

Propagation Loss Optimization in Metal/Dielectric Coated Hollow Flexible Terahertz Waveguides

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ABSTRACT

Low-loss hollow flexible metal/dielectric coated polycarbonate waveguides have been designed and fabricated for the maximal transmission of Terahertz radiation (THz). Attenuation characteristics of Terahertz radiation in Ag/Au coated waveguides with bore diameters 4.1 mm, 3.2 mm, 2 mm were studied at 215 μm wavelength and the maximal transmission was obtained by coupling the lowest loss TE_{11} mode from an optically pumped terahertz laser. Transmission loss can be reduced substantially by adding a dielectric layer to the metal coated waveguide and by coupling HE_{11} mode into it. Polystyrene (PS) was chosen to be the dielectric, due to its low extinction coefficient, which enhances the transmission through the waveguide. A propagation loss of less than 1 dB/m was achieved with these metal/dielectric coated waveguides.

Keywords: Continuous-wave THz imaging, flexible terahertz waveguide, metal/dielectric coating, closed loop method, LFC process

1. INTRODUCTION

The Terahertz (THz) frequency regime of an electromagnetic spectrum extends from 0.1 to 10 THz and lies between the microwave and infrared regions of the spectrum. At wavelengths from millimeters to microns, this spectral region is rapidly developing in source/detection technologies. The low energy THz radiation interacts with the low-frequency motion in molecular systems such as the flexing of individual molecules or intermolecular interactions through either weak Vander Waals forces or the stronger hydrogen bonds. This interactivity makes THz radiation highly sensitive to water concentration and has led to the applications in imaging¹ and spectroscopy. Researchers have chosen to investigate THz radiation instrumentation for Bio-sensing applications, due to its non-ionizing property, unlike X-rays. For many applications like Bio medical imaging², it is necessary to transport the THz radiation with minimal transmission loss. A flexible THz waveguide³ is an essential tool for guiding the radiation in the field of in-vivo medical imaging. The primary goal of this study is to design and fabricate flexible terahertz waveguides with maximal transmission.

2. DESIGN

A waveguide is a cylindrical or rectangular structure that confines and guides the electromagnetic radiation. Waveguides are classified into two types, solid core and hollow core. If the inner volume of the waveguide is filled with some dielectric, it is called a solid core waveguide. Hollow core waveguides (HCW) provide several advantages over solid core optical fibers because they transmit light in an air core. As the radiation is coupled from free space into the air core, there are no Fresnel reflections from the end face of the HCW, resulting in low insertion loss. Since most of the radiation is confined to the air core, HCWs have low non-linear effects, high laser damage threshold, and low absorption. These advantages have led to the use of HCWs in applications requiring the transmission of radiation at wavelengths

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Terahertz Technology and Applications V, edited by Laurence P. Sadwick, Cr idhe M. O'Sullivan,
Proc. of SPIE Vol. 8261, 82610P · © 2012 SPIE · CCC code: 0277-786X/12/\$18 · doi: 10.1117/12.906833

where it is difficult to find materials that have the optical, thermal, and mechanical properties required for use in solid core fibers. HCWs have also been used to transmit the Infrared⁴, Ultraviolet, and soft X-ray radiation. The main objective of this study is to design and fabricate a low loss, hollow, flexible waveguide for the propagation of THz radiation. Generally electromagnetic radiation is guided by dielectric or metal waveguides. Metals are highly reflective but not flexible. Dielectrics are flexible but not lossless in terahertz regime. Hence depositing a layer of highly reflective metal inside a flexible tube will make an efficient waveguide for the transmission of terahertz radiation. A dielectric layer is then added over the metal to provide enhanced reflectivity within specific wavelength ranges due to an interference effect. These waveguides are called metal/dielectric HCWs. The schematic cross section of a metal/dielectric coated waveguides is shown in Figure 1.

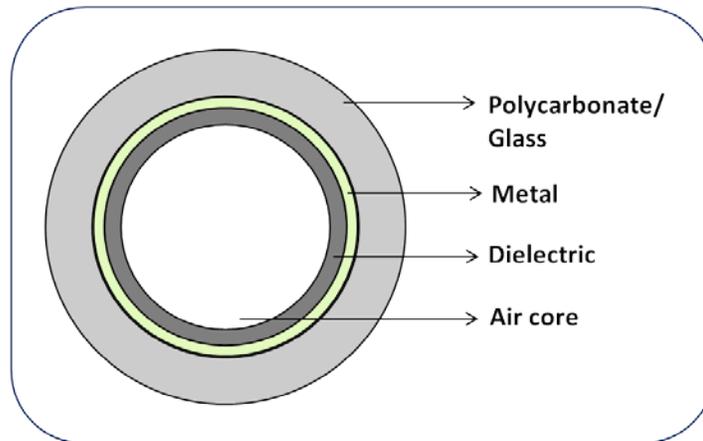


Figure 1. Schematic cross section of metal/dielectric coated hollow core waveguide

3. THEORY

Terahertz radiation propagated through the waveguide suffers two types of losses, namely, propagation loss and bending loss. Propagation loss is due to the dissipation of energy in the walls and bending loss is due to flexing the waveguide. Therefore, the total transmission loss of a waveguide can be represented as the sum of propagation and bending loss given by,

$$\alpha_{tot} = \alpha_{propagation} + \alpha_{bending} \quad (1)$$

or

$$\alpha_{tot} = \alpha_{pq}L + \alpha_{\theta}L_R \quad (2)$$

where α_{pq} represents the propagation loss and α_{θ} is the bending loss of a THz waveguide in dB/m. L represents the length of the waveguide and L_R is the bend length of waveguide.

The power attenuation coefficient⁵, α_{pq} , for hollow metal terahertz waveguides is given by,

$$\alpha(\text{TE}_{0q}) = 10 * \frac{u^2}{(n_0k_0)^2r^3} \left[\frac{n}{n^2+k^2} \right], \quad \alpha(\text{TM}_{pq}) = 10 * \frac{1}{r} \left[\frac{n}{n^2+k^2} \right]$$

and

$$\alpha(\text{TE}_{pq}) = 10 * \frac{u^4}{(u^2-p^2)} \left[\frac{n}{n^2+k^2} \right] \left(\frac{1}{k_0^2r^2} + \frac{p^2}{u^4r} \right) \quad (3)$$

where r represents the inner radius of the waveguide, $N = n - ik$ is the complex refractive index of the metal, p is the

mode index, $k_0 = 2\pi / \lambda_0$ is the wavenumber, and u is the phase constant representing q^{th} zero of the Bessel function $J_1(x)$ for TE_{0q} , $J_{p-1}(x)$ for TM_{pq} and $J_p(x)$ for TE_{pq} modes.

The power attenuation coefficient⁶, α_{pq} , for hollow metal/dielectric terahertz waveguides is given by,

$$\alpha(\text{HE}_{pq}) = 10 * \frac{u^2}{(n_0 k_0)^2 r^3} \frac{n}{n^2 + k^2} F_{\text{diel}}; \quad F_{\text{diel}} = \frac{1}{2} \left(\frac{N_d^2}{\sqrt{N_d^2 - 1}} \right)^2 \quad (4)$$

where $n-ik$ represents the complex refractive index of metal, $N_d = n_d - ik_d$ is the complex refractive index of the dielectric, and u is the q^{th} zero of $J_{p-1}(x)$.

From Equations (3) and (4), it is evident that the attenuation constant α_{pq} is directly proportional to the square of the wavelength and inversely proportional to the cube of the waveguide's radius. The numerical value for u_{pq} is higher for higher order modes. The losses for higher order modes increase quite rapidly due to the quadratic dependence of α_{pq} on u_{pq} . Hence, only lower order modes tend to propagate.

The bending loss depends on the radius of curvature of bending (R), angle of bending (θ), and inner radius (r) of the waveguide. Theoretically, it varies as r^3/R^2 .

The total transmission loss of a waveguide can be found experimentally by using Beer-Lambert's law as follows,

$$\alpha = \frac{10}{2L} \log \left(\frac{P_{\text{in}}}{P_{\text{out}}} \right) \text{ dB/m} \quad (5)$$

where L represents the length of the waveguide, P_{in} and P_{out} are the input and output powers of the terahertz beam respectively. The theoretical and experimental values of the attenuation coefficients can be obtained from Equations (4) and (5) respectively.

4. MATERIAL SELECTION

4.1 Selection of Base Material

In general, Silica glass tubing is the base material for terahertz waveguides. Since glass is not flexible at large bore diameters. Polycarbonate (PC) was chosen to be the base material for flexible terahertz waveguides, due to its flexibility even at large bore diameters (up to 6.3 mm). PC has an advantage over Polyethylene and Teflon as the extrusion process for glass and polycarbonate capillaries creates a smooth inner surface with surface roughness (order of nm).

4.2 Selection of Metal

The complex refractive index of metal is defined as $N = n - ik$, where n is the refractive index and k is the extinction coefficient. In the THz regime, metals have high reflectivity because of their high refractive index and extinction coefficients. Commonly used metals for high reflection coatings are silver, gold, aluminum and copper. The values of refractive index and extinction coefficient of these metals were calculated as a function of wavelength using the Drude parameters⁷. The figure of merit, $F = n / (n^2 + k^2)$, is the quantity used to characterize the performance of the waveguide, which is a measure of metal's reflectivity. For metals, the figure of merit is inversely proportional to its reflectivity. The figure of merit has been calculated and plotted as a function of wavelength in Figure 2. In this study the wavelength of interest is 215 μm with the figure of merit values for various metals depicted in Table 1.

Minimum loss at a wavelength of 215 μm can be achieved by choosing either silver (Ag) with $F = 0.684 \times 10^{-3}$ or gold (Au) with $F = 0.878 \times 10^{-3}$ due to their lower figure of merit values.

Table 1. Calculated figure of merit values for Ag, Al, Au, Cu, W, and Pb at a wavelength of 215 μm

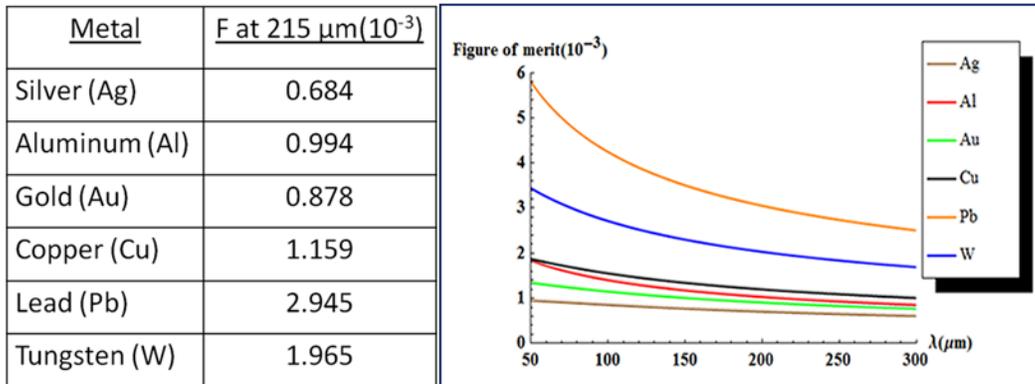


Figure 2. Figure of merit for Silver (Ag), Gold (Au), Aluminum (Al), Copper (Cu), Tungsten (W), and Lead (Pb) as a function of wavelength in μm

4.3 Selection of Dielectric

Hollow core metal coated waveguides utilize a highly reflective wall to confine light to the air core. By increasing the wall's reflectivity, the degree to which light is confined to the waveguide's air core increases, thus decreasing the attenuation of radiation propagating along its axis. Since the wall is not a perfect reflector, leaky modes tend to propagate in these hollow silver or gold coated waveguides. A dielectric layer is then added over the metal to improve reflectivity⁸ within specific wavelength ranges due to the interference effect. From Equation (4), it is evident that the attenuation coefficient varies directly as the figure of merit F_{diel} , which is a function of the dielectric material's complex refractive index ($N_d = n_d - ik_d$). The optimum value of n_d can be calculated by assuming a lossless dielectric (extinction coefficient $k_d = 0$). Shown in Figure 3, is the graph for F_{diel} as a function of n_d , indicating the optimum n_d as 1.414, with a value of k_d that corresponds to optimum n_d of no more than 5×10^{-3} . Polystyrene (PS) was chosen to be the dielectric⁹, due to its low extinction coefficient, which enhances the transmission through the waveguide. Also polystyrene has a complex refractive index of $1.58 - i3.58 \times 10^{-3}$, within the dielectric's optimal range.

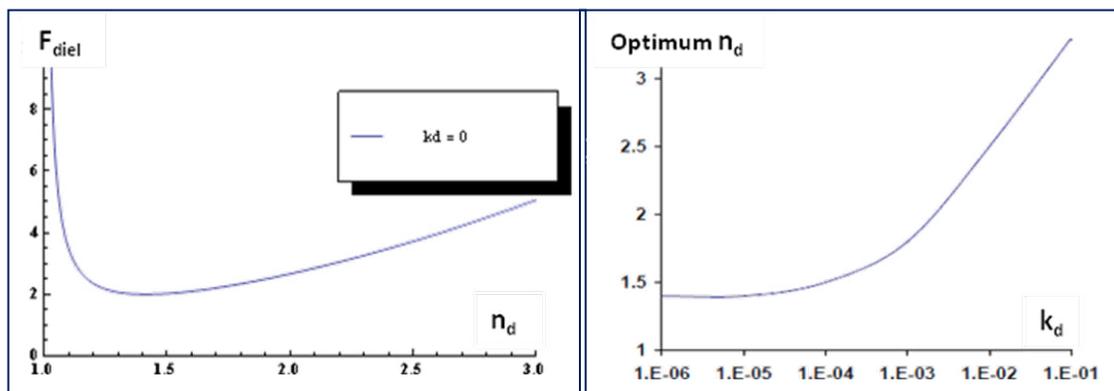


Figure 3. F_{diel} as a function of n_d by assuming a loss less dielectric, Optimum n_d as a function of k_d to obtain optimum complex refractive index (N_d) of dielectric

5. FABRICATION

5.1 Metal Coated Waveguide

Waveguides to transmit terahertz radiation were fabricated by depositing thin metal/dielectric films inside glass and plastic tubes. The maximal power transmission occurs not only by choosing a high reflective metal, but also by achieving a uniform layer. There exists an additional loss due to scattering, if the surface roughness of the metal/dielectric layer is comparable with the wavelength. The waveguides were fabricated in diameters of 4.1 mm, 3.2 mm, and 2 mm.

Silver or gold can be coated on the inner surface of the polycarbonate tube using a liquid phase chemical deposition process. The formation of metal film involves the kinetic process of condensation, nucleation, and growth. In order to achieve the maximum transmission of THz radiation through the waveguide, the metal film thickness should be greater than its skin depth at the propagated wavelength. The thickness of the silver film is chosen to be at least 10 times its skin depth, $0.5\mu\text{m}$, to get a reflection of 99%. For gold it is $0.7\mu\text{m}$ at $215\mu\text{m}$ wavelength.

Adhesion of the silver coating was achieved using a wetting solution and palladium stannous based sensitizer prior to the deposition of silver. For the deposition of the silver film inside a polycarbonate tube, diluted silver solution (HE-300 obtained from Peacock Laboratories) and reducing agent (HE-400R) were used. The silver solution consists of silver nitrate, ammonium hydroxide and water. The reducing agent consists of dextrose and disodium salt of ethylenediamine tetra acetic acid and sodium hydroxide. Diluted HE-300 and HE-400R were pumped through tygon tubing (2.29 mm ID) at a rate of 10 – 15 ml/min for 2 – 4 minutes using a peristaltic pump. Initially, small clusters of Ag atoms precipitate on the inner surface of the PC tube. The clusters grow and coalesce, ultimately forming a continuous silver coating. The experimental arrangement for the deposition of silver on the inner surface of PC tubing is shown in Figure 4a.

Figure 4b shows the experimental arrangement for the deposition of gold on the inner surface of the polycarbonate tubing using a closed loop electroless liquid phase chemical deposition process. For depositing gold inside a polycarbonate tube, gold solution (EG-2000 A from Peacock Laboratories), gold addition agent (EG-2000 B), and reducer solution (EG-2000 C) were used. Gold solution consists of gold tri chloride and selenium dioxide. The gold addition agent consists of sodium carbonate. Formaldehyde was used as the reducing agent. 5 ml of each solution was mixed together and pumped at a rate of 35 – 40 ml/min for 5 minutes. This process was repeated until the desired thickness of gold layer was achieved.

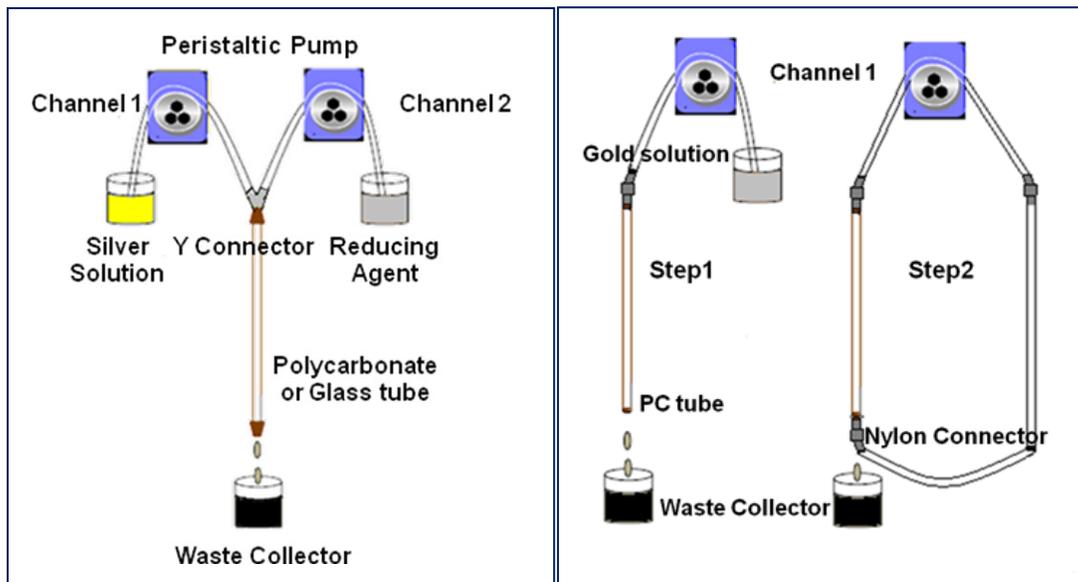


Figure 4. Experimental setup for the deposition of a) Silver, b) Gold inside a hollow polycarbonate tube by electroless liquid phase chemical (ELPCD) process

5.2 Metal/Dielectric Coated Waveguide

To deposit the dielectric film inside a silver coated hollow waveguide by using a liquid flow coating process, polystyrene has to be dissolved in a solvent. Polystyrene can be dissolved in many solvents, but the most commonly used ones are toluene ($C_6H_5CH_3$), tetrahydrofuran (THF- C_4H_8O), chloroform ($CHCl_3$), and cyclohexane (C_6H_{12}). All of these solvents have good compatibility with metals, due to silver or gold's microns thickness, chemical compatibility¹⁰ with polycarbonate tubing has to be tested. Toluene, THF, and chloroform had severe effect with polycarbonate tubing. Hence, cyclohexane was used as the solvent due to its excellent compatibility with all polycarbonate, Viton and Nylon tubing as shown in Table 2.

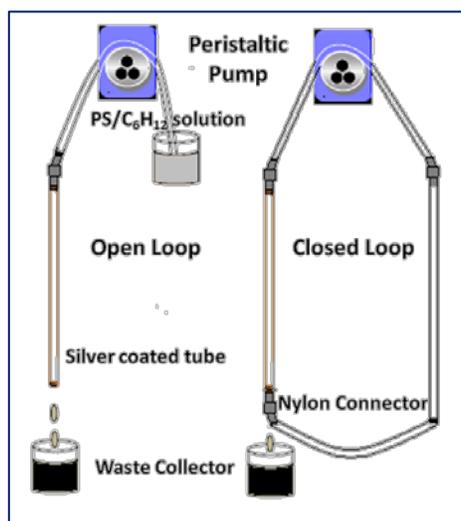


Figure 5. LFC process for polystyrene deposition in metal coated waveguides by conventional open loop method - closed loop flowing method

To obtain a uniform dielectric layer inside the metal coated waveguide, a closed loop method¹¹ was used as the airtight environment depresses the evaporation of cyclohexane in the PS solution. The system was designed to draw polystyrene/cyclohexane solution through the silver coated waveguides at a constant coating velocity using an Ismatec peristaltic pump. Figure 5 shows the schematic diagram of the system used to deposit the PS film inside the silver coated waveguide. 5 – 8 ml of a 16 – 20 wt% polystyrene/cyclohexane solution was pumped into the PC tube with a maximum flow rate of 40 ml/min. The optimum thickness¹², d_{diel} , at glancing incidence of a non-absorbing ($k_d = 0$) dielectric film on a metal substrate at a wavelength of 215 μm was obtained as 26.8 μm . As the thickness of the polymer coating increases linearly with time, the desired 26 μm thickness was achieved by coating the PC tube up to 10 minutes.

Table 2. Chemical Compatibility of tubing material with solvents of Polystyrene

MATERIAL	Toluene	THF	Chloroform	Cyclohexane
Tygon	D	C	B	D
Viton	C	D	A	A
Silicone	D	D	D	D
Nylon	A ¹	A	A	A
PVDF	A ¹	B	A	A
Polycarbonate	D	D	D	B
Poly Propylene	C	C	C	D

(A – Excellent, A¹ – satisfactory to 22^oC, B – Good, C – Fair, D – Severe effect)

6. EXPERIMENTAL SETUP

The source used for this experiment was a CO₂ optically pumped far-infrared gas (FIR) laser operating at 1.39 THz. The 214.6 μm line in CH₂F₂ was pumped by the 9R34 transition of the CO₂ laser. Near the laser face, the measured terahertz output power was 370 mW. A liquid helium cooled silicon bolometer was used to detect the terahertz signal. Since the beam emerging from the FIR laser¹³ is a few millimeters in diameter and expands fairly rapidly as it propagates, an optical system was designed to focus the radiation into the waveguide.

The coupling coefficients (η_m) for HE_{1m} modes¹⁴ were plotted as a function of ω_0/r , where $2\omega_0$ represents the minimum beam waist diameter. From Figure 6 and Table 3, it is evident that the maximum transmission in metal/dielectric coated waveguides occurs by coupling 98.1% of lowest loss HE₁₁ mode into it. Therefore, the considerations for the system design were that the ratio of ω_0/r should be 0.77, 0.64 mm to couple 90.3%, 98.1% of the lowest order transverse electric TE₁₁, HE₁₁ mode into silver and silver/polystyrene coated waveguides respectively.

Table 3. Calculated coupling coefficients of HE_{1m} modes at ω_0/r as 0.64

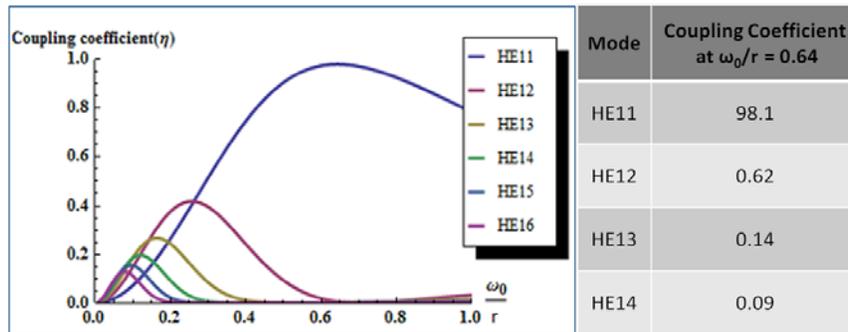


Figure 6. Theoretical coupling coefficients of lower order HE_{1m} modes for a Gaussian input beam of waist $2\omega_0$

The measured waist of the terahertz beam coming out of the dielectric waveguide was 2.36 mm. This beam was allowed to expand in free space before being collimated by a 30" focal length TPX lens. The resulting beam waist was 22.14 mm. The collimated beam was propagated 50" to a 7" focal length, 3" diameter, off-axis parabolic mirror (OAP) that focused the beam onto the dielectric tube. This 3.2 mm polycarbonate/Pyrex tube was used to clean higher order modes¹⁵. Figure 7 shows a schematic of the optical layout.

The output beam was focused using a polyethylene lens of 3" focal length and 3" diameter. The focused Gaussian beam of 0.77 mm beam waist was coupled into a metal coated waveguide of 2 mm inner diameter, to excite the TE₁₁ mode, by maintaining the ratio of ω_0/r as 0.77. As the optimum coupling to the HE₁₁ mode¹³ occurs at an f -number launch of $f/35$, the output beam was focused onto a polyethylene lens of 2.76" focal length with 3" diameter. The focused output Gaussian beam of polyethylene lens with 1 mm beam waist was coupled into the metal/dielectric coated waveguide of 3.2 mm inner diameter, to excite the HE₁₁ mode, by maintaining the ratio of ω_0/r as 0.64. A Spiricon Pyroelectric camera was used to observe and adjust the mode pattern.

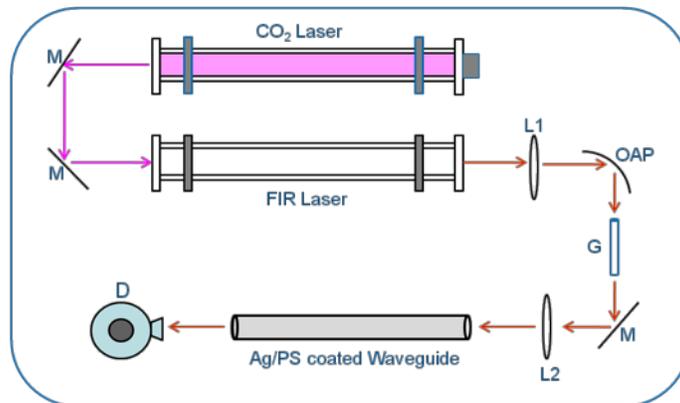


Figure 7. Experimental setup for the measurement of attenuation coefficient for metal/dielectric coated THz waveguides (M – Mirror, L – Lens, OAP – Off axis parabolic mirror, G – Glass tube to clean the higher order modes, and D – Detector)

7. RESULTS AND DISCUSSION

The total transmission loss of a waveguide was found experimentally by using Beer-Lambert's law. In this study Equation (4) and Equation (5) were used to calculate the theoretical and experimental values of attenuation coefficient respectively. The system signal to noise (SNR) ratio using a lock-in amplifier was estimated to be 68 dB. The input and output Gaussian modes of metal coated hollow terahertz waveguides were shown in Figure 8. From the Figure 8a, it is evident that the vertical polarized terahertz radiation was efficiently coupled as the HE_{11} mode into the Ag/PS coated waveguide. Figure 8b confirms that there was no change in the modal properties of the transmitted radiation. Hence this study concludes that the low loss HE_{11} mode was preserved for straight metal/dielectric coated waveguides.

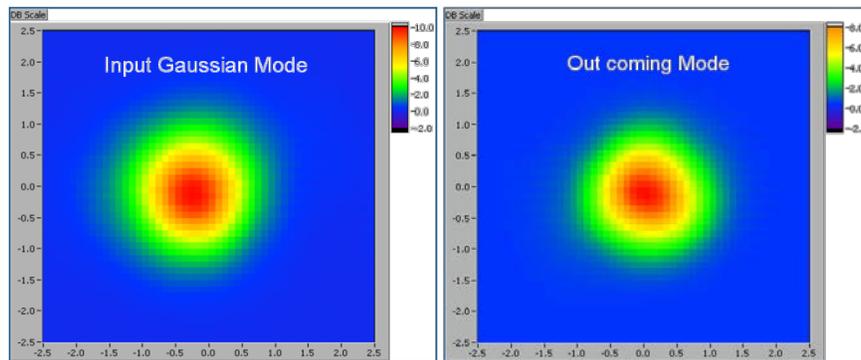


Figure 8. a) Input and b) output beam profiles of the propagated beam through a 3.2 mm silver/polystyrene coated THz waveguide

Theoretical minimal loss in metal/dielectric coated waveguides was achieved by coupling TE_{11} mode. The theoretical and experimental attenuation coefficients of both silver and gold coated waveguides were plotted as a function of bore diameter “d”. Figure 9a shows a good agreement between experimental and theoretical results. The experimental propagation losses of the waveguides were generally higher than the theoretical losses, but the predicted trend of decreased loss with increasing bore diameter and the reduction in the difference of silver and gold coated waveguide attenuation coefficients are clearly evident from Figure 9a. The higher agreement between experimental and theoretical losses of gold coated waveguides can be due to their lower surface roughness resulted from a closed loop LFC method. A propagation loss of less than 2 dB/m was achieved with 4 mm inner diameter silver coated waveguides.

Theoretical minimal loss in metal/dielectric coated waveguides was achieved by coupling HE_{11} mode. The theoretical and experimental results for attenuation coefficient of both silver and silver/polystyrene coated waveguides were plotted as a function of bore diameter “d” as shown in Figure 9b. The predicted trend of decreased loss with increasing bore diameter and the reduction in the loss silver/polystyrene coated waveguides than that of silver only coated waveguides were clearly visible from Figure 9b.

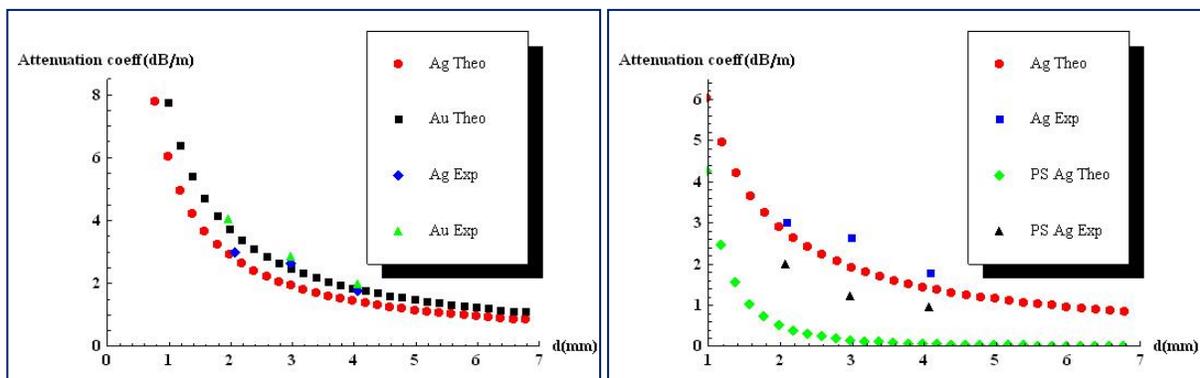


Figure 9. Theoretical and Experimental attenuation coefficients as a function of inner bore diameter of silver and a) gold coated and b) silver/polystyrene coated waveguides

The discrepancy between theoretical and experimental values of attenuation coefficient can be due to the roughness or non-uniformity in the metal or dielectric coating, edge coupling effect, and also due to the difference in theoretical and experimental mode coupling efficiencies. Also, the theoretical attenuation coefficients for metal/dielectric coated waveguides were obtained by assuming an ideal lossless dielectric ($k_d = 0$). The green diamonds shown in Figure 9b should be at a higher level due to the nonzero absorption coefficient of polystyrene. A propagation loss of 0.96 dB/m was achieved with 4 mm inner diameter silver/polystyrene coated THz waveguides.

8. CONCLUSION

A low loss hollow flexible waveguide for the propagation of terahertz radiation was fabricated by coating silver and polystyrene inside the polycarbonate tubes of 4.1 mm, 3.2 mm, and 2 mm. A propagation loss of 1.77 dB/m, 2 dB/m was measured by coupling lower order TE_{11} mode into the silver, gold coated waveguide of 4 mm inner diameter respectively. The lower order hybrid HE_{11} mode was successfully coupled into the silver/polystyrene coated waveguides. A propagation loss of 0.96 dB/m was achieved with 4 mm inner diameter Ag/PS coated THz waveguide. The promising results obtained in this work demonstrate that these metal/dielectric coated waveguides have the potential for transporting the THz radiation over distances of several meters with acceptable transmission loss.

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