MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

Illustration by Mike Avitabile

How free does a test need to be? Does it really matter that much?
This is an important item to discuss.

Alright – now here is an item that I think everyone gets confused about. It really stems from the fact that we all are familiar with rigid body dynamics and the concepts surrounding that. But we don't let go of those concepts easily.

What I mean is that rigid body dynamics is a good approximation when the body is described by a center of mass concept and all the points on the structure can be described in terms of that one point. What we then have is a point in space that is the only point we need to describe the entire motion of the structure associated with that one point.

And that is a pretty powerful statement. It means that every point on the geometry of the structure can be defined completely by that one point on the structure. Therefore, if this was the case then we would say that we have a rigid body.

So now what do we mean when we say we have a free-free system and we have rigid body modes that describe that system. That implies that we have no constraint whatsoever to ground and that the structure is essentially floating freely in space. If that is the case, then we would have rigid body modes describing the six independent ways that the body can move in space. There would be three separate translation motions in the three principle directions as well as three separate rotations in the three separate directions. Of course we have to realize that the six independent motions could possibly be comprised of linear combinations of each other as another possibility. Just because we tend to think in x,y,z directions doesn't mean that the rigid body motion needs to be isolated to those three directions – any linear combinations are also valid.

OK – so now we have this rigid body mode concept down. Now let's talk about a simple beam that we might possibly model with a finite element model. Let's assume to start that the beam is a uniform cross section and uniform weight distribution so there is nothing fancy about this beam. To further simplify the discussion we will only consider planar motion but there is no reason we couldn't extend it to six degrees of freedom to be general.

So let's first describe the first few modes of this planar system. Figure 1 shows the first four modes of the planar beam system. Notice that the first two modes are the rigid body modes and that the next two modes are the flexible modes of the system.

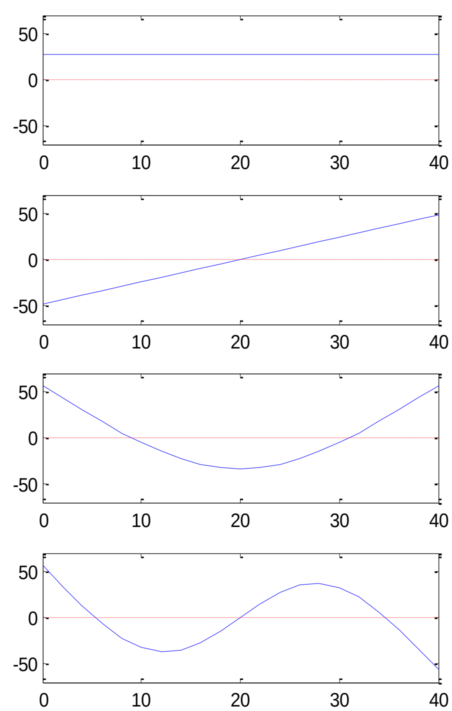

Figure 1 –Modes of Free-Free Beam

Notice that the first rigid body mode is a bounce mode with up and down motion and that the second rigid body mode is a rocking mode about the geometric center of the beam. This is what is expected for the free-free modes of a fully unconstrained beam structure.

Now let's consider that the beam really can't float in space unconstrained when we test it in the lab. And let's consider a range of spring stiffnesses to apply to the two ends of te beam. And let's further let the stiffnesses range from close to zero all the way up to a very high stiffness approaching a pinned condition or perfectly constrained. This is schematically shown in Figure 2.

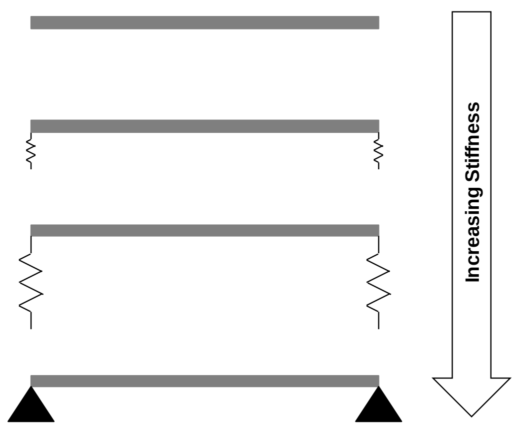

Figure 2 – Elastic Support for Beam

Now to make this simple, we are just going to look at how the first mode changes as we add increasing spring stiffness to the ends of the beam. We are going to look at what happens to the mode shape as the stiffness increases. This is shown in Figure 3 with the mode shape shown from top to bottom with increasing stiffness.

The first shape plotted is the free-free beam first mode shape. So as we increase the stiffness at the end of the beam, the natural frequency will shift upwards because there was an increase in stiffness as expected. So if we added just a little bit of stiffness the mode shape may not change appreciably. And we will notice that in the second plot from the top that the mode shape is still very similar to a rigid body mode but that there is slight amount of curvature in the beam. As we increase the stiffness we see that in the third plot that the shape doesn't really look like a perfect rigid body mode and that the shape is starting to take on more of a curvature like the first flexible mode of the system. By the time we increase the stiffness even more, the fourth and fifth plots don't really resemble a rigid body mode any longer and basically the mode shape really resembles a flexible mode with just a tiny bit of rigid body motion.

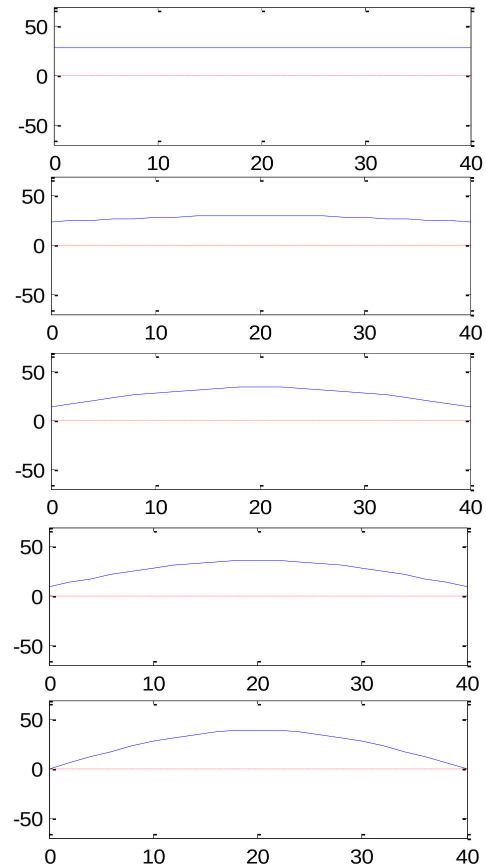

Figure 3 – Progression of Mode 1 Shape

So now the story is basically told. The rigid body mode is only truly a rigid body mode when it is completely free-free. Once any amount of stiffness is added to the ends of the beam, then the mode starts to change from a rigid body type mode to a flexible type mode and the proportions of rigid body and flexible mode is heavily dependent on the amount of stiffness added as well as the stiffness of the structure itself.

This means that when we measure any structure in the lab in the so called free-free state, the actual rigid body mode obtained will always have some of the flexible modes included and is really not a perfectly rigid body mode. Depending on how the test is set up and and how stiff the free-free suspension is will have a direct effect on just how rigid those rigid body modes are. I hope this simple explanation clears up any misconceptions that you may have had. If you have any more questions on modal analysis, just ask me.