), the LRFD partial safety factors were calibrated using FORM (First Order Reliability Method) procedure as developed by Hasofer and Lind (1974). FORM can be used to assess the reliability of a pile with respect to specified limit states, and provides a means for calculating partial safety factors \( \phi \) and \( \gamma_i \) for resistance and loads, respectively, against a target reliability level, \( \beta \). FORM requires only first and second moment
information on resistances and loads (i.e. means and variances), and an assumption of distribution shape (e.g. Normal, lognormal, etc.). The calibration of the resistance factors is performed for a set of load factors already specified in the structural code. Thus, the load factors are fixed. In this case, the following algorithm was used to determine resistance factors:

1. For a given value of the reliability index $\beta$, probability distributions and moments of the load variables, and the coefficient of variation for the resistance, compute mean resistance $R$ using FORM.

2. With the mean value for $R$ computed in step 1, the partial safety factor $\phi$ is revised as:

$$\phi = \frac{\sum_{i=1}^{n} \gamma_i \mu_{Li}}{\mu_R}$$  \hfill (7)

where $\mu_{Li}$ and $\mu_R$ are the mean values of the loads and strength variables, respectively and $\gamma_i, i = 1, 2, \ldots, n$, are the given set of load factors.

Substituting the parameters of equations (5) and (6) into equation (4) provides a relatively simple format to calculate the resistance factors knowing the bias of the calibrated design method. The FORM on the other hand, is based on iterative process, though not complex, requires the use of a computer.

Figure 7 presents the relationship between the resistance parameters developed using FORM to those developed using FOSM for 160 various calibrations categorized under three groups; driven piles static analysis, drilled shafts static analysis, and driven piles dynamic analysis, using a target reliability of $\beta = 2.33$ ($p_f = 0.01$). The data in Figure 7 suggests that FORM results in resistance factors consistently higher than those obtained by FOSM. The ratio between the two indicates that as a rule of thumb, FORM provides resistance factors approximately 10% higher than those obtained by FOSM. Two practical conclusions can be obtained from the observed data; (i) first evaluation of data can be done by the simplified closed form FOSM approach and the obtained resistance factors are on the safe (lower) side, and (ii) the obtained parameters in the study based on databases can be directly compared to the current specifications or other LRFD codes based on FOSM and back-calculated parameters from WSD.

![Figure 7. Comparison Between Resistance Factors Obtained Using the First Order Second Moment (FOSM) vs. Those Obtained by Using First Order Reliability Method (FORM) for a Target Reliability of $\beta = 2.33$.](image)
Table 2 presents the values of resistance factors recommended for selected pile capacity prediction methods based on dynamic analysis and the use of target reliability of $\beta = 2.33$ ($p_f = 0.1\%$). One advantage of using a large database is that the probability of failure (or the risk) can be directly calculated from the available data, rather than using the calculated distribution function. The procedure is done by applying a certain resistance factor to the calculated resistance (capacity), and examining the number of cases that exceed the actual capacity (nominal strength). An example of the process’s outcome as applied to some of the dynamic methods is presented in Table 3.

**Table 2.** Recommended Resistance Factors for Selected Pile Capacity Evaluation Methods Based on Dynamic Analysis.

<table>
<thead>
<tr>
<th>Method</th>
<th>Resistance Factor $\phi$ ($\beta = 2.33$, $p_f = 1%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPWAP EOD</td>
<td>0.40</td>
</tr>
<tr>
<td>AR &lt; 350</td>
<td></td>
</tr>
<tr>
<td>BC &lt; 16B/10cm</td>
<td></td>
</tr>
<tr>
<td>CAPWAP BOR</td>
<td>0.65</td>
</tr>
<tr>
<td>Energy Approach EOD</td>
<td>0.55</td>
</tr>
<tr>
<td>FHWA Eq. (mod. Gates)</td>
<td>0.40</td>
</tr>
</tbody>
</table>

**Table 3.** Calculated Probability of Failure Based on Direct Utilization of Database PD/LT 2000 for Selected Prediction Methods.

<table>
<thead>
<tr>
<th>Resistance Factor $\phi$</th>
<th>CAPWAP BOR</th>
<th>CAPWAP EOD AR &gt; 350 BL ct. &gt; 16 BP10cm</th>
<th>Energy Approach EOD</th>
<th>FHWA Mod. Gates (General Case)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0</td>
<td>2.70</td>
<td>1.56</td>
<td>10.42</td>
</tr>
<tr>
<td>0.40</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3.13</td>
</tr>
<tr>
<td>0.33</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.78</td>
</tr>
<tr>
<td># of cases used</td>
<td>162</td>
<td>37</td>
<td>128</td>
<td>384</td>
</tr>
</tbody>
</table>

The data in table 3 can be compared to the recommended resistance factors of Table 2. The comparison suggests that at times the actual probability of failure is smaller than the one the analysis was based on (e.g. CAPWAP EOD), while for other cases it exceeded the assigned $p_f$ (e.g. FHWA Mod. and Energy Approach EOD). In the cases where the “actual” probability of failure exceeded the “nominal” one, two observations can be made:

(a) “Failure” for the data in Table 3 means that the evaluated (predicted) resistance multiplied by the recommended resistance factor resulted in a value larger than the nominal resistance. The evaluation in Table 3 does not consider the fact that the loads in the design are enhanced by the load factor, hence, the values in Table 3 are conservative.

(b) The “actual” probability of failure is based on a few outliers for which the predicted capacity far exceeded the measured resistance. For example the $p_f = 2.7\%$ for CAPWAP
EOD when applying $\phi = 0.5$ is a result of one extreme case. As such, the extreme cases need to be individually examined as to the reasons of the unusual values they represent. A procedure of evaluating the extreme cases is described in the following section.

5 RESISTANCE FACTORS SENSITIVITY TO THE SIZE OF THE DATABASE AND THE EXTREME CASES

Figures 8 and 9 present the results of a procedure allowing the evaluation of the extreme cases and their relevance in the determination of the recommended resistance factors. Two methods of static pile capacity analysis are presented, the $\alpha$ method (based on API) for the evaluation of pipe piles capacity in clay, and the $\beta$ method (effective stress analysis) for the evaluation of pipe piles capacity in sand, (Figures 8 and 9 respectively). The original data set in the first case consisted of 20 piles. The limit of the data for two standard deviations (SD) around the mean required the elimination of one case only and resulted in an increase in the resistance factor by about 20%. The restriction of the data to 1.5SD around the mean required the elimination of one additional case with further similar increase in the resistance factor, thus was the case of reducing the data to 1SD. On the other hand, for the second case (Figure 9), a change in the database from all cases (20) to 1.5SD around the mean required the reduction of two case histories, but there was no effect on the COV and, therefore, on the resistance factors.

Such evaluation along with the probability of failure based on the actual data leads to careful examination of the data, and using comparisons with more inclusive databases (e.g. in reference to Figure 8, one would examine the database of all piles in clay evaluated for the $\alpha$-API method, see Figure 1). Practical conclusions can be made as to the choice of the resistance factor and the actual probability of failure associated with it.

![Figure 8](image_url)  
**Figure 8.** Sensitivity Analysis Examining the Recommended Parameters for the Design of Pile Piles in Clay Using $\alpha$ API Method.
Figure 9. Sensitivity Analysis Examining the Recommended Parameters for the Design of Pipe Piles in Sand Using the $\beta$ Method.

6 CONCLUSIONS

Some of the practical lessons learned from the development of LRFD parameters for deep foundations based on databases are presented and discussed. The major derived conclusions are:

(a) The use of the simple closed form equations based on FOSM lead to practical values consistently on the safe side relative to the values obtained by calibrating the resistance factors using FORM. These values were approximately 10% lower and, hence, first evaluation of resistance factors using FOSM is justified and reasonable.

(b) When choosing the appropriate resistance factor and evaluating the associated probability of failure, one can employ several techniques utilizing actual data beyond the calculations based on the representative probability distribution functions. These techniques include the actual probability of failure based on the database and sensitivity analysis examining the influence of the outliers on the determined resistance factors. These, along with related (more inclusive) databases and the close examination of individual case histories in the database, result in a more logical and appropriate selection of resistance factors and the evaluation of the associated probability of failure.

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REFERENCES


