MODAL SPACE - IN OUR OWN LITTLE WORLD

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I am still confused about how the SDOF in modal space is related to the physical response?
This needs some discussion.

Well - this is a concept that is actually very simple but does need some explaining to make sure it is comprehended properly. First let's start with a few summary equations that we have presented several times before in previous articles. Of course the equation of motion in matrix form is the starting point

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\}$$

This coupled set of matrix equations is then uncoupled by performing an eigensolution. The modal transformation equation is obtained from the set of modal vectors obtained from the eigensolution. The physical coordinate {x} is related to the modal coordinate {p} using the collection of modal vectors [U]

$$\{x\} = [U]\{p\} = \{u_1\}p_1 + \{u_2\}p_2 + \{u_3\}p_3 + \cdots$$
$$\text{with } [U] = [\{u_1\} \quad \{u_2\} \quad \{u_3\} \quad \cdots]$$

Substituting this into the physical equation and premultiplying by the transpose of the projection operator [U] will result in a very simple diagonal set of equations in modal space where every equation (modal oscillator) is orthogonal and linearly independent (uncoupled) from each other and is given as

$$\begin{bmatrix} \bar{m}_1 & & \\ & \bar{m}_2 & \\ & & \ddots \end{bmatrix}\begin{Bmatrix} \ddot{p}_1 \\ \ddot{p}_2 \\ \vdots \end{Bmatrix}+\begin{bmatrix} \bar{c}_1 & & \\ & \bar{c}_2 & \\ & & \ddots \end{bmatrix}\begin{Bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \vdots \end{Bmatrix}+\begin{bmatrix} \bar{k}_1 & & \\ & \bar{k}_2 & \\ & & \ddots \end{bmatrix}\begin{Bmatrix} p_1 \\ p_2 \\ \vdots \end{Bmatrix}=\begin{Bmatrix} \{u_1\}^T\{F\} \\ \{u_2\}^T\{F\} \\ \vdots \end{Bmatrix}$$

There are several important things that need to be noted about this equation. And the mode shape matrix [U] has a lot to do with this.

First is that every equation contains only one variable to describe each equation – the modal displacement for each particular mode. Second is that every equation is uncoupled from every other equation. Third is that each equation is basically a very simple single degree of freedom (SDOF) system. Fourth is that the right hand side of the equation identifies the force that is appropriated to the modal oscillator from the physical force applied to the physical system. Figure 1 shows a schematic of a multiple degree of freedom system (MDOF) where coupling between degrees of freedom exist in the physical model and the resulting equivalent set of SDOF systems representing the modal system in modal space.

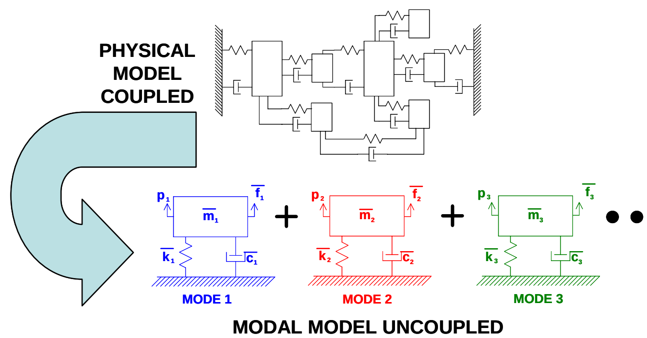

Fig 1 – MDOF System Schematic and SDOF Equivalent

So if we write out any one equation from the modal space form and use an "i" subscript the "ith" equation we would get

$$\bar{m}_i\ddot{p}_i + \bar{c}_i\dot{p}_i + \bar{k}_i p_i = \{u_i\}^T\{F\} = \bar{f}_i$$

So with this simple SDOF equation we can calculate the response due to any force applied on the equivalent system. Of course we can see that the right hand side of the equation identifies how much of the force is appropriated from physical space to the equivalent modal system through the mode shape. With this force, then the response for the equivalent system can be identified. This response can be simply found from any Vibrations textbook – usually this is one of the first four chapters in most textbooks for free response, forced sinusoidal response or arbitrary input response. For sake of the discussion here, let's assume that an impact is applied at one point on the physical system.

Now that physical force will be appropriated to each of the modal DOF in modal space. So if we look at the first mode then we could calculate the impulse response for the SDOF describing mode 1. This SDOF response is then distributed over all the physical DOF using the modal transformation equation; this essentially scales the SDOF response to all the physical DOF using each value of the first mode shape at each individual DOF. This is schematically shown in Figure 2 (for just a few DOF to illustrate the concept). Now this only provides the part of the response of the physical system that is related to the contribution that mode 1 makes over all the physical DOF in the system; the portion of the response related to mode 1 is shown in blue.

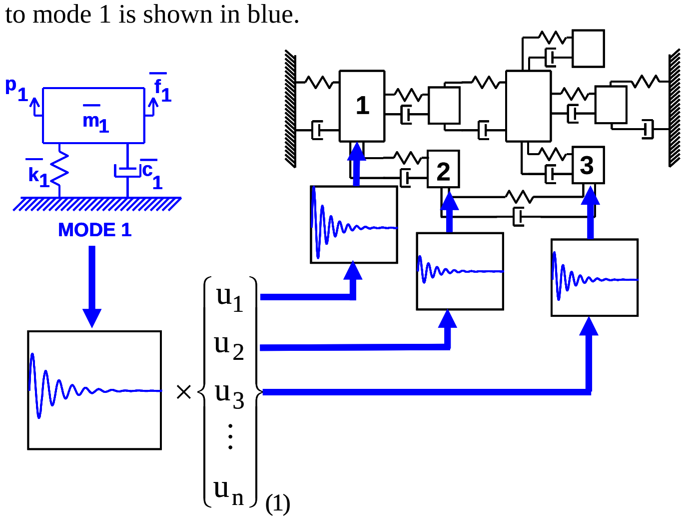

Fig 2 – Schematic Response for Mode 1 Contribution

This is not the entire physical response of the system – it is just the portion of the response that is related to mode 1. Now the contribution of the other modes needs to also be included. If we look at the second mode then we could calculate the impulse response for the SDOF describing mode 2 with the force that is appropriated to mode 2. Again this SDOF response for mode 2 needs to be distributed over all the physical DOF using the second mode shape; this only represents the portion of the response that is related to the second mode of the system. This is shown in Figure 3 (for just a few DOF to illustrate the concept); the portion of the response related to mode 2 is shown in red.

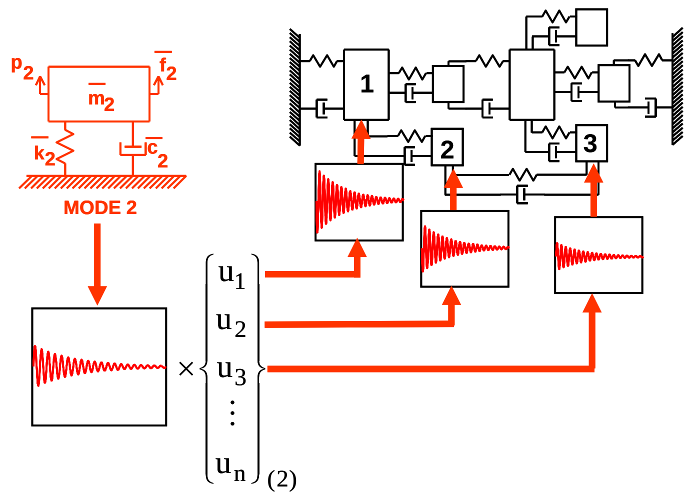

Fig 3 – Schematic Response for Mode 2 Contribution

This process is then continued for all the modes that contribute to the total response of the physical system. Of course you have to include all the modes that have a contribution to the overall response otherwise some of the solution is lost. The entire process is best seen in Figure 4. This figure shows the physical equation and the modal transformation equation which allows the coupled physical system to be written as a set of equivalent SDOF systems in modal space with the equivalent modal force applied on all the modal oscillators in modal space. It is important to realize that each mode is linearly independent from every other mode but that the total response is made up of the linear combination of the response of all the modes that participate in the response of the system.

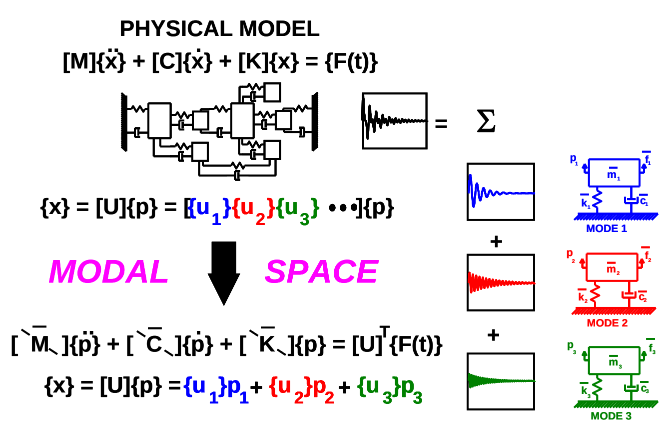

Fig 4 – Overview of the Modal Space Representation

I hope this explanation helps you to understand how the SDOF response is characterized in physical space from the modal space response. If you have any other questions about modal analysis, just ask me.