Structural Dynamics and Acoustic Systems Laboratory
University of Massachusetts Lowell

DEVELOPMENT OF DYNAMIC RESPONSE MODELING TECHNIQUES FOR LINEAR MODAL COMPONENTS INTERCONNECTED WITH NON-LINEAR CONNECTION ELEMENTS

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RESEARCH OBJECTIVE – ERMT

- Provide set of efficient computational tools for analyzing systems with highly nonlinear connection regions
- Develop physical reduced model approaches that address nonlinear contact problems for system response prediction
- Validate the analytical approaches and techniques developed using experimental data

THEORY

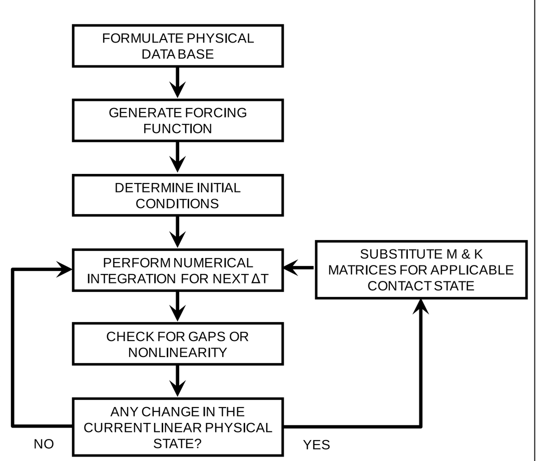
- Develop physical reduced database for potential system contact states

$$\{x_n\}=\begin{Bmatrix}x_a\\x_d\end{Bmatrix}=[T]\{x_a\}\qquad [M_a]=[T]^T[M_n][T]\qquad [K_a]=[T]^T[K_n][T]$$

- Perform direct integration of physical reduced equations of motion

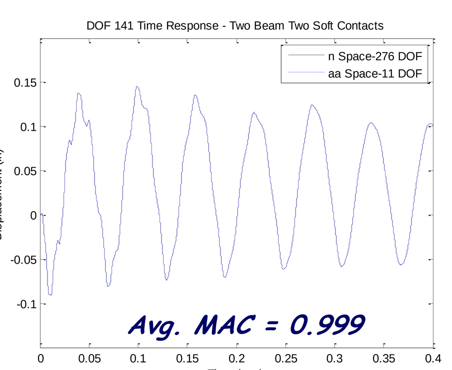
$$\begin{bmatrix}M_a^A & \\ & M_a^B\end{bmatrix}\begin{Bmatrix}\ddot{x}_a^A\\ \ddot{x}_a^B\end{Bmatrix}+\left(\begin{bmatrix}K_a^A & \\ & K_a^B\end{bmatrix}+[K_{TIE}]\right)\begin{Bmatrix}x_a^A\\ x_a^B\end{Bmatrix}=\begin{Bmatrix}f_a^A\\ f_a^B\end{Bmatrix}$$

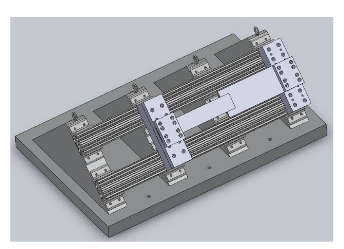
ALGORITHM



RESULTS

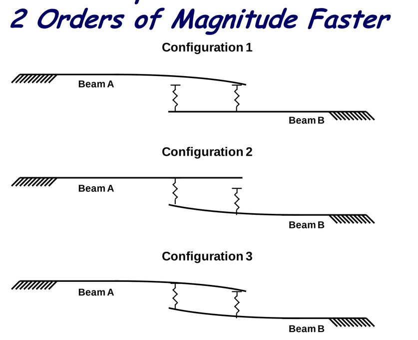
Analytical — Two Beams with Two Contacts

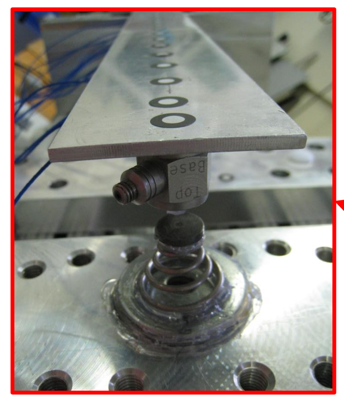




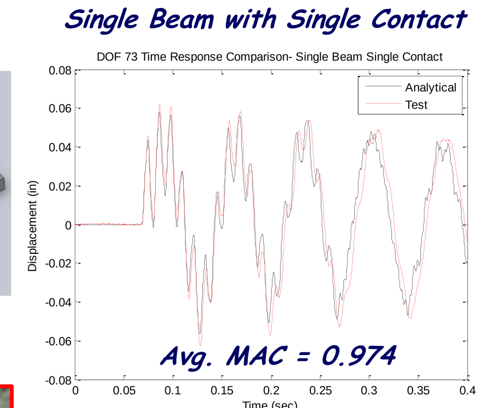
Avg. MAC = 0.999

Full 'n' Space Solution - 88.7 s
ERMT 'a' Space Solution - 1.1 s
2 Orders of Magnitude Faster



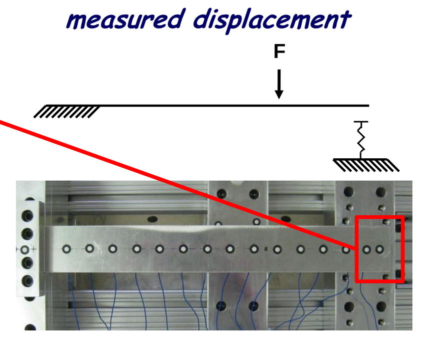


Experimental — Single Beam with Single Contact

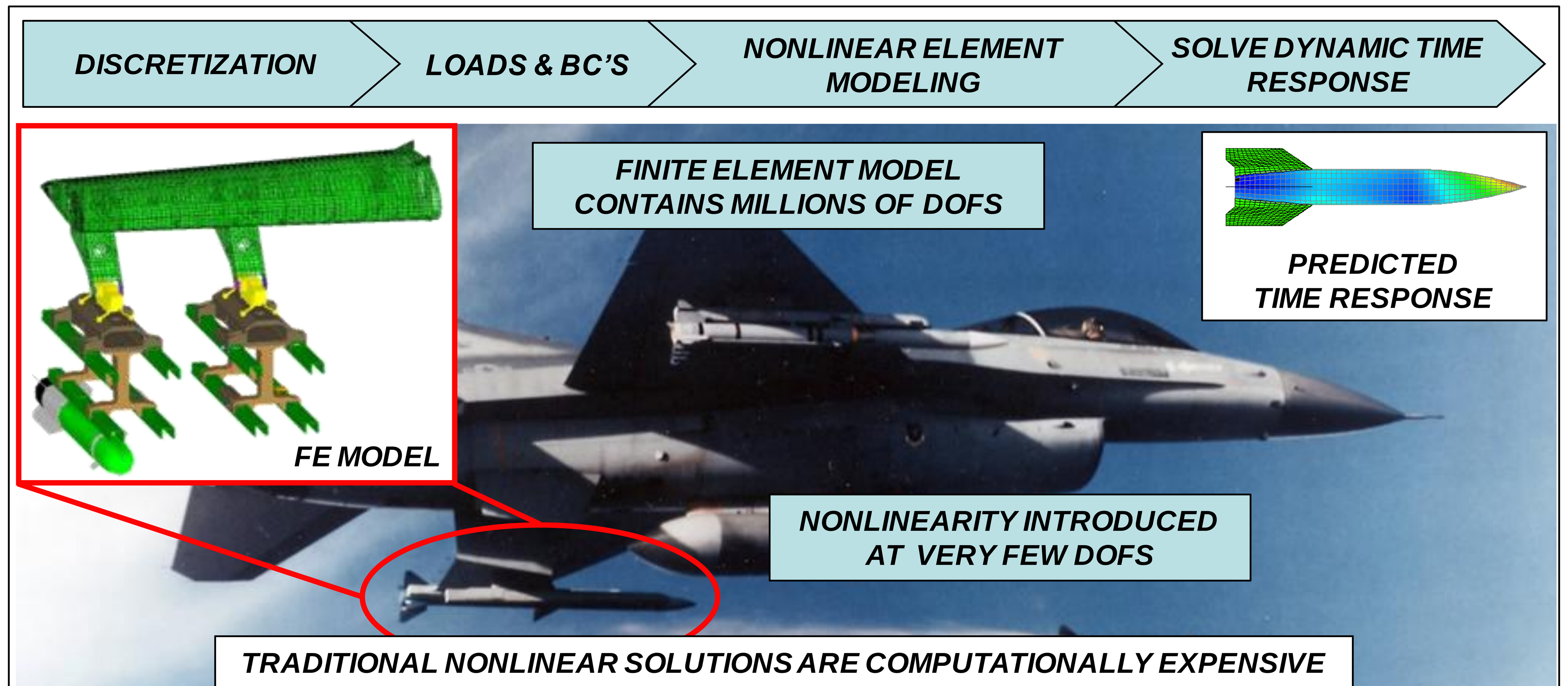


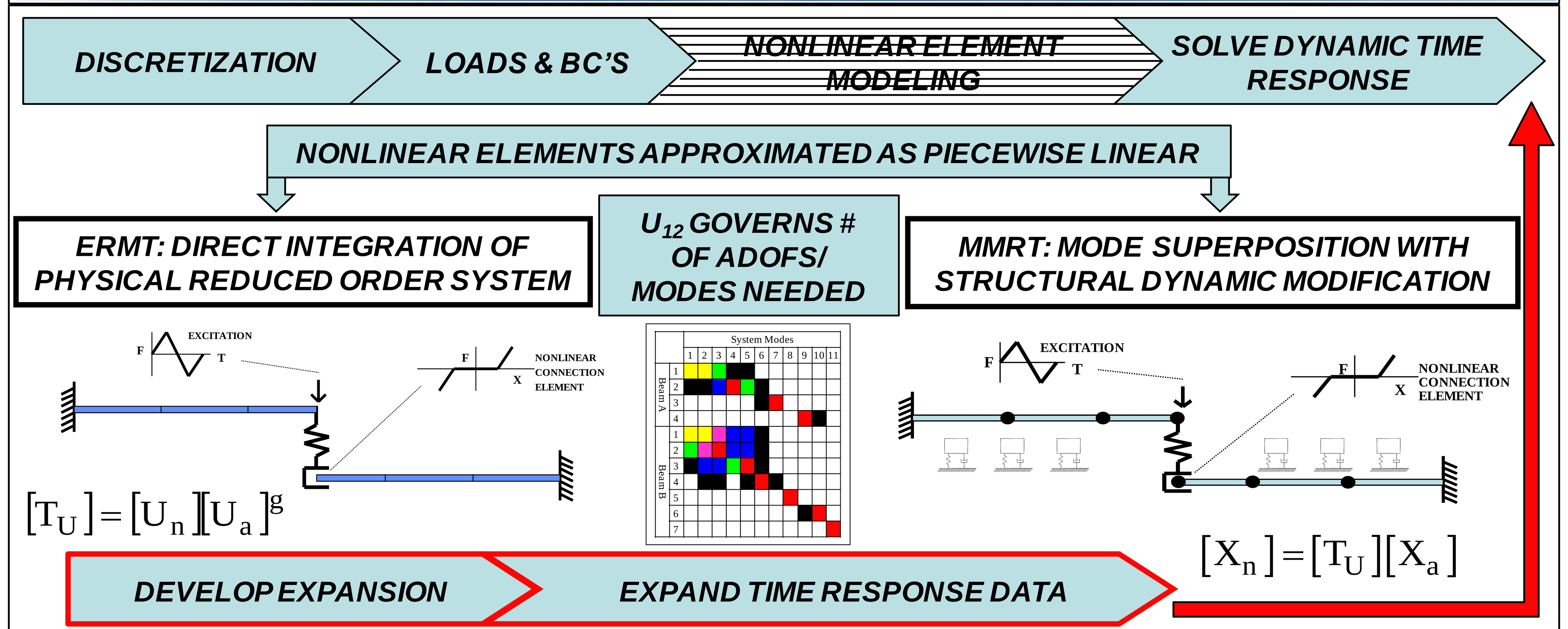
Avg. MAC = 0.974

Predicted displacement results show excellent correlation with measured displacement



Mon 4:00 PM 121 Equivalent Reduced Model Technique Development for Nonlinear System Dynamic Response





EFFICIENT MODELING DRASTICALLY REDUCES COMPUTATION TIME

RESEARCH OBJECTIVE – MMRT

- Provide set of efficient tools for analyzing systems with highly nonlinear connections
- Develop modal space model approaches to be used for response prediction
- Validate the analytical approaches through use of experimental data

THEORY

- Structural changes are projected from physical to modal space to yield a drastically reduced set of matrices

$$\left[\bar{M}_1\right]+\left[\Delta\bar{M}_{12}\right]\{\ddot{p}_1\}+\left[\bar{K}_1\right]+\left[\Delta\bar{K}_{12}\right]\{p_1\}=[0]$$

- The direct equation of motion used is

$$\begin{bmatrix}\bar{M}^A & \\ & \bar{M}^B\end{bmatrix}\begin{Bmatrix}\ddot{p}^A\\ \ddot{p}^B\end{Bmatrix}+\left(\begin{bmatrix}\Omega^{2A} & \\ & \Omega^{2B}\end{bmatrix}+[U]^T[K_{TIE}][U]\right)\begin{Bmatrix}p^A\\ p^B\end{Bmatrix}=\begin{Bmatrix}[U^A]^T\{f^A\}\\ [U^B]^T\{f^B\}\end{Bmatrix}$$

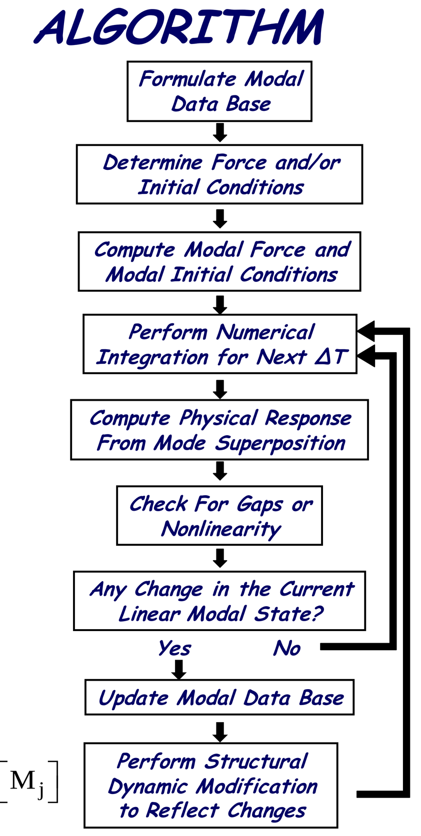
- The initial condition for the given state

$$\{p\}_j=[U]_j^g\{x\}^{(i-1)}\qquad \{\dot{p}\}_j=[U]_j^g\{\dot{x}\}^{(i-1)}$$

- The generalized inverse can be written two ways

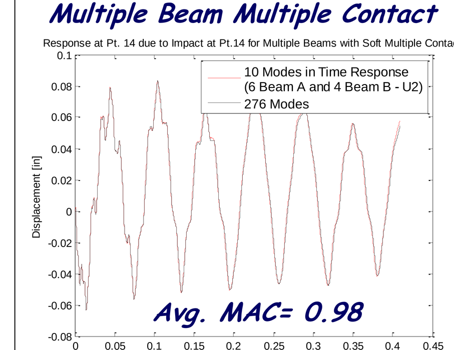
$$[U]_j^g=\left[[U_j]^T[U_j]\right]^{-1}[U_j]^T\qquad [U]_j^g=\left[\bar{M}_j\right][U_j]^T[M_j]$$

ALGORITHM



RESULTS

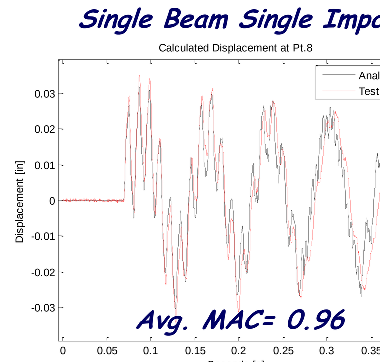
Analytical — Multiple Beam Multiple Contact



Avg. MAC= 0.98

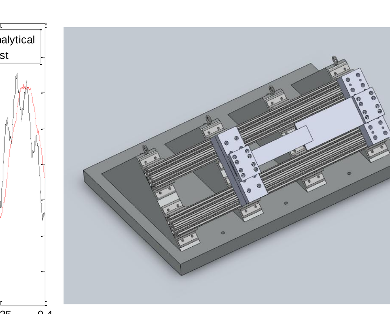
Full System Solution - 915 s
MMRT - 11.75 s
2 Orders of Magnitude Faster!

Experimental — Single Beam Single Impact



Avg. MAC= 0.96

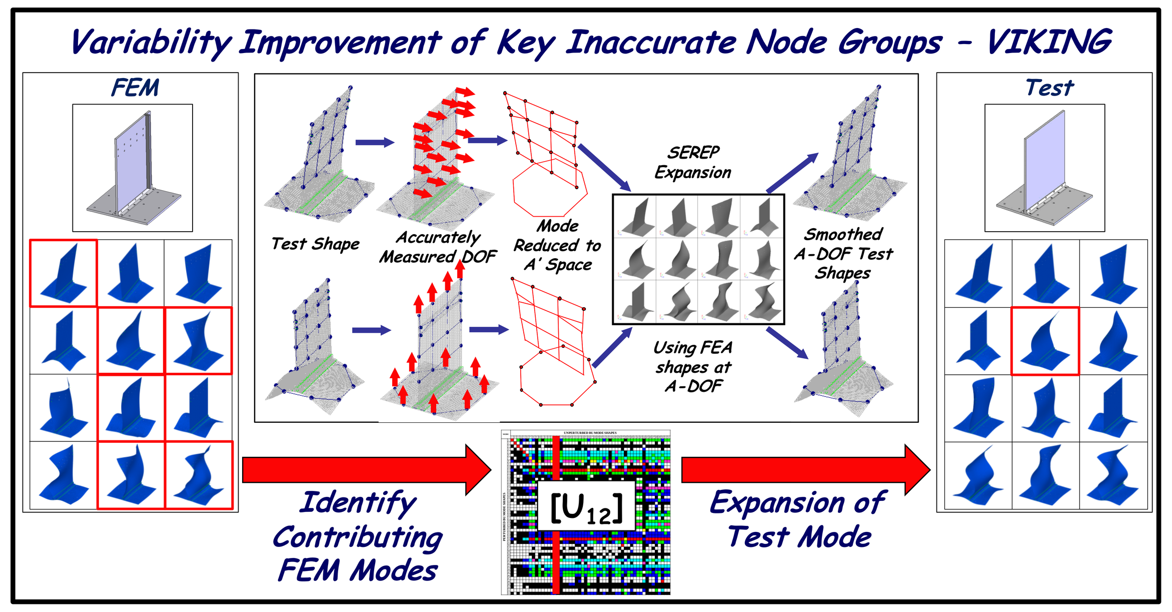
Predicted displacement results show excellent correlation with measured displacement



Mon 4:30 PM 120 Efficient Computational Nonlinear Dynamic Analysis Using Modal Modification Response …

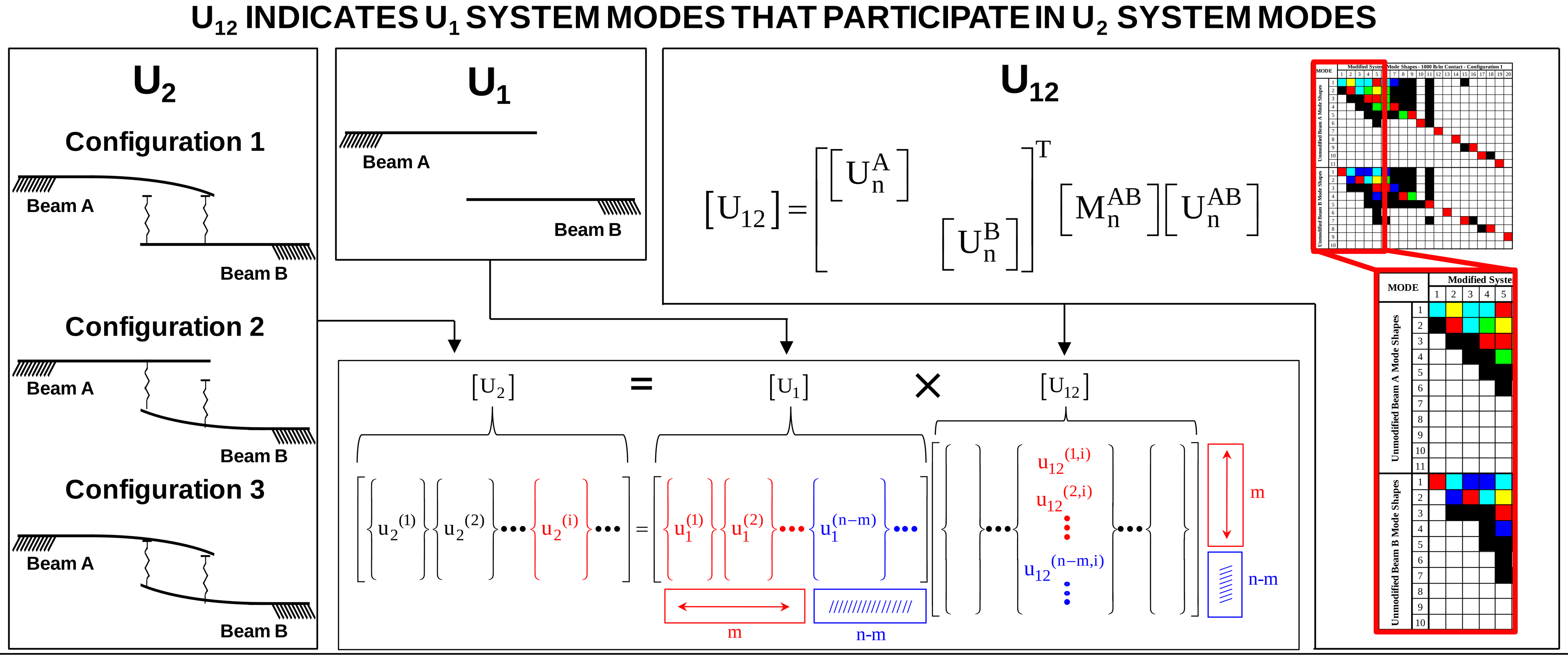
Experimental data is always plagued with measurement contaminants. In addition, accurate expansion of test data is often restricted to modes with adequate correlation to the analytical model. A new data conditioning technique referred to as Variability Improvement of Key Inaccurate Node Groups (VIKING) has been developed for processing data in these situations.

Variability Improvement of Key Inaccurate Node Groups – VIKING



Wed - 9:30 AM 123 Variability Improvement of Key Inaccurate Node Groups - VIKING

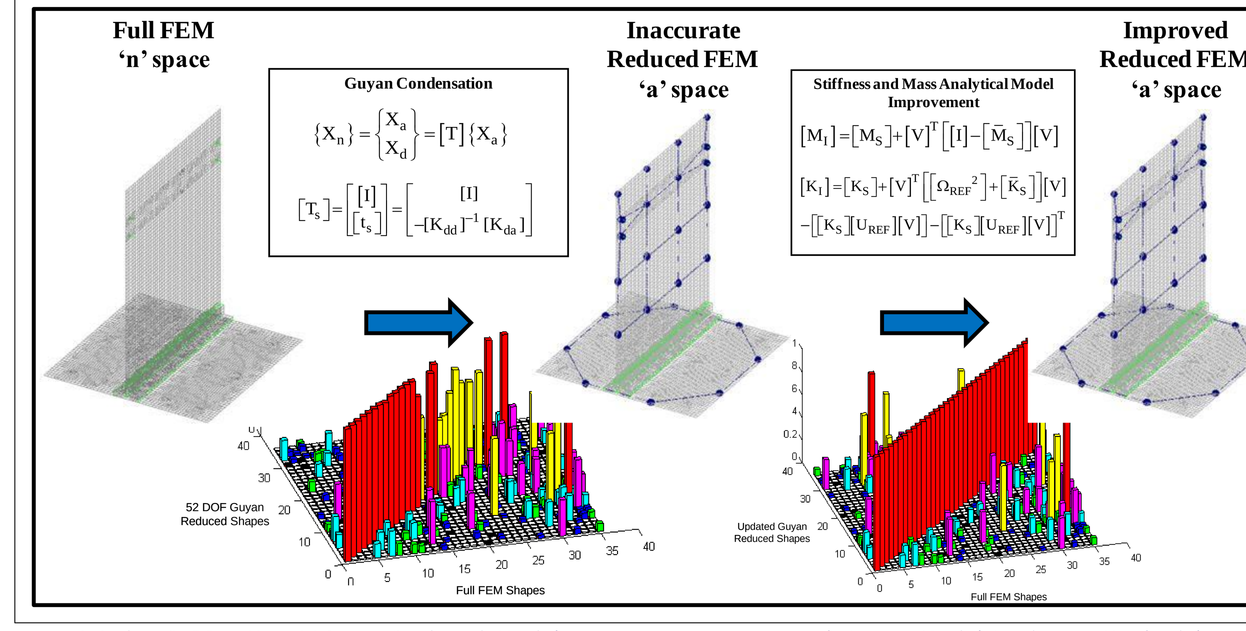
U_{12} INDICATES U_1 SYSTEM MODES THAT PARTICIPATE IN U_2 SYSTEM MODES



$$[U_{12}]=\left[\begin{bmatrix}U_n^A & \\ & U_n^B\end{bmatrix}\right]^T\left[M_n^{AB}\right]\left[U_n^{AB}\right]$$

$$[U_2]=[U_1]\times[U_{12}]$$

Traditional reduction techniques have utilized static equations in order to obtain the transformation matrix while more recent techniques have utilized the mode shapes of the model. An alternative approach combines the advantages from both schemes, yielding an exact reduced order model with fully rank matrices.



Wed - 9:00 AM 118 A Reduced Model Approximation Approach Using Model Updating Methodologies

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