

Convolution Tutorial

INTRODUCTION

This document is designed to overview the theory of the convolution integral and its applications. It introduces the convolution integral and then demonstrates its use with a detailed example. The Convolution Integral Labview GUI can be used to follow along with the example shown here.

ARBITRARY SIGNALS

The convolution of two time domain signals is given by the following equation:

$$y(t) = \int_{-\infty}^{\infty} x_1(\tau) * x_2(t - \tau) d\tau$$

To calculate the convolution at a specific time, t , first the integrand $x_1(\tau) \cdot x_2(t - \tau)$ is computed as a function of τ , and then the integration step is performed with respect to τ , resulting in a function of t . In discrete time domain data, this is the same as point by point multiplication of $x_1(\tau)$ and $x_2(t - \tau)$ and taking the sum.

This is demonstrated graphically in the Labview GUI “ConvolutionArb.vi”. The overall GUI is shown in Figure 1. The top two plots on the far left and far right are two arbitrary signals, $x_1(t)$ and $x_2(t)$. The top middle plot shows $x_1(\tau)$ overlaid with $x_2(t - \tau)$ at time t , specified by the calculation point. The second signal is flipped by the minus sign and shifted by the calculation point t . The middle plot shows the result of multiplying the two overlaid signals together at the time specified. Finally, the bottom plot is designed to build the resulting convolution of the two original signals by dragging the calculation point slider-bar from time equals zero to the end time. As the calculation point is changed, note the position of the second signal on the top middle plot, as well as the area under the curve in the middle plot.

This GUI is designed for any two arbitrary signals convolved together. The default signals (when the GUI is initially opened) are a sine wave and a square wave, each with 1 v amplitude, 20 Hz frequency, 0° phase, and are computed for one cycle. It should be noted that the order of operations does not matter for convolution, since it is equivalent to multiplication in the frequency domain. That is $X_1(\omega) \cdot X_2(\omega) = X_2(\omega) \cdot X_1(\omega) \rightarrow x_1(t) * x_2(t) = x_2(t) * x_1(t)$.

To prove this drag the calculation point slider bar all the way to the right so that the entire convolution integral is calculated. Now reverse the two signals by changing signal 1 to a square wave and signal 2 to a sine wave. The result of the convolution is the same.

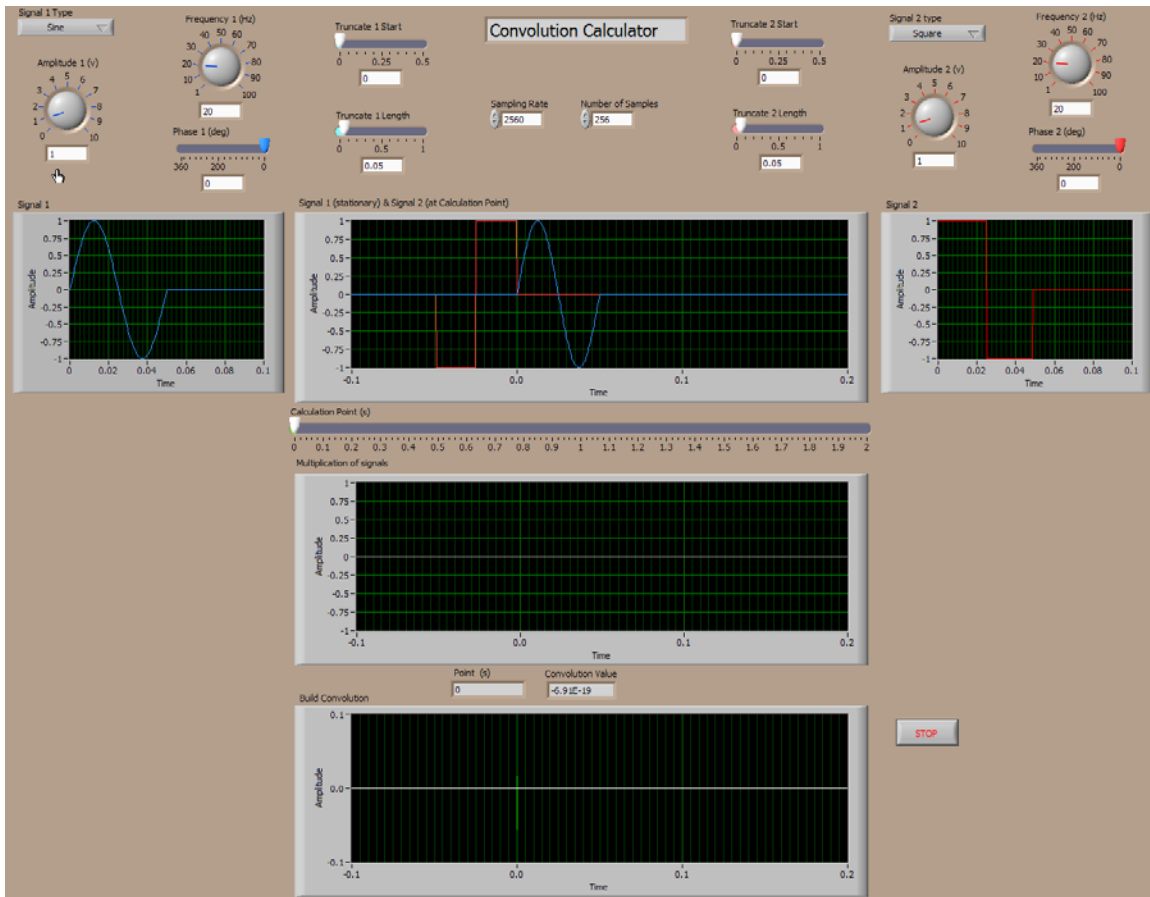


Figure 1 Labview GUI:ConvolutionArb

The GUI gives you the ability to construct signals X1 and X2 from several basic waveforms including, sine, square, triangle and sawtooth. The Amplitude, Frequency, and Phase properties can be controlled. Additional waveform shapes can be made by using the Truncate Start, and Truncate Length features along with the basic waveform functions. The signal construction tools for X1 are shown in Figure 2.

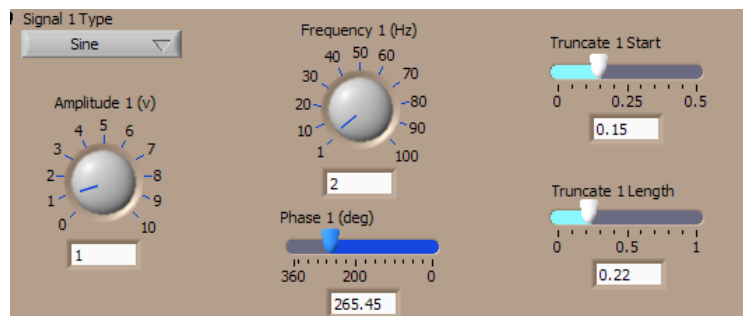


Figure 2 Signal Construction Tools

To further understand the convolution integral, explore different signals by adjusting the construction tools for each signal. What happens when one signal is a sawtooth? What happens if one of the signals is increased in amplitude or changed in frequency? What

happens when phase is introduced, or if the truncation window is adjusted to calculate longer periods?

One interesting aspect to look at is when one signal becomes a unit impulse. To do this change one of the signals to a square wave of amplitude = 1 v. Then adjust the truncation window length for as short as possible and then use the calculation point slider to compute the convolution function. The resulting convolution is the same as the other signal.

UNIT IMPULSE RESPONSE

As we've seen the response of any continuous, linear system can be described by the convolution integral. Consider a system with the unit impulse response $h(t)$, which is assumed to be known. The response signal $x(t)$, to input $f(t)$, is found by the convolution of $f(t)$ with $h(t)$. This is given by the following equation:

$$y(t) = \int_{-\infty}^{\infty} h(\tau) * x(t - \tau) d\tau$$

This concept is demonstrated by the Labview GUI "ConvolutionSDOF.vi". This GUI is very similar to the GUI for two arbitrary signals, Figure 3. The second signal has been replaced by the unit impulse response of a single degree of freedom system, $h(t)$. The first signal therefore becomes the input function, $f(t)$. The remaining plots are the same as the previous GUI, with the final plot being the result of the convolution integral, or the system response, $x(t)$. Drag the calculation point slider-bar across to build the convolution integral for the default settings.

Note how the response of the system is affected by changing the values of mass, damping, and stiffness of the single degree of freedom system. What happens when mass is increased or decreased? How does the response change if some of the damping is removed or if some of the stiffness is removed the system? Next see how the response of the system is affected by different inputs into the system by changing the signal type, amplitude, frequency, and phase.

Change the input to a unit impulse signal as before, and see how the response of the system becomes the same as the computed transfer function, $h(t)$.

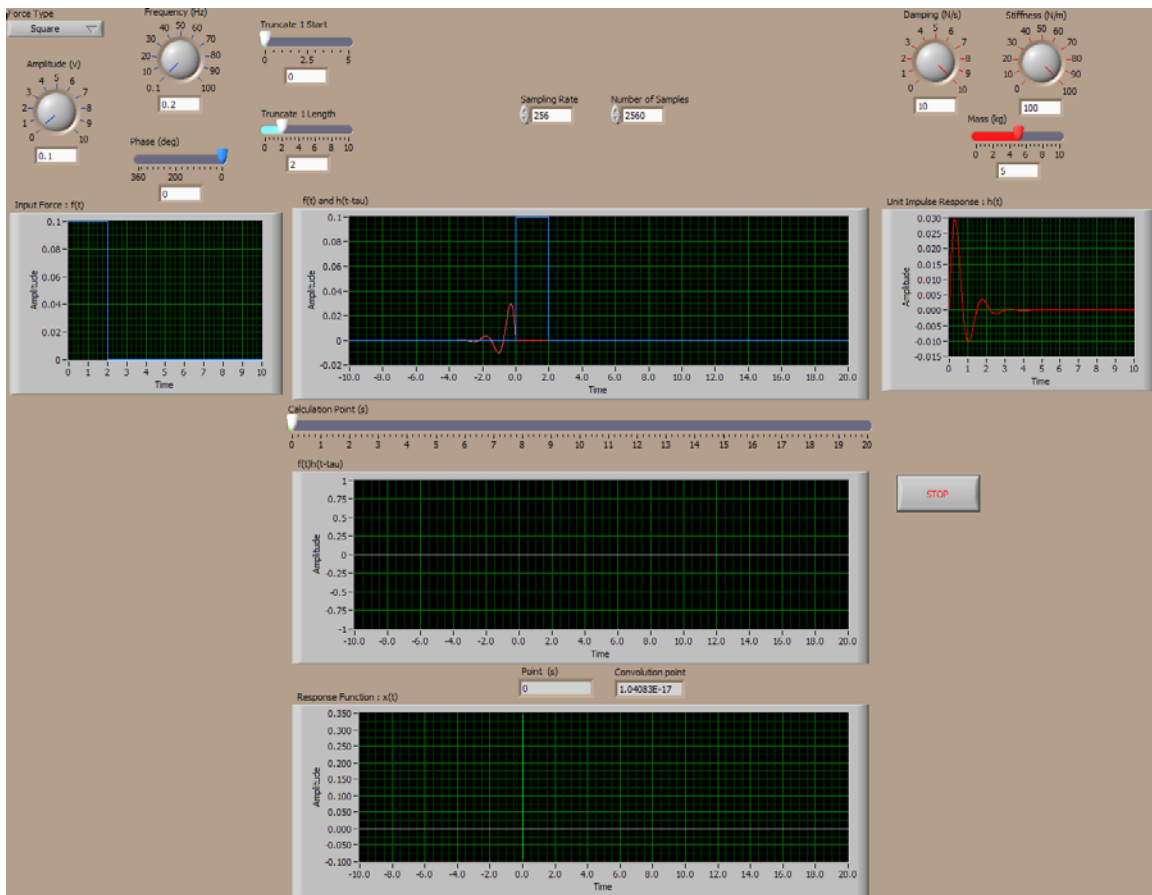


Figure 3 Labview GUI: ConvolutionSDOF

Change the input to a unit impulse signal as before, and see how the response of the system becomes the same as the computed transfer function, $h(t)$.

Now try dragging the calculation point slider-bar back to the left to hide the convolution result. Make a few changes to the system and to the input function, and see if you can predict what will happen to the convolution result. Rebuild the convolution by dragging the slider-bar to the right again to see if you were right.