ELEMENTARY MATHEMATICS FOR TEACHERS: NUMBER AND OPERATIONS

FALL 2012  92.107/201

Mondays and Wednesdays, 11:00-12:15 pm

Instructor: Dr. Roser Giné

Office Hours: Mondays, 12:30-2:00 pm and Thursdays, 2:00 pm – 3:30 pm, or by appointment

Office: OL 533  e-mail: Roser_Gine@uml.edu

Phone: (978) 934-4355

COURSE OVERVIEW/RATIONALE

The Number and Operations course for elementary and middle school teachers examines the three main categories in the Number and Operations strand of Principles and Standards of School Mathematics (NCTM) — understanding numbers, representations, relationships, and number systems; the meanings of operations and relationships among those operations; and reasonable estimation and fluent computation. There are two types of goals in this course: cognitive and affective. To meet the cognitive goals, you will revisit familiar concepts in a more in-depth manner. We will investigate the foundations of the arithmetic in grades 1 through 6 and strengthen your ability to recognize mathematical connections across different representations. The course will focus upon the real number system, place value, the behavior of zero, meanings and models of basic operations, percentages, and modeling operations with fractions, often with the aid of concrete, physical models that enhance understanding. It also examines basic Number Theory topics, such as factors and multiples, as well as divisibility tests, at both practical and abstract levels. The affective goals are to encourage you to value mathematical thinking as a means of exploring and learning about the world and foster a positive attitude about doing mathematics.
COURSE OBJECTIVES

Throughout the course, students will be encouraged to use Polya's four-step approach as a framework for problem solving. Students will also be asked to articulate their problem-solving approaches verbally, in written form, and through the use of drawings and/or diagrams, when appropriate. The course will emphasize multiple representations of every concept discussed as well as connections among them.

INTENDED LEARNING OUTCOMES

Upon successful completion of this course, you will be able to:

1. Explain and use set notation and Venn diagrams.
2. Use set and number line models with various approaches to perform operations on whole numbers.
3. Recognize and use numeration and place value.
4. Name, classify, and use various properties and algorithms for whole number addition, subtraction, multiplication, and division.
5. Recognize the differences between measurement and partitive division and understand how context determines the approach.
6. Identify and use various techniques for estimation.
7. Identify and work with the sets of prime and composite numbers.
8. Find all factors and the prime factorization of a number.
9. Find the greatest common factor and least common multiple between numbers using various methods.
10. Recognize the set of integers and its properties.
11. Use set and number line models to perform operations on integers.
12. Recognize the set of rational numbers and its properties and understand how fractions express relationships of quantities.
13. Recognize how the unit or whole in fraction operations is determined by the context of the story and by the operations.
14. Compare, order, and perform operations on rational numbers.
15. Use ratios, proportions and percents to solve problems.
ASSESSMENT OF LEARNING OUTCOMES

Rubrics that outline expectations for homework, group work/class activities, the project, and your participation in this course will be handed out in class.

HOMEWORK

Homework will be assigned regularly to reinforce and extend concepts learned in class. At the beginning of each class period, time will be allotted to review some questions from the previous night’s assignment. However, if you need more extensive support, please visit the instructor during the office hours or make an appointment if you cannot make any of these times. Homework will be collected and assessed an average of once per week.

Homework will comprise 20% of your final grade in this course.

GROUP WORK/CLASS ACTIVITIES

There will be class activities during which you will have an opportunity to work with one another. Some of these activities will require a formal write up, to be turned in as a group.

Group work/Class activities turned in will comprise 5% of your final grade in this course.

SERVICE COMPONENT

As part of the course, there is a service component of tutoring 5th and 6th grade students at the Innovation Academy Charter School in Tyngsboro. Each student is responsible for tutoring students 3 times during the semester at the school, preferably 3 consecutive sessions. After school hours are 3-4 pm. Prior to entering the school setting, you must go to the school with proper identification and fill out a CORI form. Each student will need to do this during the first week of the term since approval to enter the setting will take an average of 2-3 weeks. Additional information will be provided in class.

Service will comprise 10% of your final grade in this course.

PROJECT

You will prepare a project on a topic covered in class, with an emphasis on ways to deepen one’s conceptual understanding of the particular topic chosen. In preparing this project, you will create and organize materials to reflect the following:

- A deep understanding of the concept studied;
- Ways in which the concept might be taught in school, through activities, use of manipulatives, or class problems/projects;
The real-life relevance of the concept studied, including a brief historical overview documenting its origin;

Possible misconceptions of the concept along with strategies for addressing them.

The project will comprise 20% of your final grade in this course.

PARTICIPATION

You are expected to actively engage in class by attending regularly and coming on time; contributing your ideas; asking questions when needed; collaborating with your peers, and coming prepared to class.

Participation will comprise 5% of your final grade in this course.

MIDTERM EXAM

The midterm exam is scheduled for October 17, 2012, and will assess your understanding of the course material explored up to this date.

The midterm exam will comprise 20% of your final grade in this course.

FINAL EXAM

Our comprehensive final exam is scheduled during the final exam period (December 12-December 20).

The final exam will comprise 20% of your final grade in this course.

COURSE MATERIALS

The materials for this course will be handed out in class. We will be using five chapters from a text developed by Dr. Stan Dick, a retired professor from UMass Boston, who has given permission for the use of his materials. These will be given to you as a course packet. However, these chapters only comprise a portion of our materials, and will be used primarily as resources.

Note that in the course sequence matrix below, there are several references to TMKT. The fractions component of this class draws from the *Introducing Mathematical Knowledge for Teaching: Exploring Fractions* course materials from the University of Michigan. The TMKT materials include approximately twelve hours of the instruction in this class, and, along with additional readings, class problems, and problem sets, will also be available to you as we work towards our course objectives.
COURSE POLICIES

Attendance

You are expected to attend all of the class sessions and to participate fully in class. In the event that you must miss a class or part of a class, please let your instructor know in advance so that we can set up a plan for making up work missed. If you have poor attendance or are consistently late to class, the assessment of your contribution to the class will be affected (points deducted from the overall participation grade).

_I expect you to attend each class, be punctual, and engage meaningfully in our class activities._

Late Work

All assignments must be submitted on time. If you believe you will need to turn in an assignment after it is due, please be sure to let me know in advance. Points may be deducted from the overall grade of the assignment for lateness.

GRADES

Specific rubrics will be provided for the assignments outlined above. However, the overall grading system for the course is specified below.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Superior Work: Highest Quality</td>
<td>4.0</td>
</tr>
<tr>
<td>A-</td>
<td>High Honors Quality</td>
<td>3.7</td>
</tr>
<tr>
<td>B+</td>
<td>High Quality</td>
<td>3.3</td>
</tr>
<tr>
<td>B</td>
<td>Basic Honors Quality</td>
<td>3.0</td>
</tr>
<tr>
<td>B-</td>
<td>Below Honors Quality</td>
<td>2.7</td>
</tr>
<tr>
<td>C+</td>
<td>Above Satisfactory Quality</td>
<td>2.3</td>
</tr>
<tr>
<td>C</td>
<td>Satisfactory</td>
<td>2.0</td>
</tr>
<tr>
<td>C-</td>
<td>Below Satisfactory Quality</td>
<td>1.7</td>
</tr>
<tr>
<td>D+</td>
<td>Above Minimum Passing</td>
<td>1.3</td>
</tr>
<tr>
<td>D</td>
<td>Minimum Passing</td>
<td>1.0</td>
</tr>
<tr>
<td>F</td>
<td>Failed</td>
<td>0.0</td>
</tr>
<tr>
<td>FX</td>
<td>Failed due to Academic Misconduct (may not be replaced or deleted)</td>
<td>0.0</td>
</tr>
</tbody>
</table>

For additional information on the university’s grading policies, see [http://www.uml.edu/catalog/undergraduate/policies/grading_policies.htm](http://www.uml.edu/catalog/undergraduate/policies/grading_policies.htm)

Note that UMass Lowell’s policy on [Academic Dishonesty, Cheating and Plagiarism](http://www.uml.edu/catalog/undergraduate/policies/grading_policies.htm) applies to this course.

PREPARATION FOR THE MTEL

The purposes of this course are: to highlight the mathematical content that is used in
teaching elementary school; and to provide pre-service teacher candidates the opportunity to learn and understand mathematical concepts needed to effectively teach mathematics at the elementary school level. However, this is not an MTEL (Massachusetts Test for Educator Licensure) prep course. The MTEL is designed to test the knowledge you will acquire through this course, but this course is not designed, specifically, for preparing you for that test.

Below are the MTEL requirements for understanding of number and operations, along with the NCTM standards relevant for this course.

**MTEL REQUIREMENTS FOR NUMBER AND OPERATIONS**

**Understand number theory, the structure of numeration systems, and the properties of the real number system.**

For example: place value (including base ten and other bases); order relations; the relationships between operations (e.g., multiplication as repeated additions); factors and divisibility; prime and composite numbers; prime factorization; rational and irrational numbers; properties (e.g., closure, distributive, associative) of the real number system; operations and their inverses; the real number line; roots and powers; the laws of exponents; scientific notation; and proof of theorems using number properties (e.g., the product of two even numbers is even).

**Understand principles and operations related to fractions.**

For example: multiple representations of fractions (e.g., area and set models); multiple representations of operations involving fractions; equivalent fractions; addition, subtraction, multiplication, and division of fractions; comparison, ordering, and estimation of fractions; placement on a number line; simplification of fractions; mixed numbers and improper fractions; the relationship between properties of fractions and the development of algebraic concepts; and problems involving fractions.

**Understand principles and operations related to decimals and percents.**

For example: multiple representations of decimals; addition, subtraction, multiplication, and division of decimals; ordering decimals and placement on a number line; terminating and repeating decimals; rounding and estimation; conversion among decimals, percents, and fractions; the relationship between properties of decimals and the development of algebraic concepts; and problems involving decimals and percents.

**Understand principles and operations related to integers.**

For example: order of operations; identity and inverse elements; associative, commutative, and distributive properties; absolute value; operations with signed numbers; multiple representations of number operations (e.g., area models); rounding and estimation; standard algorithms for addition, subtraction, multiplication, and division; number operations and their inverses; the origins and development of standard computational algorithms; the relationship between properties of integers and the development of algebraic concepts; and problems involving integers.

**NCTM STANDARDS—NUMBER AND OPERATIONS**
Number and Operations with Procedural Fluency

Proficiency with number and operations requires the deep and fundamental understanding of counting numbers, rational numbers (fractions, decimals, and per cents), and positive and negative numbers, beginning in the elementary and middle grades. This understanding is extended to other number systems. Students must demonstrate understanding of numbers and relationships among numbers with a focus on the place-value system. Students must develop understanding of number operations and how they relate to one another.

Written mathematical procedures—computational procedures in the elementary grades and more symbolic algebraic procedures as students move into the secondary level—continue to be an important focus of school mathematics programs. Equally important is the ability to be comfortable and competent with estimation and mental math. As students develop number sense, they acquire abilities to estimate and perform mental calculations quickly and proficiently. Students should become proficient at using mental math shortcuts, performing basic computations mentally, and generating reasonable estimates for situations involving size, distance, and magnitude.

**IMPORTANT DATES**

Tuesday, Sept. 11: Last day to add a course without a permission number

Thursday, Sept. 18: Last day to add or drop a course or to change enrollment status

Monday, October 8: Columbus Day

Midterm Exam: October 17, 2011

Monday, November 5: Enrollment for Spring Courses begins

Monday, November 12: Veteran’s Day

Thursday, November 22: Thanksgiving Recess

Projects due Wednesday, Nov. 28

Monday, December 10: Last day of Fall Semester classes

Final Exam: scheduled between December 12 and December 20
**Course Sequence**

<table>
<thead>
<tr>
<th>Class Date</th>
<th>Topics</th>
<th>Elaboration</th>
<th>Assignments</th>
</tr>
</thead>
</table>
| Wednesday, Sept. 5 | Introduction to Problem Solving             | Analyze the structures and properties of the base-10 and other numeral systems (e.g., expanded form of a number, visual representations of place value, number systems of ancient cultures). Begin to see teaching as highly intricate and precise work. Learn to see and attend to the work of teaching (which is often invisible to observers of classroom interactions), including the mathematical knowledge, skills, and dispositions involved in teaching mathematics. Begin to appreciate the high levels of skill, coordination, and precision involved in teaching. | Due Monday, Sept. 10:  
Diagnostic Assessment  
Homework #1: Problem Solving (complete problems from class)  
Reading (to be distributed in class): Polya’s four steps for problem solving; Berlinski, *One, Two, Three*, Introduction |
| Monday, Sept. 10 | Whole Number Operations                     | Evaluate the validity of nonstandard or unfamiliar computational strategies. Recognize and analyze various representations (e.g., graphic, pictorial, verbal) of number operations. Recognize relationships among operations (e.g., addition and subtraction, addition and multiplication, multiplication and exponentiation). Identify and apply the arithmetic properties and the transitive properties of equality and inequality. | Due Wednesday, Sept. 12:  
Homework #2:  
Part 1: Problem Solving, applying four step problem solving process  
Part 2: Numeration systems & sets  
Reading: A short history of mathematics |
<p>|                 | Addition and Subtraction                    |                                                                                                                                             |                                                                                                  |
|                 | Multiplication and Division                 |                                                                                                                                             |                                                                                                  |
|                 |                                             |                                                                                                                                             |                                                                                                  |</p>
<table>
<thead>
<tr>
<th>Class Date</th>
<th>Topics</th>
<th>Elaboration</th>
<th>Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Integrate the order of operations with arithmetic properties. Demonstrate fluency in arithmetic computation. Apply appropriate strategies (e.g., proportional thinking, ratios) to estimate quantities in real-world situations. Solve problems using arithmetic operations with various representations of numbers.</td>
<td>Dick, CHAPTER 2</td>
</tr>
<tr>
<td><strong>Monday, Sept. 17</strong></td>
<td></td>
<td></td>
<td>Due Monday, Sept. 17:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Homework #3: Whole number operations</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dick, CHAPTER 2 (cont.)</td>
</tr>
<tr>
<td><strong>Wednesday, Sept. 19</strong></td>
<td>Algorithms for Whole Number Operations</td>
<td>Analyze and justify standard and nonstandard computational algorithms and mental math techniques (e.g., by application of the arithmetic properties, such as commutative, associative, distributive).</td>
<td>Due Wednesday, Sept. 19:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Homework #4: Whole number operations (cont.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dick, CHAPTER 2 (cont.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Arithmetic properties as well as operations with whole numbers (including various representations of operations and connections between them).</td>
</tr>
<tr>
<td><strong>Monday, Sept. 24</strong></td>
<td></td>
<td></td>
<td>Due Monday, Sept. 24:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Homework #5: Algorithms for whole number operations</td>
</tr>
<tr>
<td>Class Date</td>
<td>Topics</td>
<td>Elaboration</td>
<td>Assignments</td>
</tr>
<tr>
<td>------------------</td>
<td>--------------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| **Wednesday, Sept. 26** | Integers                | Analyze and move among various representations of numbers (e.g., graphic, numeric, symbolic, verbal). Identify subsets of the real numbers (e.g., integer, rational, irrational) and their properties. Interpret the concept of absolute value. | **Due Wednesday, Sept. 26:**  
Homework #6: Algorithms for whole number operations (cont.)  
*Return to Dick, CHAPTER 2, pp. 24-35* |
| **Monday, Oct. 1**   | Integers                | Understand the meanings and models of integers, fractions, decimals, percents, and mixed numbers and apply them to the solution of word problems. | **Due Monday, Oct. 1:**  
Homework #7: Integer operations  
*Return to Dick, CHAPTER 2, pp. 24-35* |
| **Wednesday, Oct. 3** | Integers                | Identify prime and composite numbers and their characteristics. Find the least common multiple (LCM) and greatest common factor (GCF) of a set of numbers. Use Venn diagram as one of | **Due Wednesday, Oct. 3:**  
Homework #8: Integer operations (cont.)  
Homework #9: Integer operations (cont.) |
| **Wednesday, Oct. 10** | Factors: GCF and LCM   |                                                                            | **Due Wednesday, Oct. 10**  
Homework #9: Integer operations (cont.)  
Homework #10:  
Part 1: Factors  
Part 2: Review for Midterm |
| **Monday, Oct. 15**   | Standards and alternative algorithms |                                                                            | **Due Monday, Oct. 15:**  
Homework #10:  
Part 1: Factors  
Part 2: Review for Midterm |
<table>
<thead>
<tr>
<th>Class Date</th>
<th>Topics</th>
<th>Elaboration</th>
<th>Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wednesday, Oct. 17</td>
<td></td>
<td>the methods for solving problems.</td>
<td>MIDTERM EXAM</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Demonstrate knowledge of the divisibility rules and why they work.</td>
<td></td>
</tr>
<tr>
<td>Monday, Oct. 22</td>
<td>Estimation and mental math</td>
<td>Analyze and determine the reasonableness of estimates.</td>
<td>Due Monday, Oct. 22:</td>
</tr>
<tr>
<td></td>
<td>Representations and language for fractions</td>
<td>Begin to understand that fractions can be represented in different ways and can be invested with different meanings (e.g., as parts of a whole, as division, as operators, as points on the number line, etc.).</td>
<td>No homework due</td>
</tr>
<tr>
<td>Wednesday, Oct. 24</td>
<td></td>
<td><em>Note that this class is designed to develop language for talking about features of representations and to develop skill in using such representations.</em></td>
<td>Due Wednesday, Oct. 24:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Homework #11: Estimation and mental math</td>
</tr>
<tr>
<td>Monday, Oct. 29</td>
<td>Fractions: Exploring Units</td>
<td>Learn the importance of attending to the whole or unit when working with fractions.</td>
<td>Due Monday, Oct. 29:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Develop representations for solving a problem and attend to and make sense of a variety of explanations. Build correspondences among representations; construct explanations and begin to develop criteria for what counts as a</td>
<td>Homework #12: Fraction representations</td>
</tr>
</tbody>
</table>

2 TMKT Class 1
3 TMKT Class 2 and 3
<table>
<thead>
<tr>
<th>Class Date</th>
<th>Topics</th>
<th>Elaboration</th>
<th>Assignments</th>
</tr>
</thead>
</table>
| Wednesday, Oct. 31 |                                             | “good” mathematical explanation.  
This class is organized around a complex multi-stage story problem involving fractions in which the unit or whole changes at each stage of the problem. Work on the problem surfaces a range of representations and solution methods. | Due Wednesday, Oct. 31:  
Homework #13: Fractions-representations & exploring units |
| Monday, Nov. 5   | Fractions: Focusing on units<sup>4</sup>  
Division of fractions<sup>5</sup> | Develop a sense of connection between division and fractions; develop an explicit understanding of the idea that fractions can be interpreted as division.  
Evaluate and write story problems for expressions such as 134 <div>12, as way to explore the meaning of division of fractions and to carefully map the context of a story to the numbers and operations in an expression.  
Solidify understanding of the importance of the unit or whole in fractions, manage units that are composites of individual objects, and explore the subtleties of implicit language about | Due Monday, Nov. 5:  
Homework #14: Fractions-exploring units |

<sup>4</sup> TMKT Class 4  
<sup>5</sup> TMKT Class 5
<table>
<thead>
<tr>
<th>Class Date</th>
<th>Topics</th>
<th>Elaboration</th>
<th>Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wednesday, Nov. 7</td>
<td>units. <em>One important goal is to develop skills in opening fraction problems and seeing and naming what makes them confusing.</em> Attend to language in the context of fractions and division (e.g., dividing in half vs. dividing by half.)</td>
<td>Due Wednesday, Nov. 7: Homework #15: Fractions-working with units; fraction division</td>
<td>-------------</td>
</tr>
<tr>
<td>Wednesday, Nov. 14</td>
<td>Partitive and measurement interpretations of division Fractions: Explaining algorithms⁶</td>
<td>Explore why multiplying the numerator and denominator by the same non-zero number generates an equivalent fraction; explore how to represent the multiplication of fractions; and explain the standard algorithm for multiplying fractions. <em>This class provides an opportunity for prospective teachers to develop an understanding of why some of the standard algorithms for operating with fractions “work.”</em></td>
<td>Project Work</td>
</tr>
<tr>
<td>Monday, Nov. 19</td>
<td></td>
<td></td>
<td>Due Wednesday, Nov. 19: Homework #16: Interpretations of division</td>
</tr>
</tbody>
</table>

---

⁶ TMKT Class 6
<table>
<thead>
<tr>
<th>Class Date</th>
<th>Topics</th>
<th>Elaboration</th>
<th>Assignments</th>
</tr>
</thead>
</table>
| Wednesday, Nov. 21 | Fractions on the number line⁷               | Locate a range of numbers on the number line as a way to articulate and understand properties of the number line. Continue to explore properties of the number line, focusing on the property of density. Understand the number line as a unifying representation for locating number systems: counting numbers, whole numbers, integers, rational numbers, and real numbers. Understand that fractions begin to fill in the spaces between integers on the number line, and that between any two fractions on the number line, there is always another fraction (so there are infinitely many fractions between any two) | Due Wednesday, Nov. 21:  
Homework #17: Explaining algorithms for fraction operations                                             |
| Monday, Nov. 26   | Units and the interpretation of fractions⁸  | 4 examined earlier, practice explaining and representing other fractions in multiple ways, and make connections across the work in this class. Recognize and name what you have learned (as a group and individually) about fractions and the skills developed for representing and explaining mathematical ideas. | Due Monday, Nov. 26:  
Homework #18: Fractions on the number line                                                                                                         |
| Wednesday, Nov. 28 |                               |                                                                                                                                            | Due Wednesday, Nov. 28:  
Homework #19: Fractions on the number line (cont.)  
PROJECT DUE                                                                |
| Monday, Dec. 3    |                               |                                                                                                                                            | Due Monday, Dec. 3:  
Homework #20: Interpretations of fractions                                                                     |

⁷ TMKT Class 7 and 8  
⁸ TMKT Class 9
<table>
<thead>
<tr>
<th><strong>Class Date</strong></th>
<th><strong>Topics</strong></th>
<th><strong>Elaboration</strong></th>
<th><strong>Assignments</strong></th>
</tr>
</thead>
</table>
| Wednesday, Dec. 5   | Decimals and Percent     | Recognize decimal expansions. Recognize equivalent representations of numbers (e.g., fractions, decimals, percents). | Due Wednesday, Dec. 5:  
Dick, CHAPTERS 5(decimals) & 7(Percents)                                                 |
| Monday, Dec. 10     |                          |                                                                                 | Due Monday, Dec. 10:  
Homework #21: Decimals and percents  
Final Review                                                                                      |
| Date To Be Announced| Comprehensive Final Exam |                                                                                 | FINAL EXAM (scheduled between December 14 and December 23)                     |
COURSE RESOURCES

Below is a short list of resources that might be useful to you in preparing your project. Some also provide additional explanations of concepts that complement the work we do in class.

NCTM Illuminations. Available online: http://illuminations.nctm.org/

Problems with a Point (problems for grades 6-12). Available online: http://www2.edc.org/mathproblems/


The National Library of Virtual Manipulatives. Available online: http://nlmv.usu.edu/

Archytech. This is a website with virtual manipulatives that include a virtual integer bar (Cuisenaire rod) utility. Available online: http://www.archytech.org/java/