Laboratory measurements of scattered electromagnetic radiation from two-dimensional metallic and dielectric rough surfaces

Z. Fried, G. Phillips, and J. Waldman

Department of Physics, University of Massachusetts at Lowell, Lowell, Massachusetts 01854

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Bistatic- and nonspecular-scattering cross-section measurements of CO₂ laser radiation from roughened metal and dielectric surfaces were made and compared with predictions given by the facet (tangent-plane approximation) model. The incident radiation was linearly polarized perpendicular to the incident plane. The scattered polarization state was analyzed along two directions, perpendicular (HH) and parallel (HV) to the scattering plane. A scattering apparatus was designed especially for this work. This apparatus allowed us to make polarization-dependent bistatic measurements over the entire hemisphere, which to the best of our knowledge has not been done before. The average slopes and radii of curvature of the roughened surfaces utilized in this study were determined with a mechanical profilometer from surface tracings. The $(\sigma_{\text{HH}})$ nulls predicted by the facet model have been verified at steep incident angles for both metallic and dielectric surfaces. Agreement is generally better for metals than for dielectrics. At shallower angles measurements diverge from theoretical predictions. Our data indicate that the departure from the predictions of the facet model is most likely associated with surface curvature. A number of calculations of polarization-dependent scattering amplitudes from metal and dielectric cylinders of radius $r$ as a function of $r/A$ have been performed. These calculations suggest that, even for surfaces with large radii, rapid amplitude and phase variations are responsible for the onset of depolarization at shallower incident and scattering angles, which leads to the disagreement with the tangent-plane model predictions. These calculations also clearly demonstrate why the facet model is a better approximation, in the region of validity, for metals than for dielectrics. The role of curvature in determining the operable regions of the tangent-plane model is further illuminated through a series of scattering measurements from metal wires with radii of curvature of the order of $A$. The experimental measurements are described in detail, and data for both roughened metals and dielectrics are presented for several scattering aspect and depression angles.

1. INTRODUCTION

Scattering of electromagnetic radiation by rough surfaces has been a subject of great interest for several decades. This interest derives from a need to study terrain characteristics of inaccessible sites and to detect and characterize small deviations from optically smooth surfaces. In either case, the goals are to relate electromagnetic-scattering data to the properties of the scattering surface. In principle, knowledge of the exact shape of the scattering surface and of the total electric field $E$ at the surface allows one to compute the electric field $E_s$ at the point of observation. The exact solution is given by Eq. (1), where $k$ is the magnitude of the wave vector of the incident radiation and $R$ is the distance between some point on the illuminated surface and the observation point:

$$E_s(P) = \frac{1}{4\pi} \int_S \left( E \frac{\partial\psi}{\partial n} - \psi \frac{\partial E}{\partial n} \right) dS,$$  

where

$$\psi = \exp(i|k| R).$$  

A cursory look at Eq. (1) immediately confronts one with the two fundamental obstacles to finding $E_s(P)$. One problem is that the exact shape of the surface is unknown. Second, given the detailed geometry of the surface, there still remains the problem of knowing the value of the total $E$ on the surface, as given by Eq. (3):

$$E = E_i + E_s.$$  

$E_i$ and $E_s$ represent the incident and the scattered electric fields, respectively, that are on the surface. To obtain the value of $E$, one must solve the boundary conditions for both $E$ and $H$ at the surface. Unfortunately, closed-form solutions for the latter problem exist only for plane surfaces. In the high-frequency or geometrical-optics (GO) limit, the ratio of scattered to incident electric field is the same as the corresponding quantities obtained from the Fresnel coefficients. Consequently, in the GO limit, this part of the problem is eliminated. The need to characterize the surface, however, remains. An exact mapping of the surface is an insurmountable task and may not be necessary. It is usually assumed that different microscopic surface shapes when they are illuminated over a sufficiently large area of the rough surface will yield similar scattering patterns. The statistical characterization of rough surfaces is motivated by the need to obtain closed-form expressions for $E_s(P)$ without detailed knowledge of the surface shape. A model in which one assumes the random distribution of hemispherical bosses, the tangent-plane model (TP), and the two-scale roughness model are commonly used. The TP approximation is the most straightforward approach. In the GO limit, all shapes can be handled by the TP approximation in which only the average surface slopes are of interest. To obtain a statistical
Another important observation of PLT is that in the TP and scattered planes were different. The copolarization of the conventionally defined field angles is presented in Appendix A. Data were obtained in the GO limit, provided that the correlation length $T$ is much larger than the wavelength and the average slopes are significantly less than one. Another important observation of PLT is that in the TP regime the polarization of the scattered wave is independent of the detailed statistical properties of the surface.

Previous measurements of rough-surface scattering were performed in the monostatic configuration or under conditions where the scattered radiation was in the plane of incidence. Such an arrangement precludes the experimental study of some interesting features of polarization dependence. These manifest themselves only in configurations in which the scattering plane is different from the incident plane. The apparatus utilized in this study permits the measurement of scattered radiation in both the polar and the azimuthal directions.

Unlike smooth surfaces, roughened surfaces scatter in all directions. This requires the detector to be movable over a hemisphere. Practical considerations, especially for a liquid-nitrogen-cooled IR detector, limit the motion of the detector to the horizontal plane in the laboratory. To ensure accessibility of arbitrary scattering angles, the polar angle $\theta$, and particularly $\phi$, the azimuthal angle, require that the target surface be free to rotate around two perpendicular axes. At the same time the polarization state of the transmitted radiation must be adjusted to maintain a well-defined polarization with respect to the surface normal. A description of the scattering system and the correspondence between laboratory angles, in which the mean surface normal changes direction, and the conventionally defined field angles is presented in Section 2.

An algorithm to compute laboratory angles for a specific set of field angles is presented in Appendix A. Data were collected in a bistatic configuration in which the incident and scattered planes were different. The copolarization and cross-polarization scattering cross-section measurements were compared with the theoretical predictions of PLT. Using a form of the scattering cross section given by Barrick [Eq. (4)], PLT describe the angular dependences of $\sigma_s$ as a function of polarization, surface roughness, and dielectric constant. The scattering cross section is given by:

$$\sigma_s = |\beta_{pq}|^2 JS.$$  \hspace{1cm} (4)\]

In Eq. (4), $S$ is the shadowing function and $J$ is the probability-density function for surface slopes. $J$ is proportional to $(T/\sigma)^2$ times an exponential function of $T/\sigma$, where $T$ is the average facet spacing and $\sigma$ is the average facet depth.

As described by Barrick, a specular point in the surface reflects as a tilted-plane tangent to the surface at that point. The $\beta_{pq}$ in Eq. (4) are given below in terms of the Fresnel coefficients for $s$ and $p$ waves and the spherical-scattering coordinates (see Fig. 1):

$$\beta_{VV} = \frac{a_2a_3 R(i) + \sin(\theta_s)\sin(\theta_i)\sin(\phi_i)R_l}{a_3a_4},$$  \hspace{1cm} (5)\]

$$\beta_{HV} = \frac{\sin \phi_i [-\sin \theta_2 a_3 R(i) + \sin \theta_3 a_3 R(i)]}{a_3a_4},$$  \hspace{1cm} (6)\]

$$\beta_{VH} = \frac{\sin \phi_i [\sin \theta_2 a_3 R(i) + \sin \theta_3 a_3 R(i)]}{a_3a_4},$$  \hspace{1cm} (7)\]

$$\beta_{HH} = \frac{-\sin \theta_3 \sin \theta_s \sin^2 \phi_i R(i) - a_2a_3 R(i) - a_2a_3 R(i)}{a_3a_4}.$$  \hspace{1cm} (8)\]

with

$$a_1 = 1 + \sin \theta_s \sin \theta_i \cos \phi_s - \cos \theta_i \cos \theta_s,$$  \hspace{1cm} (9)\]

$$a_2 = \cos \theta_i \sin \theta_s + \sin \theta_i \cos \theta_s \cos \phi_s,$$  \hspace{1cm} (10)\]

$$a_3 = \sin \theta_i \cos \theta_s + \cos \theta_i \sin \theta_s \cos \phi_s,$$  \hspace{1cm} (11)\]

$$a_4 = \cos \theta_i + \cos \theta_s.$$  \hspace{1cm} (12)\]

The angle $i$ is the angle of incidence with respect to the local normal of the facet. It is defined below in terms of the scattering angles of Fig. 1:

$$\cos i = \frac{1}{\sqrt{2}}(1 - \sin \theta_i \sin \theta_s \cos \phi_s + \cos \theta_i \cos \theta_s)^{1/2}.$$  \hspace{1cm} (13)\]

In Eq. (4), the $J$ term is proportional to the average number of facets having slopes that scatter into the observation direction. The shadowing function $S$ gives the fraction of the total number of specular points not shadowed. In the analysis of PLT, it is pointed out that neither the shadowing function nor the slope statistical $J$ term has an influence on the position of nulls in the scattering cross section. Thus the predicted angular position of nulls in $\sigma_s$ as given by these authors is based solely on the behavior of $|\beta_{pq}|^2$ in Eq. (4). Furthermore, their analysis shows that for a given incident signal polarization there exist nulls in the copolarization scattering cross section for various angles.

In this study the predicted polarization-dependent behavior of the scattering cross section as given above was observed for a subset of incident scattering angles. For steep incident and receive polar angles, the data given in
Table 1. Surface Slopes Depths and Radii of Curvature for EBT 4 + 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass bead (diameter)</td>
<td>50-500 μm</td>
</tr>
<tr>
<td>Average slope</td>
<td>0.081</td>
</tr>
<tr>
<td>Rms slope</td>
<td>0.118</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.086</td>
</tr>
<tr>
<td>Average facet depth</td>
<td>2.88 μm</td>
</tr>
<tr>
<td>% of surface with slope</td>
<td></td>
</tr>
<tr>
<td>≤0.05</td>
<td>92.8</td>
</tr>
<tr>
<td>≤0.10</td>
<td>79.5</td>
</tr>
<tr>
<td>≤0.20</td>
<td>95.4</td>
</tr>
<tr>
<td>≤0.30</td>
<td>98.7</td>
</tr>
<tr>
<td>≤0.40</td>
<td>99.2</td>
</tr>
<tr>
<td>≤0.50</td>
<td>99.9</td>
</tr>
<tr>
<td>% of surface with radius</td>
<td></td>
</tr>
<tr>
<td>of curvature (in wavelengths)</td>
<td></td>
</tr>
<tr>
<td>≤3</td>
<td>3.6 μm</td>
</tr>
<tr>
<td>≤4</td>
<td>5.5 μm</td>
</tr>
<tr>
<td>≤5</td>
<td>7.8 μm</td>
</tr>
<tr>
<td>≤6</td>
<td>10.5 μm</td>
</tr>
</tbody>
</table>

this paper conform to predictions of the TP approximation. Deficiencies in the TP model appear for larger angles. The inability of the TP approximation to describe accurately the scattering of radiation for all angles $i$ has been noted in other, monostatic, measurements. The departure from the TP model has been attributed to the neglect of small-scale roughness of the surface and to neglect of multiple scattering from facets with large slopes. A convincing argument can be made that curvature inherent in rough surfaces is a major cause of the observed deviation from the facet model.

A good way to test the adequacy of the TP approximation is to juxtapose the TP predictions with exact calculations. For example, we can compare the reflection coefficients computed for the case of radiation incident upon a cylinder with the results obtained if we approximate the cylinder by a set of tangent planes. It is possible to describe analytically the polarization-dependent scattering amplitude and phase for perfect curved geometries, such as cylinders, in terms of the parameter $r/A$. The onset of depolarization for certain scattering geometries and the departure of data from the TP model are shown to be critically dependent on this parameter. A series of calculations of scattering from metal and dielectric cylinders is given in Section 4. These calculations disclose the rapid phase and amplitude variation of the $p$ and $s$ components of the scattered field as a function of the incident angle and the surface curvature. A description of these calculations and their significance in our understanding scattering from rough surfaces is also contained in Section 4.

A degree of insight into polarization-dependent scattering can be attained from these calculations by characterizing the roughened surfaces according to slope and radius of curvature.

Metal and dielectric rough surfaces utilized in this study were prepared in the following manner. The surfaces were blasted with glass beads under a pressure of approximately 4000 Torr. The bead diameters ranged from 50 to 500 μm. Profilometer measurements were performed on both metal and dielectric rough surfaces. A digitization technique in which the plotted profilometer data were converted into computer-readable data files was employed, and programming was developed to calculate the average surface slope and radius of curvature. Utilizing these techniques, we determined the average values of these surface parameters for the entire surface as well as that percentage of the surface falling within a certain range of the parameters. In Table 1, which displays results of such measurements for the surface utilized in this study, the radii of curvature are given in units of the wavelength.

Finally, results from randomly arranged metal wires with $r \sim \lambda$ are reported, yielding further evidence that the departure from the facet-model predictions is due to surface curvature.

2. EXPERIMENTAL SETUP AND DISCUSSION OF LABORATORY ANGLES

The source of radiation is a CO$_2$ laser (Ultra Lasertech Model 5122) providing 10 W of linearly polarized radiation with a choice of 48 lines between 9.2 and 10.8 μm (see Fig. 2). The ability to rotate the linearly polarized radiation to any desired angle is provided by a K rotator. The rough surface is allowed to rotate through two degrees of freedom, which, in conjunction with the position of the detector, simulates the environment of a bistatic radar in the field. At the receive end, discrimination between polarization states is accomplished by rotating a Brewster plate analyzer and wire grid polarizer analyzer. The combination of the two yields an extinction ratio of better than 1000:1. The radiation is collected by a 2-in. (5.08-cm), diameter ZnSe lens with a focal length of 20 in. (50.8 cm). A liquid-nitrogen-cooled HgCdTe detector (detectivity 10$^{10}$ cm Hz$^{1/2}$/W), mounted upon a rotatable detector arm in the focal plane of the lens, scans the resultant intensity pattern. Scanning over a finite angular spread and averaging the data is necessitated by the scintillation pattern that results from the narrow-band laser radiation scattering from the rough surface. The detector output is fed to a lock-in amplifier and the angle-averaged signal displayed on a Mac II computer.

Fig. 2. Component diagram of scatterometer system.

The rough surface is mounted upon a goniometer, which can rotate through a range of 90°. This rotation is about an axis that is parallel to a horizontal surface. The goniometer is mounted on a turntable that rotates through a vertical axis. By means of these two components, the target surface can be rotated around two axes. The detector has a range of 270° and is rotated independently of the target surface. The setup can simulate the three inde-
where \( \mathbf{r}_i \) is a unit vector along the direction of propagation and \( \mathbf{n} \) is a unit vector along the surface normal. Similarly,

\[
\mathbf{r}_i \cdot \mathbf{n} = \cos(\theta_i),
\]

(15)

where \( \mathbf{r}_i \) is a unit vector along the detector direction and

\[
\frac{\mathbf{r}_i - \mathbf{n}(\mathbf{r}_i \cdot \mathbf{n})}{\sin(\theta_i)} \cdot \frac{\mathbf{r}_i - \mathbf{n}(\mathbf{r}_i \cdot \mathbf{n})}{\sin(\theta_i)} = \cos(\phi_i).
\]

(16)

In the laboratory frame (Fig. 3), the propagation direction is fixed and is taken along the negative z axis. Thus

\[
\mathbf{r}_i(L) = -\mathbf{i}_3,
\]

(17)

where \( L \) designates the laboratory frame and \( \mathbf{i}_3 \) is a unit vector along the z axis (Fig. 3). The detector arm is in the \( \mathbf{xz} \) plane and can be rotated through 270° (see Fig. 4). Hence

\[
\mathbf{r}_D(L) = \cos(\psi)\mathbf{i}_1 + \sin(\psi)\mathbf{i}_3.
\]

(18)

The target surface normal can be rotated around two perpendicular axes. One rotation is around the y axis (Fig. 4), the axis perpendicular to the \( \mathbf{xz} \) plane that contains the transmitter and receiver. The angle describing this rotation is designated \( \xi \). The range of \( \xi \) is 0–90°. The detector arm requires a 180° range for any fixed \( \xi \). Hence the full range of the detector has to be 270°. Figure 5 shows the case for \( \xi \neq 0 \). The relation between \( \delta_s \) and \( -\phi \) is equal to the incident-field depression angle \( \lambda \) when the target is rotated around the y axis only.

A second rotation can be performed around the line that is the intersection of the target plane and the \( \mathbf{xz} \) plane (see Fig. 3). The first rotation fixes \( \xi \), and the second rotation fixes \( \psi \). When the target surface is rotated through \( \xi \) and \( \psi \) in succession, the normal to the target surface, expressed in the unit vectors of the laboratory coordinate system, is

\[
\mathbf{n'} = -\mathbf{i}_1 \sin(\xi) \cos(\psi) + \mathbf{i}_2 \sin(\psi) + \mathbf{i}_3 \cos(\xi) \cos(\psi).
\]

(19)

The transformation of the field angles into the corresponding laboratory angles will now be presented.

In the field frame, the target-surface orientation is fixed, and the transmit and the receive directions (vectors) can vary independently over a hemisphere (see Fig. 2). In this frame, we designate the following angles: \( \theta_i, \theta_s, \) and \( \phi_s \). \( \theta_i \) stands for the incident angle, the angle between the incident propagation direction and the target normal.
While \( \xi \) is now not equal to the incident depression angle, we will, for convenience, refer to it as the laboratory depression angle and, similarly, refer to \( \psi \) as the laboratory aspect angle.

We note that the scalar products [see Eqs. (14)–(16)] defining the various field angles retain the same form when \( n^* \) is substituted for \( n \). The following expressions relate laboratory angles (\( \xi, \psi, \eta \)) to field angles (\( \theta_1, \theta_2, \phi \)):

\[
\begin{align*}
\cos(\theta_1) &= -r_1 \cdot n^* = \cos(\xi)\cos(\psi), \\
\cos(\theta_2) &= r_2 \cdot n^* = \sin(\eta - \xi)\cos(\psi), \\
\cos(\phi) &= \frac{r_1 \cdot r_2 - (r_1 \cdot n^*)(r_2 \cdot n^*)}{\sqrt{1 - (r_1 \cdot n^*)^2}[1 - (r_2 \cdot n^*)^2]^{1/2}}.
\end{align*}
\]

Substituting Eqs. (17), (18), (20), and (21) into Eq. (22) yields

\[
\cos(\phi) = \frac{-\sin(\eta) + \cos^2(\psi)\sin(\xi - \eta)\cos(\xi)}{[1 - \cos^2(\xi)\cos^2(\psi)]^{1/2}[1 - \cos^2(\psi)\sin^2(\xi - \eta)]^{1/2}}.
\]

An algorithm for computing laboratory angles \( \xi, \eta, \) and \( \psi \) for a given set of field angles \( \theta_1, \theta_2, \) and \( \phi \), can be found in Appendix A.

3. SCATTERING DATA

Scattering data were obtained from roughened aluminum surfaces, roughened dielectric surfaces, randomly arranged wires, and a randomly distributed set of glass beads. A sandblaster was used to roughen aluminum surfaces. The abrasive material used to prepare the surfaces was a collection of glass beads ranging from 50 to 500 \( \mu \)m in diameter. The results of profilometer tracings indicate that the roughening produces randomly distributed valleys ranging from 3 to 9 \( \mu \)m with an average spacing of 70–100 \( \mu \)m. Examination of the surfaces under a microscope indicated the existence of sharp-edged patches. To remove these sharp edges, the surfaces were electropolished; approximately 1 \( \mu \)m of material was removed from the upper surface. Statistical data given in Table 1 indicate radius of curvature, slope, and average facet depth for the surface (EBT 4 + 13) studied. The histogram characterizing this surface according to distribution of facet slopes is plotted in Fig. 6.

Data are presented for both copolarization, \( \sigma_{\phi}(HH) \), and cross-polarization, \( \sigma_{\phi}(HV) \), measurements as well as for ratios of \( \sigma_{\phi}(HH)/\sigma_{\phi}(HV) \). The results are displayed in both laboratory and field coordinates. In the laboratory coordinates, the transmit and receive laboratory depression angles, \( \xi \) and \( \xi' \), are set equal and remain fixed. The target surface is rotated around a horizontal axis. This rotation defines the laboratory aspect angle. Changing the laboratory aspect angle induces changes in both \( \theta_1 \) and \( \theta_2 \), the incident and scattered polar angles. However, there is an intrinsic advantage to performing measurements in the laboratory coordinate system since the laboratory aspect angle is identical to the angle subtended by the global normal and the facet normal (recall that, according to the TP model, the scattering at a given angle is due to facets for which the specular condition is fulfilled). Figures 7 and 8 display ratios of both \( \sigma_{\phi}(HH)/\sigma_{\phi}(HV) \) and \( \sigma_{\phi}(HH) \) and \( \sigma_{\phi}(HV) \) in laboratory coordinates. The data points plotted for \( \sigma_{\phi}(HH) \) and \( \sigma_{\phi}(HV) \) have been ratioed in such a way that the data at some small-aspect angle coincides with the model prediction. The average slope parameter, \( T/\lambda \), that enters into the TP model is obtained from the results of profilometer measurements (Table 1). Figures 9 and 10 show data for \( \sigma_{\phi}(HH)/\sigma_{\phi}(HV) \) and for \( \sigma_{\phi}(HH) \) and \( \sigma_{\phi}(HV) \) for configurations in which the field angles \( \theta_1 \) and \( \theta_2 \) and the azimuthal scattering angle \( \phi \) are varied.

Figures 11 and 12 display scattering data for roughened plastic surfaces with \( n = 1.6 \) and \( k = 0.002 \).

Roughening of the plastic surfaces was accomplished in the following manner. The plastic surfaces were softened in acetone and compressed with C clamps between roughened aluminum surfaces. After hardening, the roughened plastic surfaces were measured by a mechanical profilometer and found to have surface depths and slopes that were similar to the metal surfaces used in their preparation.

Figure 13 displays scattering data from randomly distributed glass beads. Figure 14 displays scattering data from a glass surface roughened with glass beads. Figures 15 and 16 show scattering data from aluminum wires randomly distributed on an absorbing flat surface.

To obtain a better understanding of both the success and the failure of the tangent-plane approximation, we prepared a sample target surface consisting of randomly distributed aluminum wires. Wire from a spool was continuously wound two or three layers deep in a random fashion on a flat absorbing plate. The radius \( a \) of the wire was 12.5 \( \mu \)m, and the corresponding \( \lambda a \) was 0.85. For such a large value of \( \lambda a \), the tangent-plane approximation should not work at all. Interestingly, however, the data for the ratio \( \sigma_{\phi}(HH)/\sigma_{\phi}(HV) \) at a 60° laboratory depression angle are remarkably similar to those obtained for the randomly roughened surfaces. A plausible reason for this similarity is presented in Section 4.

4. DISCUSSION OF RESULTS

For the sake of clarity we divide our discussion into four parts. We start with scattering data from metals and di-
vide these into two groups: data in laboratory angles (Figs. 7 and 8) and data in field angles (Figs. 9 and 10).

As we noted previously, the facet model implies that the laboratory aspect angle is identical to the angle subtended by the facet normal with the surface (global) normal. The facets that radiate into the detector all have normals in the horizontal plane, the plane defined by the transmit and receive directions. Figures 7 and 8 show that agreement with the TP model is excellent up to a 15° aspect angle and that the onset of significant deviation from the TP model occurs at an aspect angle somewhere between 15° and 20°.

The exception to this statement is the deviation of HH/HV at small aspect angles. According to the TP model, $\sigma_\alpha(HV) \to 0$ as the aspect angle $\to 0$. Tentatively, this deviation can be attributed to a small amount (a few percent) of diffuse and depolarized scattering, leading to a nonzero $\sigma_\alpha(HV)$ at all angles including small aspect angles. A plausible explanation for the origin of this behavior is presented below.

Figures 9 and 10 exhibit data in field coordinates for fixed and identical transmit and receive polar angles as a function of the azimuthal angle. A zero azimuthal angle thus corresponds to the specular configuration. In the field representation, the deviation from the TP model predictions occurs at different azimuthal angles. Of course, the underlying source of the discrepancy between the TP
Fig. 9. Ratio of HH/HV for different field polar angles versus azimuthal angle.

Fig. 10. HH and HV intensities for different field polar angles versus azimuthal angle.

Fig. 11. $\sigma_{\phi}(\text{HH})/\sigma_{\phi}(\text{HV})$ for field polar angles $\theta_i = \theta_s = 7.5^\circ$.

Fig. 12. $\sigma_{\phi}(\text{HH})$ and $\sigma_{\phi}(\text{HV})$ for field polar angles $\theta_i = \theta_s = 7.5^\circ$. 
model and the data must be the same in both laboratory and field coordinate systems. By plotting the angle between the global and local facet normal as a function of the azimuthal angle, a set of curves is generated in which each curve corresponds to a given polar angle. Looking at the data, one can find the azimuthal angle at which there is a significant departure between the TP model and the experimental data. These points are plotted as X's in Fig. 17. The X's lie approximately on a straight line and show that in field coordinates the departure of the data from the predictions of the TP model occurs when the facet normal subtends an angle of approximately 20° from the global normal.

It has been noted before that there are limits to the validity of the TP approximation because of curvature. Brekhovskikh derived the following criterion for the validity of the TP approximation:

$$4\pi r_c \cos \theta > \lambda,$$

(24)

where $$r_c$$ is the radius of curvature and $$\theta$$ is the local angle of incidence. Unfortunately, the inequality is not specific enough to be of use in data analysis.

We present here a somewhat different argument. In a rough surface for which the average slope is small (of the order of 0.1 as is the case here), larger-than-average slopes can occur in several ways: the depth is larger than average and the spacing between neighboring high and low points has an average value, the depth is average and the spacing between high and low points is smaller than average, or, finally, both depths and spacings are significantly different from the average value. We consider the case indicated in Fig. 18, in which the depth is average and the spacing between high and low points is smaller than average.

Let the circle be centered at a point inside the surface that is at 3$$\lambda$$ beneath a high point on the surface such that

$$d = \sqrt{R^2 - (R - h)^2} = (2Rh - k^2)^{1/2}.$$

(25)
If we set $R = 3\lambda = 31.8 \mu m$ and $\bar{h} = 5.76 \mu m$, we obtain

$$d = 18.24 \mu m, \quad \theta = 17^\circ.$$  

Thus approximately half of the facets corresponding to slope angles larger than $17^\circ$ will have radii of curvature smaller than $3\lambda$, the onset of the resonance regime\textsuperscript{11} for curved surface (cylinder, sphere) scattering according to the Mie theory.\textsuperscript{11}

Surface characteristics, such as depth, facet spacing, and radius of curvature, were obtained by means of a mechanical profilometer, Mitutoyo Surftest 201, with a 5-\mu m-diameter diamond stylus. Some of the parameters, such as the diameter of the glass bead and the duration and pressure of the blast, were varied during the sandblasting operation. Five differently prepared surfaces were used in this study.\textsuperscript{12} After sandblasting, the surfaces were examined under a microscope and observed to display many sharp points and edges. Consequently the samples were electropolished to remove these sharp features, as has been the practice of other researchers in the field.\textsuperscript{13} The surface data given in Table 1 are tabulated according to bead size (BT#). The data displayed in Figs. 7 and 8 were obtained for sample EBT 4 + 13, an electropolished sample blasted with a mix of BT4 and BT13 beads. According to Table 1, the average facet depth for this surface was 2.88 \mu m. This average was measured from the mean surface level. The quantity $\bar{h}$ that enters into Eq. (25) is double the value of the mean depth found in Table 1. Table 1 also gives information about the radius of curvature of facets for all surfaces studied. Radius-of-curvature calculations were performed by fitting a parabola to groupings of data points from the digitized profilometer tracings (five data points each, spanning approximately 40 \mu m on the surface).

Some of the earlier measurements obtained in the course of this work were made with a rough surface, referred to as original EBT4, which has an average facet depth of 8.68 \mu m. The slope angle associated with this depth is approximately $22^\circ$. At larger angles, the facet model deviates from the measured values. Figure 19 is consistent with our prediction and displays data for $\sigma_0(HH)/\sigma_0(HV)$ at a $60^\circ$ laboratory depression angle. This plot shows that there is an excellent fit up to a laboratory aspect angle of $24^\circ$.

As a further check on our hypothesis that curvature effects reduce the effectiveness of the TP approximation, we prepared a surface consisting of randomly wound
aluminum wires upon an absorbing substrate. These measurements are displayed in Figs. 15 and 16. Among the interesting features of these data is the observation that at small aspect angles the fit to the TP model is good, and at large aspect angles (Fig. 16) the data follow the same pattern as the roughened surface at a 60° laboratory depression angle (Fig. 19).

Figures 11 and 12 show plots of measurements for plastic in comparison with the facet model predictions. The roughened plastic surface was prepared by pressing an acetone-treated, smooth plastic surface against a roughened aluminum surface. After hardening, the plastic surface had a roughness that was verified by profilometer to be similar to that of the rough metal. The complex index of refraction of this plastic at 10.6 μm was \( n = 1.6 \) and \( k = 0.002 \). The penetration depth is given by \( \alpha^{-1} = \lambda / 4\pi k = 422 \mu m \). This is too large given that the theoretical calculations assume that all scattering takes place on the surface without transmission through the facet. The scattering in the data points shown in Figs. 11 and 12 may be due to the transmission associated with small \( k \).

To remedy this situation, we performed additional measurements on rough glass surfaces. Glass has a complex index of refraction of \( n = 2.2 \) and \( k = 0.1 \) at 10.6 μm. The penetration depth of glass at 10.6 μm is 8.4 μm, sufficiently small to justify the neglect of transmission through facets. Two types of rough glass surface were prepared. Figure 13 displays scattering data on a glass substrate from randomly stacked glass beads of varying radii. Here the deviation from the facet model is at small laboratory aspect angles. The other set of glass-scattering data is displayed in Fig. 14. This target surface was prepared by abrating a flat glass surface with glass beads under pressure. Rough glass surfaces prepared in this manner displayed many deep fissures when observed under a microscope.

This surface did not quite satisfy the surface criteria necessary to test the theory. Nevertheless, at steep angles (Fig. 14), the data seemed to validate the facet-model prediction up to a laboratory aspect angle of 15°. It is instructive to compare Fig. 14 with Fig. 7 (10/10 data). Both appear to have the same range of validity. Both display an HH null with a depth of 0.1. When comparing Figs. 13 and 14 with Figs. 7 and 8, one observes that at larger laboratory depression angles the deviation from theory is more pronounced for dielectrics than for metals.

To obtain a better understanding of the limitations of the TP approximation, we present some numerical results of polarization-state-dependent scattering amplitudes for both metal and dielectric cylinders of different radii. The underlying rationale for doing this is to juxtapose the TP approximation with exact calculations for curved surfaces where solutions to Maxwell's equations with associated boundary conditions can be obtained numerically. If the tangent-plane approximation were to be valid independently of the surface curvature, one should be able to approximate the scattering amplitude from any cylinder of arbitrary radius by the Fresnel coefficients of the tangent plane. Thus in the case of a metal cylinder one would conclude that the scattering intensity for a cylinder is independent of the azimuthal angle \( \phi \) and of the orientation of the incident linear polarization. Such, however, is not the case. An understanding of the structure of the polarization-dependent scattering amplitudes as a function of \( \alpha / \lambda \) is one way to put the tangent-plane approximation in its proper perspective.

The validity of the tangent-plane approximation will depend on how close the ratio of \( s / p \) for a cylinder divided by \( s / p \) for a plane is to unity. An additional requirement is that the phase differences for \( s \) and \( p \) amplitudes for planes and cylinders remain the same. It is of interest to note that the \( s-p \) phase differences for cylinders and planes are the same except in the immediate vicinity of the Brewster angle.

Figure 20 displays the amplitude ratios for metal cylinders of increasing radii. The radius ranges from 3.18A to 9.54A. The plots confirm qualitatively the inequality of Eq. (A1) below, i.e., the larger the radius of curvature, the better the TP approximation. One observes that for large angles of incidence the TP approximation fails even for a radius of curvature of the order of 10A. On the other hand, for steep angles of incidence, the TP approximation is effective even for small radii of curvature. This is consistent with our measurements on roughened aluminum surfaces, dielectric surfaces, and metal-wire surfaces at steep angles of incidence.

Similar results are displayed in Fig. 21 for dielectric cylinders with \( n = 1.6 \) and \( k = 0.1 \). These differ from the cylinder plots for metals and indicate that the TP approximation for dielectrics fails at smaller angles than it does.
for metals with the same radius. One notes that the inequality derived in Ref. 10 does not distinguish between dielectrics and metals.

Reviewing our data in light of these model calculations, we reach the following conclusion. The bulk of the scattered radiation (~95%) is accounted for by the facet model. There is, however, a small but finite component of diffuse radiation with a polarization that varies rapidly as a function of angle. It is the latter that gives rise to deviations from the predictions of the TP approximation. These deviations are especially pronounced in polarization states and at angles where the TP model predicts no scattering or small amounts of scattering. Thus, at an azimuthal angle of 0°, where the TP model predicts \( \sigma_0(HV) = 0 \), a small but finite \( \sigma_0(HV) \) appears consistently. The other area of pronounced deviation from the TP model is at scattering angles that are significantly different from the specular direction. Here, the intensity of the scattered radiation should fall off exponentially. This, however, is not the case, owing to the underlying diffuse background. The most likely cause for this is the existence of areas on the rough surface with small radii of curvature. This hypothesis is consistent with the differences observed between metals and dielectrics and with the scattering observed from randomly distributed wires.

Other investigators have attributed depolarization effects to multiple scattering. Another plausible source of depolarization may be found in two-scale roughness. These issues will be addressed in a subsequent publication.

APPENDIX A: ALGORITHM FOR TRANSFORMING FIELD ANGLES INTO LABORATORY ANGLES

Having defined laboratory angles \( \xi, \eta, \) and \( \psi \) in Section 2, we proceed with further analysis to obtain an algorithm.
that will yield a unique set of $\xi$, $\eta$, $\psi$ for a given set of $\theta$, $\theta_b$, and $\phi_r$.

First we show that for a fixed $\theta$ and $\theta_b$, both larger than 0° (0° is a point of degeneracy), $\cos(\phi_m)$ can assume all values between -1 and +1. We denote $\cos(0\psi) = x$ and $\cos(\theta_b) = y$; Eq. (23) can be written as

$$\cos(\phi_m) = \frac{-\sin(\eta) + yx}{(1 - x^2)^{1/2}(1 - y^2)^{1/2}} \quad (A1)$$

For a fixed $\psi$, $\xi$, and $\eta$, $\eta$ is not an independent variable. We therefore express $\sin(\eta)$ in terms of $x$, $y$, and $\phi_m$ as

$$\sin(\eta) = \sin(\eta - \xi + \xi) = \sin(\eta - \xi) \cos(\xi) + \cos(\eta - \xi) \sin(\xi) \quad (A2)$$

or

$$\sin(\eta) = \frac{x + [\cos^2(\psi) - y^2] \cos^2(\phi_m) - x^2]}{\cos^2(\phi_m)} \quad (A3)$$

A minus must be inserted before the square root when $(\eta - \xi) > 90°$. Combining Eqs. (23) and (A1)–(A3), one writes

$$\cos(\phi_m) = \frac{-xy \tan^2 \psi \pm [1 - (y^2 - \cos^2 \psi)]]^{1/2}[1 - (x^2 - \cos^2 \psi)]^{1/2}}{(1 - y^2)^{1/2}(1 - x^2)^{1/2}} \quad (A4)$$

First we show that $\cos(\phi_m)$ can assume the extreme values of ±1. This can be seen by setting $\psi = 0$. The plus in front of the square root yields +1, and the minus yields -1. It is useful to rewrite Eq. (14) as two equations, each valid in a given range of $\cos(\phi_m)$. Assuming that $y > x$, then

$$\cos(\phi_m) = \frac{-xy \tan^2 \psi + [1 - (y^2 - \cos^2 \psi)]]^{1/2}[1 - (x^2 - \cos^2 \psi)]^{1/2}}{(1 - y^2)^{1/2}(1 - x^2)^{1/2}} \quad (A5)$$

Equation (A5) is valid for

$$-\frac{x}{y} \left(1 - \frac{y^2}{x^2}\right)^{1/2} \leq \cos(\phi_m) \leq 1,$$

$$\cos(\phi_m) = \frac{-xy \tan^2 \psi - [1 - (y^2 - \cos^2 \psi)]]^{1/2}[1 - (x^2 - \cos^2 \psi)]^{1/2}}{(1 - y^2)^{1/2}(1 - x^2)^{1/2}} \quad (A6)$$

Equation (A6) is valid for

$$-1 \leq \cos(\phi_m) \leq -\frac{x}{y} \left(1 - \frac{y^2}{x^2}\right)^{1/2}.$$

12. Five surfaces were prepared and analyzed in this study; to keep the paper within bounds, only statistics for the surface EBT 4 + 13 are reported.
Persistent Spectral Hole Burning

FEATURE EDITORS

W. E. Moerner
IBM Research Division
Almaden Research Center
K95/801, 650 Harry Road
San Jose, California 95120-6099

Gerald J. Small
Department of Chemistry
Ames Laboratory
U.S. Department of Energy
Iowa State University
Ames, Iowa 50011