The Rising Australian Household Debt: Results from A Bayesian VAR Analysis

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Abstract

Using Bayesian vector autoregressions (VAR) and sign restriction identifications, we find that while the impact of unemployment rate on household debt significantly increases, the interest rate does not play an important role in the rising of household debt as documented in the literature. Comparisons between Bayesian structural VAR and OLS structural VAR results have been made and it is shown that the OLS structural VAR over estimates the impact of housing price on the debt level. The inefficiency of VAR estimations is identified as a reason for the significant differences between the two models.

Keywords: Bayesian estimation, structural VAR, household debt, housing price

JEL classification: E24; C32; H60
1 Introduction

Rising household debt in recent decade is a common phenomenon in developed countries. This phenomenon raised considerable concerns. Notably, the Economist 2003 commented: The profligacy of American and British households is legendary, but Australians have been even more reckless, pushing their borrowing to around 125 per cent of disposable income. This comment was justified by the outbreak of subprime mortgage crisis in the US and the global financial crisis followed. After the 2008 global financial crisis, the level of household debt in most countries tends to decrease, but this is not the case in Australia. The real household debt per person has continued to grow after 2008 albeit at a slower pace (see Figure 1). The household debt income ratio did decrease significantly in 2008 but it started to rise afterwards, approaching 180 in 2013 (see Figure 2).

![Figure 1: Real (a) household debt per person](image)

Notes: (a) All dollar values presented in this graph have been converted into December quarter 2013 dollars using the All Groups Consumer Price Index. Source: Australian National Accounts: Financial Accounts, December Quarter 2013 (ABS cat. no. 5232.0); Australian Demographic Statistics, September Quarter 2013 (ABS cat. no. 3101.0); Unpublished projected resident population for 31 December 2013; Consumer Price Index, Australia, March Quarter 2014 (ABS cat. no. 6401.0)

The growing trend of Australian household debt renewed the concerns about the sustainability of rising Australian household debt. The ABC News program (2014) reported the concern by David Skutenko from the Australian Bureau of Statistics that the figure is not just high in historical terms, but by global standards our debt burden is among the highest in the developed world. Consumer group Choice is warning that borrowers are being offered
new home loan products that create huge financial risks for the individual. Bank of America Merrill Lynch’s chief economist for Australia Saul Eslake says the risk of a financial crisis is small, but the slower growth in debt is likely to mean less household consumption and slower economic growth. These concerns indicate that rigorous research on the rising household debt is much needed.

This paper attempts to explain the reason of the rising of Australia rising debt. We argue that interest rate does not affect very much on the debt fluctuations, but unemployment rate has a much larger impact. Contradicting with common knowledge and previous results, a rise in housing price affect negatively on household debt. The fact that unemployment rate in Australia keep decreasing for a long time plays a significant role in the context of rising household debt. The remainder of the paper is organised as follows. The next section reviews the surveys and studies on Australian household debt. Section 3 describes the data set used for the study. Section 4 is aimed at specifying the methodology approach and setting up the model. Section 5 present the major estimation results, interpretations and discussion on the main findings from the empirical models. Section 6 summarises the main conclusions.
2 Previous studies

There are a large body of studies on household debt, so it is not necessary (even if it is possible) to review all studies in this area. This section will review only the representative studies on household debt, with an emphasis on studies on Australian household debt. On the factors influencing household debt, Barnes and G. (2003) employ a calibrated partial equilibrium overlapping generation (OLG) model to explain the indebtedness of the US households in terms of a consumption-income motive and housing-finance motive. They find that the substantial rise of household debt in the 1990s can be explained by real interest rate, income growth expectations, demographic changes, and the removal of credit constraint. Martins and Villanueva (2003) construct a data set combining household survey data and administrative record of debt to estimate the responsiveness of long-term household debt to the interest rate change in Portugal and they find that the elasticity of the probability of mortgage borrowing to a change in the interest rate is large and negative. Kearns (2003) employs household-level data to explore the reasons why households fall into mortgage arrears during the 1990s in Ireland. His study suggests that a modest rise of interest rates would result in a substantial repayment burdens for significant number of newly mortgaged households and concludes that the continuing strong growth of mortgage lending, caused by relaxed lending criteria and households accepting higher repayment burdens, may lead to a higher rate of mortgage arrears among households. Hull (2003) uses the ordinary least square (OLS) method to estimate a long-run consumption function derived from the life-cycle model and finds that the financial deregulation and the resulting improved access to borrowing have a positive effect on household consumption decisions. Jacobsen (2004) employs a flexible dynamic model and the Norwegian quarterly data from 1994 Q1 to 2004 Q1 to estimate the effects of various factors on household debt and claims that many factors influence the household debt such as the housing stock, interest rates, the number of house sales, the wage income, the housing prices, the unemployment rate, and the number of students. Yilmazer and DeVaney (2005) use the data from the Survey of Consumer Finances in the USA to examine whether the types and amounts of household debt changes over a life cycle and they discover that the likelihood of holding each type of debt and the amount of each type of debt compared to total assets decrease with age. Crawford and Faruqui (2011) investigated the causes of upward trend of Canadian household debt and concluded that favourable income growth and low interest rates support significant increases in mortgage debt while rising house prices and financial innovation lead to the expansion in consumer credit. On the effect of rising household debt, the Hong Kong Monetary Authority (2002) uses an error correction
model (ECM) to examine the relationship between consumer credit, the debt-service burden, and consumer spending in Hong Kong and uncovers that the changes in debt-service burden is negatively associated with consumption in the short run, but the quantity of consumer credit has little explanatory power for consumption in addition to that of income and property prices. Ogawa and Wan (2005) utilise the re-sampled data from the National Survey of Family Income and Expenditure in Japan to estimate the influence of household debt on consumption during and after the bubble period in Japan. They claim that when a household is trapped into borrowing constraints, debt has independent negative effects on consumption besides conventional wealth effects and that debt has adverse effects especially on consumption on semi-durables, non-durables, and luxuries. Rinaldi and Snachis-Arellano (2006) also use an error-correction framework to model the household arrears on payment obligations. Their results suggest that the financial conditions of households with debt might become more vulnerable to adverse shocks in their income and wealth. OECD (2013) investigated the impact of household debt on macroeconomic stability. It is suggested that the household debt in OECD countries has risen to high levels and the high levels of household debt created a number of vulnerabilities and raised the risk of recession. To retain the household debt level and reduce vulnerabilities, the paper suggested three lines of defence: micro-prudential regulation, macro-prudential regulation and monetary policy. A few papers address both the reasons for and the effects of household debt. Maki (2000) summarizes some of the relevant facts and the research concerning the growth of consumer credit and the household debt service burden. Crook (2003) compares the household debt and the results of studies on it across countries and finds that the debt holding by age follows the life cycle pattern in all countries observed and that there are considerable variations in the determinants of desired stock of debt and in marginal effects of household debt within countries as well as between countries. Debelle (2004) uses the data across countries to analyse the possible determinants and the macroeconomic implications of rising household debt. According to him, the rise of household debt reflects the response of households to lower interest rates and an easing of liquidity constraints. The increased household debt itself is not likely to be the source of negative shocks to the economy but will amplify shocks from other sources. Thaicharoen and Chucherd (2004) find that low interest rates, demographics, and declining borrowing constraints, have contributed to indebtedness of Thai households and that the current debt levels in Thailand do not pose a threat to financial stability and the macro-economy. Mason and Jayadev (2012) studied the long term dynamic of household debt in the US. By extending the standard decomposition of public debt to household debt, the paper showed that
interest rates, real GDP growth rates and inflation rates are important factors for explaining rising household debt levels. The paper also claimed that the high leverage of household debt hampered the economic growth and the deleveraging cannot be achieved through reducing expenditure relative to income level. Without substantial reduction in interest rates, large-scale debt forgiveness is required to deleverage household debt. In Australia, the RBA has published a number of papers and speeches on this topic. For example, Stevens (1997) emphasizes the positive effect of low inflation on household borrowings. The Reserve Bank of Australia, RBA (1999) attributes the quick growth of personal credit to the innovations in products offered by banks, the increasing household preference toward the use of credit cards, and the continuing economic expansion with low inflation and low interest rates. The RBA (2003) illustrates the composition and distribution of household debt and suggests that low interest rates, low inflation rate, and financial deregulations may have led to the rising household debt. Other studies air an opinion similar to that of RBA. For example, the Australian Consumers Association (2003) believes that financial deregulation, retailing credit, and the new financial products and channels account for the rising household debt. The treasury (2005) suggests that the increase in household debt partly reflects increased house prices, due to the sustained low inflation and interest rates, and partly reflects the improved product choice and reductions in borrowing costs due to the deregulation of the financial sector in the 1980s and 1990s. The ANZ bank (2005) claims the rising household debt level is due to the sustained boom in house prices and the sustainability of household credit depends on the growth of household disposable income and employment. Keen (2009) adopted a Minskian approach to construct a model of endogenous money creation. Using this model, he demonstrated that borrowing based on speculation on asset prices can lead to household debt bubble and concluded that the government should retain the household debt level. Meng et al. (2013) employ a cointegrated VAR model also found that interest rate is the major factor that influences the level of household debt in Australia.

3 Dataset

The household debt level is jointly determined by supply and demand. That is, the availability of funding, and the households decision to take on debt. However, the macroeconomic environment ultimately determines both supply and demand. Consequently, the determinants of household debt must lie in these macroeconomic factors. By analysing factors affecting borrowing and/or lending, we can therefore identify potential determinants of household
debt. With regard to demand, household desire for borrowing is subject to the level of household disposable income as well as the purpose/s for borrowing. Household disposable income is comprised of household gross income plus social transfer, less income tax payable and other outlays. We focus here on household gross income, since income tax rates in Australia have undergone minimal change in recent decades, while social transfer and other outlays are small relative to household gross income. Household gross income includes wage income and gross mixed income, as well as domestic and overseas investment income. At the macro level, these factors can be approximated by Gross Domestic Product (GDP). The purposes of household borrowing include smoothing consumption and investing. It may be related also to the macros affecting consumer confidence, such as unemployment rate and GDP. Investment decisions are typically related to interest rates. Moreover, from the disaggregation of Australian household debt, we found housing to be the main investment vehicle for Australian households. In considering the large amount of housing debt, housing price may be an important factors. With regard to supply, the availability of funding and ease of obtaining finance are largely indicated by interest rates. However, to reduce credit risk, lenders may take into account household income level and other macroeconomic variables, including unemployment rate. Among these factors, the household income level can be approximated by GDP. Including possible variables affecting Australian household debt yields the following dataset:

\[
X = (\text{DEBT}, \text{GDP}, \text{HPI}, R, U)
\]

- **DEBT**: accumulated household debt
- **GDP**: gross domestic product
- **HPI**: housing price index
- **R**: interest rate
- **U**: unemployment rate

It may be argued that real per capita data are preferable because they can reduce heteroscedasticity in the model. Nonetheless, this study uses nominal values for a number of reasons. First, the White tests show the heteroscedasticity problem in the model is not serious when nominal values are used. Second, measurement errors are often a problem in macro time series, and transforming nominal value to real and/or per capita value (divided by CPI or POP) may magnify these errors. Finally, some international studies show that population and inflation have a significant influence on household debt. Thus, a nominal value is required to test if this holds true for Australia as well. The quarterly time series data from 1988Q2 to 2011Q2 for the variables in the information set were collected from...
the Australian Bureau of Statistics ABS (2011), the RBA (2011), and other institutions. The majority of data were obtained from the ABS, including unemployment rates, housing prices, and the GDP. Data for household debt and official interest rates were provided by the RBA. Due to different sources and different measurement of data, some data sequences were adjusted before use. Specifically, the dataset in this study is described as follows:

Household debt (DEBT): seasonally adjusted quarterly data, measured in billion Australian dollars (AUD billion), at the end of quarter.

GDP: seasonally adjusted quarterly data, measured in billion Australian dollars (AUD billion).

Housing price index (HPI): CPI on housing 1989/90=100.

Interest rate (R): official interest rate, quarterly averaged monthly data.

Unemployment rate (U): quarterly averaged monthly data.

4 Methodology Approach

4.1 Bayesian VAR

Since the work by Sims (1980), vector autoregressive (VAR) model is a popular model to study the interdependence between macro variables. VAR treats all variables as endogenous and by this way, it allows to capture all the possible relationships among the variables. By its nature, VAR is not a parsimonious model. For example, a VAR model with 5 variables and 4 lags contains 105 parameters to be estimated. Macro variables are observed with time dimension and it is rarely to find a data set of macro indicators with more than a couple hundreds observations. For most of OECD countries, the data contains between 100 to 200 observations. This raises a serious question in VAR estimation as with the number of parameters as many as number of observations, the estimates are inefficiency and it is inaccurate to use those estimates in producing impulse response functions and other references. The limited data availability is a problem that is hard to be solved in most of macro research. Bayesian approach is a big hope to overcome this issue. In the view of Bayesian researchers or experimenters, the true values of the model parameters are random numbers as they don’t know and never know about the exact values of the parameters. They can use out of sample information to guess about the distributions of the parameters, these are called prior distributions. Based on the observed data, the prior distributions would be adjusted to be posterior distributions. Inferences about the parameters are made using the
posterior distributions. The posterior distributions conditional on the observed data could be obtained from the prior distributions using the Bayesian theorem applied in statistics:

\[
\pi(\theta|y) = \frac{f(y|\theta)\pi(\theta)}{f(y)}
\]

\[\theta\] : the model parameters
\[\pi(\theta)\] : prior distributions
\[\pi(\theta|y)\] : posterior distributions
\[f(y|\theta)\] : distribution of \(y\) given \(\theta\), this is the likelihood function
\[f(y)\] : marginal distribution of \(y\), this is the unconditional distribution of \(y\)

The posterior distributions express the researchers’ updated belief about \(\theta\) in the light of the observed data \(y\). Data will change the researchers’ belief about the unknown parameters. Because the unconditional distribution of \(y\) is constant so that the posterior distribution is a fraction of the likelihood function multiple by the prior distribution, we can write

\[
\pi(\theta|y) \propto f(y|\theta)\pi(\theta)
\]

We follow the Bayesian VAR discussion by Koop (2003) and Koop and Korobilis (2009). The VAR(\(p\)) model in reduced-form could be written:

\[
y_t = a_0 + \sum_{j=1}^{p} A_j y_{t-j} + \epsilon_t
\]

\(y_t\) is \(M \times 1\) vector
\(\epsilon_t\) is \(M \times 1\) vector of errors
\(a_0\) is \(M \times 1\) vector of intercepts
\(A_j\) is an \(M \times M\) matrix of coefficients
\(\epsilon_t \sim N(0, \Sigma)\)

The VAR can be written in matrix form in different ways:

\[
Y = XA + E
\]

or

\[
y = (I_M \otimes X)\alpha + \epsilon
\]
where \( x_t = (1, y'_{t-1}, \ldots, y'_{t-p}) \)

\[
X = \begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_T
\end{bmatrix}
\]

\( A = (a_0, A_1, \ldots, A_p)' \). If we denote \( K = 1 + Mp \) then \( \alpha = vec(A) \), \( \alpha \) is a \( KM \times 1 \) vector which stacks all the VAR coefficients and the intercepts into a vector. \( X \) is a \( T \times K \) matrix, \( Y \) and \( E \) are \( T \times M \) matrices. \( y \) and \( \epsilon \) are \( MT \times 1 \) vectors, \( \epsilon \sim N(0, \Sigma \otimes I_M) \). OLS estimates of the model are:

\[
\begin{align*}
\hat{A} &= (X'X)^{-1}X'Y \\
\hat{\alpha} &= vec(\hat{A}) \\
S &= (Y - X\hat{A})' \left(Y - X\hat{A}\right) \\
\hat{\Sigma} &= \frac{S}{T - K}
\end{align*}
\]

4.2 Priors

4.2.1 The diffuse prior

The diffuse or Jeffreys’ prior for \( \alpha \) and \( \Sigma \) takes the form

\[
p(\alpha, \Sigma) \propto |\Sigma|^{-(M-1)/2}
\]

The conditional posteriors in analytical forms could be derived

\[
\begin{align*}
\alpha|\Sigma, y &\sim N(\tilde{\alpha}, \Sigma) \\
\Sigma|y &\sim IW(S, T - K)
\end{align*}
\]

4.2.2 The natural conjugate prior

The natural conjugate prior has the form

\[
\begin{align*}
\alpha|\Sigma &\sim N(\alpha, \Sigma \otimes V) \\
\Sigma^{-1} &\sim W(\nu, S^{-1})
\end{align*}
\]
where $\alpha, V, \nu$ and $S$ are prior hyperparameters chosen by the researcher. A noninformative prior could be set as: $\nu = 0, S = V^{-1} = cI$ and let $c \to 0$

The posterior for $\alpha$ could be derived in analytical form as:

$$\alpha|\Sigma, y \sim N(\overline{\alpha}, \Sigma \otimes V)$$

where $\nu = (V^{-1} + X'X)^{-1}$ and $\overline{\alpha} = vec(A)$ with $A = V(V^{-1}A + X'X\hat{A})$

The posterior for $\Sigma$ is

$$\Sigma^{-1}|y \sim W(\nu, S^{-1})$$

where $\overline{\nu} = T + \nu$ and $\overline{S} = S + S'X'X\hat{A} + A'V^{-1}A - \hat{A}'(V^{-1} + X'X)A$

$A$ is a $K \times M$ matrix made by unstaking the $KM$ vector $\alpha$

Posterior inference about the VAR coefficients can be derived using the property that the marginal posterior for $\alpha$ is a multivariate t-distribution with the mean is $\overline{\alpha}$, the degree of freedom is $\overline{\nu}$ and the covariance matrix is

$$\text{var}(\alpha|y) = \frac{1}{\overline{\nu} - M - 1} \overline{S} \otimes \overline{V}$$

### 4.2.3 The Minnesota prior

The Minnesota priors deal with restrictions on the parameters $\alpha$. It is assumed $\Sigma$ is known and replacing by $\hat{\Sigma}$. Because $\Sigma$ is fixed, only the prior for $\alpha$ is needed and it is supposed:

$$\alpha \sim N(\underline{\alpha}_{Min}, V_{Min})$$

The traditional choices of $\underline{\alpha}_{Min}$ and $V_{Min}$ are:

$$\underline{\alpha}_{Min} = 0_{KM}$$ except for the elements corresponding to the first own lag of the dependent variable in each equation. These elements are set to one.

The prior covariance matrix, $V_{Min}$, is assumed to be diagonal. If $V_i$ denotes the block of $V_{Min}$ associated with the $K$ coefficients in the equation $i$ and $V_{i,jj}$ be its diagonal elements, it is usually set:

$$V_{i,jj} = \begin{cases} \frac{\sigma_{ii}}{\hat{p}^2} & \text{for coefficients on own lag } i = j \\ \frac{\underline{\sigma}_{ii}}{\hat{p}^2 \sigma_{jj}} & \text{for coefficients on lags of variable } i \neq j \\ \underline{\sigma}_{ii} & \text{for coefficients on exogenous variables} \end{cases}$$
The structure of the variance means the coefficients will shrink to zero as the lag increases. Researchers may conduct experiments with different values of $a_1, a_2, a_3$. The Minnesota prior allows the posterior has a Normal distribution form. The posterior for the Minnesota prior could be derived as

$$
\alpha | y \sim N(\alpha_{\text{Min}}, V_{\text{Min}})
$$

where

$$
V_{\text{Min}} = \left[ V_{\text{Min}}^{-1} + \left( \hat{\Sigma}^{-1} \otimes (X'X) \right) \right]^{-1}
$$

$$
\alpha_{\text{Min}} = V_{\text{Min}} \left[ V_{\text{Min}}^{-1} \alpha_{\text{Min}} + \left( \hat{\Sigma}^{-1} \otimes X \right)^{-1} y \right]
$$

The Minnesota prior requires $\Sigma$ is an known parameter. It is only a Bayesian analysis for $\alpha$ but not for $\Sigma$. The advantage of the Minnesota prior is that it allows for plausible shrinkage coefficients to be implemented in the VAR model. This would bring a more accurate estimation for the model.

### 4.2.4 The independent Normal-Wishart prior

Although the natural conjugate prior allows for analytical forms of the posteriors, it assumes each equation in VAR to have the same explanatory variables and the prior covariance of the coefficients in any two equations to be proportional to one another. The natural conjugate prior assumes $\alpha$ and $\Sigma$ are not independent. The independent Normal-Wishart prior has the coefficients and the error variance being independent. This means the analytical forms of the posteriors are not available. It will require the Gibbs sampling to carry out Bayesian inference.

Relaxing the assumption of the same explanatory variables in every equation of VAR, we could rewrite VAR as

$$
y = Z \beta + \epsilon
$$

where $\epsilon \sim N(0, I \otimes \Sigma)$, $Z$ is a $T \times K$ matrix. A general prior for this model is the independent Normal-Wishart prior

$$
p(\beta, \Sigma^{-1}) = p(\beta)p(\Sigma^{-1})
$$

where

$$
\beta \sim N(\underline{\beta}, V_\beta)
$$

$$
\Sigma^{-1} \sim W(\underline{\Sigma}^{-1}, \nu)
$$
\( V_\beta \) is not restricted to the form \( \Sigma \otimes V \) of the natural conjugate prior. \( \hat{\beta} \) and \( V_\beta \) could be set as in the Minnesota prior. A noninformative prior for the independent Normal-Wishart prior could be set as: \( \nu = S = V^{-1}_\beta = 0 \)

The posterior \( p(\beta, \Sigma^{-1}|y) \propto p(y|\beta, \Sigma^{-1})p(\beta)p(\Sigma^{-1}) \) does not have a convenient form allowing for analytical results of the mean and variance. However, conditional posterior distributions \( p(\beta|y, \Sigma^{-1}) \) and \( p(\Sigma^{-1}|y, \beta) \) could be derived analytically:

\[
\beta|y, \Sigma^{-1} \sim N(\hat{\beta}, V_\beta)
\]

\[
\Sigma^{-1}|y, \beta \sim W(S^{-1}, \nu)
\]

where \( V_\beta = \left( V^{-1}_\beta + \sum_{t=1}^{T} Z_t^\prime \Sigma^{-1} Z_t \right)^{-1} \), \( \hat{\beta} = V_\beta \left( V^{-1}_\beta + \sum_{t=1}^{T} Z_t^\prime \Sigma^{-1} y_t \right) \), \( \nu = T + \nu \) and \( S = S + \sum_{t=1}^{T} (y_t - Z_t \beta)(y_t - Z_t \beta)^\prime \)

This allows for applying Gibbs sampling procedure to make inferences on posterior \( p(\beta, \Sigma^{-1}|y) \)

### 4.3 Sign-restrictions identification for VAR

Sign restrictions identification for a structural VAR model was suggested by Uhlig (2005). The VAR in reduced-form could be written:

\[
y_t = a_0 + \sum_{j=1}^{p} A_j y_{t-j} + \epsilon_t
\]

To identify the structural shocks, we need to find a matrix \( A_0 \) such that \( u_t = A_0 \epsilon_t \) and \( E(u_t u_t') = I \). This means \( E(A_0 \epsilon_t \epsilon_t' A_0') = I \) or \( A_0 E(\epsilon_t \epsilon_t') A_0' = A_0 \Sigma A_0' = I \). Cholesky decomposition gives \( A_0 = P^{-1} \) where \( P = Chol(\Sigma) \) is the cholesky decomposition of \( \Sigma \).

If we could find an orthogonal matrix \( H \) such that \( HH' = I \), denote \( Q = PH \) and choose \( A_0 = Q^{-1} \) then \( u_t = A_0 \epsilon_t \) will also satisfy the orthogonal condition of the shocks as:

\[
QQ' = PHP' = PIP' = PP' = \Sigma
\]

which implies:

\[
E(u_t u'_t) = Q^{-1} E(\epsilon_t \epsilon'_t) (Q^{-1})' = Q^{-1} \Sigma (Q^{-1})' = Q^{-1} QQ' (Q')^{-1} = I
\]

\( H \) is called 'given matrix'. There will be infinite number of given matrix \( H \). For the bivariate
VAR, we can choose
\[ H = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \]

We have
\[ HH' = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]
because \( \cos^2(\alpha) + \sin^2(\alpha) = 1 \)

In general \( H_{ij} \) is formed by taking \((n \times n)\) identity matrix and setting:
\[ H_{ii} = \cos(\alpha), \quad H_{ij} = -\sin(\alpha), \quad H_{ji} = \sin(\alpha), \quad H_{jj} = \cos(\alpha) \]

the superscripts indicate the rows and columns of matrix \( H_{ij} \). For example, with 3 variable VAR, we define
\[ H_G(\alpha) = H_{12}(\alpha_1)H_{13}(\alpha_2)H_{23}(\alpha_3) \]

\[ = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \]

\( \alpha = (\alpha_1, \alpha_2, \alpha_3); \ 0 \leq \alpha_k \leq \pi \)

Obviously \( H_G(\alpha)H_G'(\alpha) = I \). If we choose \( A_0 = [H_G(\alpha)]^{-1} \), then \( E(u_tu'_t) = I \) which implies \( u_t = A_0\epsilon_t \) are orthogonal shocks The moving-average presentation of a structural VAR is
\[ y_t = \theta_0u_t + \theta_1u_{t-1} + \theta_2u_{t-2} + \ldots + \theta_su_{t-s} + \ldots \]

where \( \theta_0 = A_0^{-1}, \ \theta_1 = B_1A_0^{-1}, \theta_2 = B_2A_0^{-1}, \ldots \)

\( B_1, B_2, \ldots \) are coefficient matrices in the moving average presentation of the reduced form VAR

The contemporaneous affects in the structural VAR will be decided by the sign of elements of matrix \( A_0^{-1} = PH_G(\alpha) \). For example, with a simple macro model we can expect the signs of matrix \( A_0^{-1} \) as
\[ y_t = \begin{bmatrix} + & - & - \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \]
\[ \pi_t = \begin{bmatrix} + & + & - \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \]
\[ i_t = \begin{bmatrix} + & + & + \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \]

where \( y_t \) is GDP, \( \pi_t \) is inflation and \( i_t \) is interest rate.

We can generate a large number of matrices \( A_0^{-1} \) and retain only matrices that have the
signs of their elements agree with the postulated signs. In this paper, we employ the mean-target (MT) method suggested by Fry and Pagan (2011) to decide the matrix $A_0$ among those that satisfy the signs restrictions. The purpose is to find the a single model whose impulses are close to the mean of all the impulses from the models that satisfy the signs restrictions:
- subtracting the mean of each impulse
- divided the subtracting mean impulses by their standard errors to get standardized impulses
- standardized impulses will be placed in a vector $\Theta^k$ for each value of $\alpha_k$
- choose $k$ that minimize $(\Theta^k)(\Theta^k)'$
- use this $k$ to decide the model

5 Empirical Results

We use only the fact that when there is an unexpected positive shock on unemployment rate, the output will decrease to decide for the sign restriction. The rest of signs are unrestricted as we not really sure about the react of debt and output to other variables shock. This will allow the data speaks on the interdependence between the variables. The sign of matrix $A_0^{-1}$ then as followning:

\[
\begin{pmatrix}
\text{debt}_t \\
\text{output}_t \\
\text{housing}_t \\
\text{rate}_t \\
\text{unempl}_t
\end{pmatrix}
= A_0^{-1}u_t =
\begin{pmatrix}
+ & \times & \times & \times \\
\times & + & \times & \times \\
\times & \times & + & \times \\
\times & \times & \times & +
\end{pmatrix}
\begin{pmatrix}
\text{debt}_t \\
\text{output}_t \\
\text{housing}_t \\
\text{rate}_t \\
\text{unempl}_t
\end{pmatrix}
\]

where $\times$ means unrestricted on sign. We generate 10 thousands given matrix $H$, among those, roughly 200 given matrices produce impulse responses that match with the sign restrictions of the matrix $A_0^{-1}$

5.1 Interest rate and unemployment shocks

The major finding of this paper is presented in Figure 3 and Figure 4. Contradiction to all previous studies on the household debt issues across a range of countries which find interest rate is the main factor that has a large negative impact on the household debt
level, the impulse response in Figure 4 from Bayesian structural VAR estimation shows that an appreciation in interest rate only reduces the household debt a small amount. Impulse responses from OLS estimations of VAR indicate a much larger impact of interest rate on household debt. A positive shock of less than 0.1 percent on interest rate results in decreasing the household debt about $5 billions in the same quarter. A shock that makes an appreciation of 0.05 percent on unemployment rate will lower the debt about $2.5 billions in the same quarter. The structural VAR seems to exaggerate the impact of interest rate on household debt level. We should have a good reason for this questionable outcome from the structural VAR model. Given that the estimated VAR model has 4 lags and 5 variables, the total number of coefficients to be estimated is 105. The total number of observations in the data is 93 observations. This mean the OLS estimates are highly inefficient and the accuracy of the estimations are questionable. This is the main problem of estimating VAR model using macro data. Macroeconomics data are time series and for a good data, we could expect around 100 observations or maximum at roughly 200 observations on macro indicators. If we keep the lags number small (1 or 2 lags) to reduce the number of coefficients then there would be autocorrelations in the error terms which makes a consequence of existence of endogenous problems in the AR equations. But if we increase lags number to clear the serial correlations in error terms of VAR, we would have a large number of coefficients to be estimated and the estimates may be not trustable. Because of this problem, most of reduced-form VAR estimations using OLS or Maximum Likelihood methods should be used with a great care for the structural shocks identification. Bayesian fit very well in the context of the reduced-form VAR estimation. Bayesian estimation will reduce the worry about inefficiency by allowing to include out of sample information on the coefficients. Please note in the case we don’t have any out of sample information about the VAR coefficients, we could use the diffuse prior which contain no prior information and Bayesian estimations in this case will be identical with OLS estimations

The results from Bayesian estimation with the Normal-Wishart prior shows that a 0.2 percent positive shock on interest rate would result in decreasing less than $2 billions in the first quarter and the shock quickly converge to zero after that. The impact of unemployment shock is even larger from Bayesian estimations. A less then 0.1 percent unexpected increasing in unemployment rate will reduce the household debt level roughly $5 billions in the first quarter.

As we discuss above, the Bayesian approach is a remedy for the inefficient OLS estimates. The results from Bayesian estimation are supposed to be more accurate in the sense it
contains both in and out of sample information. It surprisingly shows that the role of interest rate on the fluctuations of household debt level is not as significant as we believe from previous studies.

How do we explain for this phenomenon? We suppose that for most of Australian the decision to make a large spending such as buying a house does not depend very much on the level of interest rate but depends largely on the situation of their finance constraints which is directly affected by their job status. For young people, they would buy their first house as soon as they can land a stable job with affordable income. They seems don’t care very much about the fluctuate of the interest rate in making such kind of decision. The context might also be similar to other OECD countries.

Figure 3: OLS estimations - impulse responses of interest rate and unemployment shocks

Notes: Structural shocks identified by sign restrictions and mean-target
Figure 4: Bayesian estimations - impulse responses of interest rate and unemployment shocks

Notes: Independent Normal-Wishart prior, structural shocks identified by sign restrictions and mean-target

5.2 Output and housing price shocks

There are also significant differences between impulse responses from OLS estimations and responses from Bayesian estimations for random shocks in output and housing price. As shown in Figure 5 and Figure 6, a 0.5 percent increasing in output will results in $1 billions increasing in debt in the same quarter according to the structural VAR model but it will result in $2 billions increasing in household debt as from the Bayesian VAR estimation. It is very interesting to see the Bayesian model shows that impulse responses of housing price shocks are in reverse signs compare to the respective impulse responses from OLS structural VAR model.

In the Bayesian structural VAR, an increasing of 0.8 percent in housing price will bring the debt level down about $4 billions in the first quarter. With the OLS structural VAR, an appreciation of 0.7 percent in housing price will indicate an increase debt level roughly $1 billion in the first quarter. We would rely more on the Bayesian inferences as it is noticed that the OLS estimation suffers a serious inefficiency problem because the total number of
coefficients are larger than the number of observations. We could think about an plausible explanation for the Bayesian results. As mortgage debt is a major debt in most of household, an increasing of housing price will discourage people in making decisions to buy properties. They may defer their decision to buy houses until the housing market is stable. It shows in figure 1 that during the period 2008-2012 where housing price increase quickly in Australia, the rising speed of debt level per person is slowing down.

Figure 5: OLS estimation - impulse responses of GDP and housing price shocks

Notes: Structural shocks identified by sign restrictions and mean-target
Figure 6: Bayesian estimation - impulse responses of GDP and housing price shocks

Notes: Independent normal-wishart prior, structural shocks identified by sign restrictions and mean-target
6 Concluding remarks

In this paper, we argue that the impact of interest rate on household debt in Australia and some other countries is not as significant as we believe. The estimations of VAR models by OLS are inefficient because the size of the sample is not large enough. Bayesian VAR models could resolve the problem and bring more accurate estimates for impulse responses analysis. Results from our Bayesian analysis imply that the rising of Australian household debt for the last two decades is mainly responsible by the reducing unemployment rate at the same time. More investigations should be implemented using different sign restrictions patterns and different data sets.
References


APPENDIX

Structural VAR estimations:

Figure 7: OLS estimations - impulse responses of all the shocks that satisfy the sign restrictions

Notes: Structural shocks identified by sign restrictions and mean-target
Figure 8: OLS estimations - impulse responses of the shocks identified by mean-target

Notes: Structural shocks identified by sign restrictions and mean-target
Figure 9: OLS estimations - forecast error variance decompositions (FEVD)

Notes: FEVD identified by sign restrictions and mean-target
Bayesian VAR estimations:

**Minnesota prior**

Figure 10: Bayesian estimations - impulse responses of all the shocks that satisfy the sign restrictions

Notes: Minnesota prior, structural shocks identified by sign restrictions and mean-target
Figure 11: Bayesian estimations - impulse responses of the shocks identified by mean-target

Notes: Minnesota prior, structural shocks identified by sign restrictions and mean-target
Natural conjugate prior

Figure 12: Bayesian estimations - impulse responses of the shocks identified by mean-target

Notes: Natural conjugate prior, structural shocks identified by sign restrictions and mean-target
Independent Normal-Wishart prior

Figure 13: Bayesian estimations - impulse responses of the shocks identified by mean-target

Notes: Independent Normal-Wishart prior, structural shocks identified by sign restrictions and mean-target