Cutoff Frequency

The cutoff frequency is defined as the frequency at which the ratio of the $\left(\frac{\text{output}}{\text{input}}\right)$ has a magnitude of 0.707. This magnitude, when converted to decibels using Eq. 1 is equal to –3dB, often referred to as the 3dB down point.

$$\text{Magnitude} = 20\log_{10}\left(\frac{\text{output}}{\text{input}}\right) \tag{1}$$

The cutoff frequency is characteristic of filtering devices, such as RC circuits. At this point, the amount of attenuation due to the filter begins increase rapidly. An example of this is shown in Fig. 1 below. This RC circuit has a time constant of 0.01 sec.

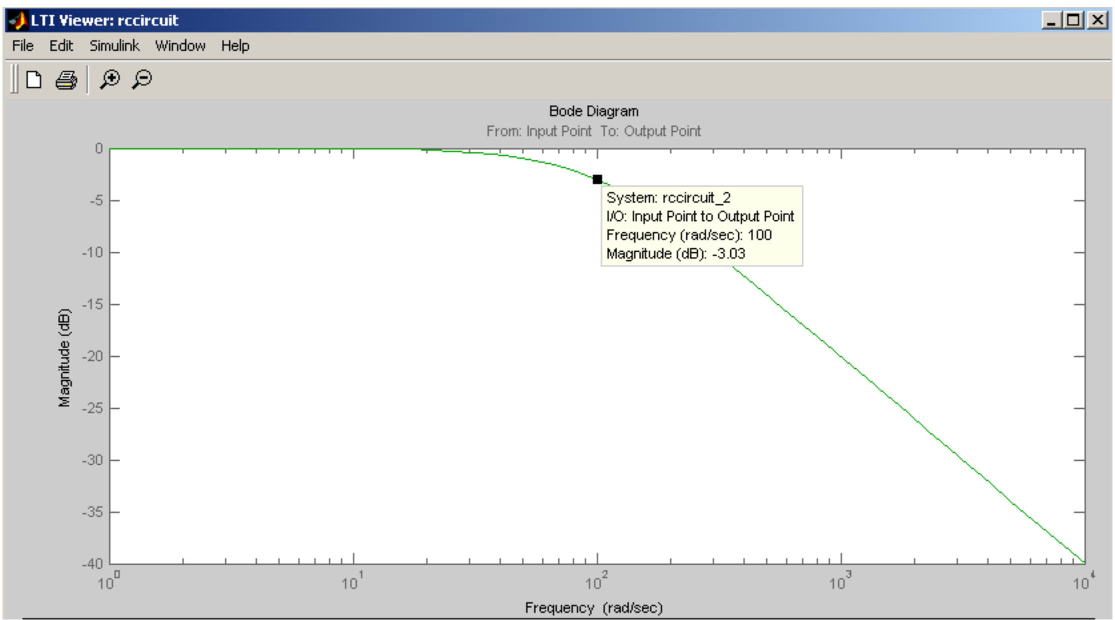

Fig. 1. Bode Diagram with Cutoff Frequency

The magnitude low frequency signals are relatively unaffected before the cutoff frequency. After the cutoff frequency, however, we see much more attenuation. Equation 2 governs the characteristics shown in Fig. 1.

$$\tau\dot{x} + x = f(t) \tag{2}$$

Where: τ =time constant

The Transfer Function (T.F.) of 2 is

$$\text{T.F.} \Rightarrow \frac{X(s)}{F(s)} = \frac{1}{\tau s + 1} \tag{3}$$

To represent 2 as a Frequency Response Function (FRF), ' $j\omega$ ' is substituted for 's' where ' ω ' is frequency in rad/sec. The T.F. then becomes

$$\frac{X(j\omega)}{F(j\omega)} = \frac{1}{\tau(j\omega)+1} \tag{4}$$

We will now work with the right side of the equation. Multiply by the complex conjugate to get the complex numbers in the numerator. The result is

$$= \frac{1}{\omega^2\tau^2+1} - \frac{\omega\tau}{\omega^2\tau^2+1} j \tag{5}$$

The magnitude of a number with real and imaginary parts is found using 6

$$\text{Magnitude} = \sqrt{(\text{Re})^2 + (\text{Im})^2} \tag{6}$$

From 6, we can expand to get

$$\sqrt{\frac{1}{(\omega^2\tau^2+1)^2} + \frac{\omega^2\tau^2}{(\omega^2\tau^2+1)^2}} \tag{7}$$

which simplifies to

$$\text{Magnitude} = \sqrt{\frac{1}{\omega^2\tau^2+1}} \tag{8}$$

Now, since we know that the cutoff frequency, ω_c, occurs at Magnitude=0.707, this can be substituted into 8 to get

$$0.707 = \sqrt{\frac{1}{\omega_c^{\,2}\tau^2+1}} \tag{9}$$

Solving for ω_c,

$$\omega_c = \frac{1}{\tau} \tag{10}$$

What significance does this have? Previously, it was stated that the time constant, τ, was 0.01 sec. This would make $\frac{1}{\tau}$ equal to 100. Let us glance back at Fig. 1 to see the cutoff frequency of the RC circuit in the Bode Diagram… 100 rad/sec! Knowing this makes it very simple to design RC circuits as filters.