MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

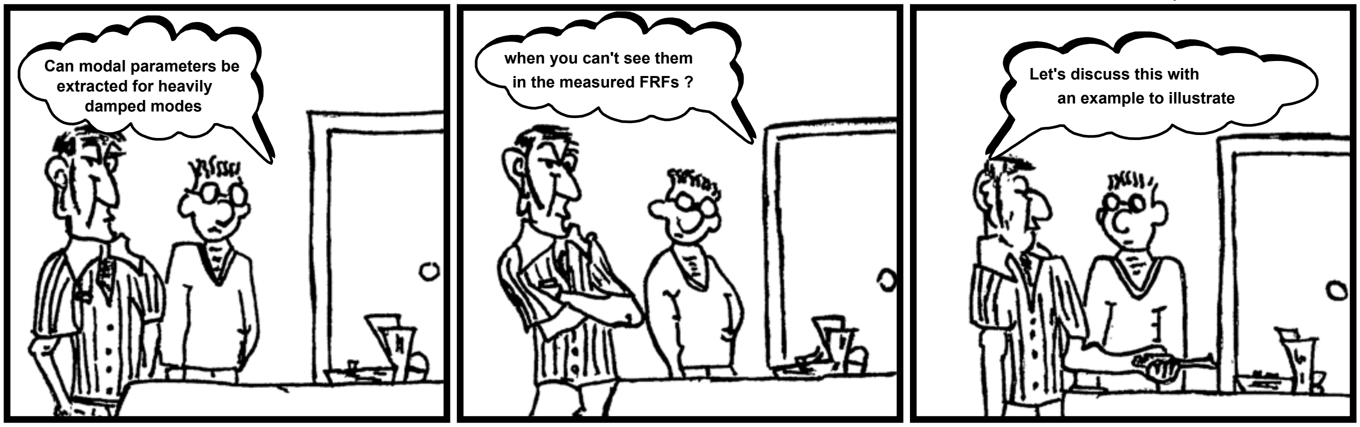

Illustration by Mike Avitabile

Can modal parameters be extracted for heavily damped modes when you can't see them in the measured FRFs? Let's discuss this with an example to illustrate.

Now this is a question that I have heard many times over the years. The answer is bittersweet in many respects. Of course you can extract heavily damped modes from FRFs! But you need to know there is a root in the FRF and make sure that you make a good measurement so that the root can be extracted. Let's elaborate on this with an example to help show that the parameter estimation algorithms are very robust and well-suited to extract heavily damped roots.

To illustrate this point, I am going to refer to a simple model that we have used for many years. It is a very simple 2 DOF mass-spring-dashpot system that has non-proportional damping. The equation of motion and mass, damping and stiffness are defined as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\}$$

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; C = \begin{bmatrix} 0 & 0 \\ 0 & 646.225 \end{bmatrix}; K = \begin{bmatrix} 428400 & -132900 \\ -132900 & 532800 \end{bmatrix}$$

These matrices can be used to extract the complex solution (frequency, damping and mode shapes). In addition, the frequency response functions can be synthesized to simulate a set of collected data. In this case, only one reference, for the first DOF will be used to generate a row of the frequency response matrix as shown in Figure 1. Now it is very clear that only one mode is observed in the peak amplitude of the magnitude of the frequency response function. If we only looked at the magnitude of the function then it would appear that there is only one mode in the system. But if we also looked at the phase, then there is an indication that maybe there is something other than a single mode in the band of interest. (In fact for this case, there are definitely two modes in this band.)

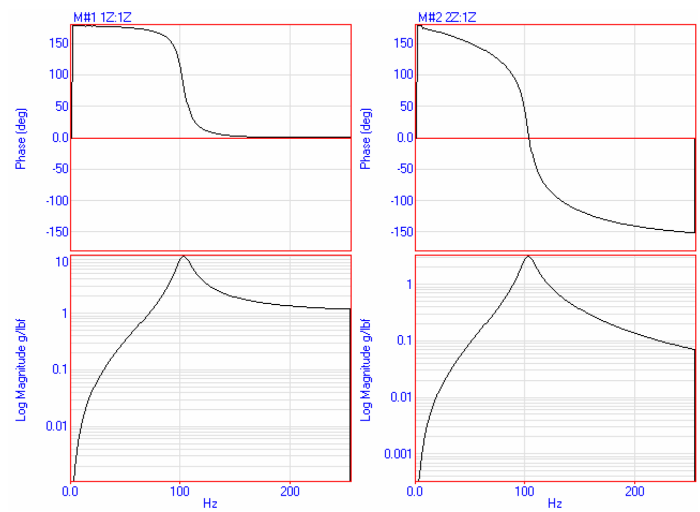

Figure 1 – H_{11} and H_{12} Frequency Response Functions

But the big question is "Can the modal parameter estimation algorithms extract reasonable (accurate) values for the residues of the system?". As a user, the proper order model must be identified for the extraction of residues. If only one mode is requested, then obviously only one mode will be estimated – and it might provide marginal estimates for one of the roots. And if the model is overspecified with too many modes, then the results may be equally distorted - possibly there will be a reasonable estimate for one or two of the modes but they are also likely to be poorly estimated.

Now if the proper order model is specified, will the correct modal parameters be estimated? Using a 2 DOF model fit with an orthogonal polynomial estimation algorithm (popular in many commercially available software packages), the poles extracted are reported in Table 1.

Table 1 – Poles Extracted from Orthogonal Polynomial

Mode	Frequency (Hz)	Damping (%)
1	103	5.31
2	103	40.7

These are actually the poles that would be obtained from a complex eigensolution for the non-proportioanl system matrices presented above. From this table, you can see that there are two roots at the same frequency with one at 5% of critical damping and another at 40% of critical damping which is very heavy damping. Now proceeding on, the residues can be extracted in a similar fashion. For H11 and H12, the residues are shown in Table 2 and 3, respectively. These residues match well with the analytical residues that were determined from the analytical model used to generate the frequency response functions.

Table 2 – H_{11} Residues from Orthogonal Polynomial

Mode	Frequency (Hz)	Damping (%)	Res Mag (g/lbf-sec)	Res Phs (deg)
1	103	5.31	745	183
2	103	40.7	108	21.8

Table 3 – H_{12} Residues from Orthogonal Polynomial

Mode	Frequency (Hz)	Damping (%)	Res Mag (g/lbf-sec)	Res Phs (deg)
1	103	5.31	257	107
2	103	40.7	309	306

Now from this simple example, it is clear that the modes can be extracted from a frequency response function and it is independent on the damping of the system - whether it be lightly damped or heavily damped and whether it be proportional or non-proportionally damped system.

Let's be very clear here… The modal parameter estimation algorithms are very capable curvefitters that are commonly used today in almost all commercial software packages. The problem does not lie with the curvefitter as much as it lies with the measurement and the engineer using the software.

Obviously, the engineer needs to have some indication that there are a certain number of roots in a given frequency band – either from the mode indicator tools or from apriori knowledge that there are a certain number of roots in a given band. While the curvefitters are generally robust, what I find many times is that the mode indicator tools, at times, do not provide a clear, concise indication of the number of roots in a band. And even more often that that, there is generally a poor set of measurements that have been collected that are not considered adequate to extract modal parameters. The problem most times lies with the actual measurements. Generally, they are not of sufficient quality to extract accurate modal parameters. That is the plain and simple fact in most cases!

Another case is a structure with relatively heavy damping and pseudo-repeated roots. The mode indicator tools in Figure 2 clearly help to identify the number of modes present for the roots extracted and shown in Table 4. This case is included here to show that all the tools must be used together to assist in the modal parameter estimation process. These measurements are reasonably good which leads to success in the extraction of modal parameters.

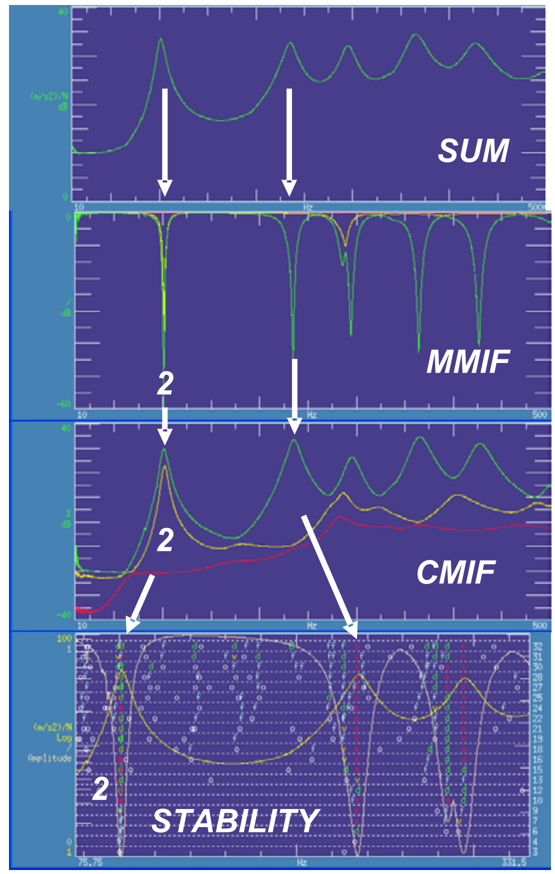

Figure 2 – SUM, MMIF, CMIF, Stability

Table 4 – Time Domain Polyreference Extraction

No.	Frequency	Damping	Stab.	DOFs	Stored
1	101.08 Hz	3.95 %	s	0	no
2	101.62 Hz	3.29 %	s	0	no
3	234.55 Hz	3.20 %	s	0	no
4	285.65 Hz	2.88 %	s	0	no
5	294.90 Hz	2.90 %	s	0	no

I hope that this little discussion has shed some light on estimation of modal parameters with heavy damping and pseudo- repeated roots. The modal parameter estimation algorithms are capable of extracting these roots provided good measurements are provided. But good measurements are the critical key. If you have any more questions on modal analysis, just ask me.