Finding a Local Maximum in a Circle of Numbers

Suppose 55 cards with distinct real numbers are arranged face down in a circle. What is the minimum number of cards which must be turned over in order to guarantee finding a “local maximum”, i.e. a card whose value exceeds its two immediate neighbors?

We show how a solution can be found by turning over 11 cards. Begin by turning over three cards equally spaced (as much as possible) around the circle. Call the smallest card #1 and spread the cards out in a line — the other turned cards are #20 and #38. So now we have

#1 (18) #20 (17) #38 (17) #1

where the numbers in parentheses show the number of hidden cards. Locate the biggest card — #20 or #38. Without loss of generality let's say it's #20. (Actually the situation is slightly different if #38 is biggest because it has fewer unknowns to its left but this only helps because the number of remaining unknown cards is one less.) Since card #20 exceeds the known cards on either side there must be a local maximum among cards #1 — #38. Turn over the card midway between the left two cards #1 and #20 — if the midpiont falls between two cards let's always choose the one closer to the middle. In this case it's #11. If #11 exceeds #1 and #20 we know there is a local maximum among the cards #1 — #20. If not we turn over #29, the middle card between #20 and #38. It's easy to see that we now have as the worst case

#11 (8) #20 (8) #29 (8) #38

in which #20 or #29 is the biggest of the four. Let's say it's #29 which leaves the sequence

#20 (8) #29 (8) #38

Now divide again — the worst case is #25 is less than both #20 and #29 and we have to do #33 also giving

#25 (3) #29 (3) #33 (4) #38

with the biggest card #29 or #33. Let's assume it's #29 again and divide once more assuming the worst

#31 (1) #33 (1) #35 (2) #38

with #35 is biggest. To finish off we try #36. If this is bigger than #35 we turn #37 and we're done, otherwise we turn #34 and we're done. So here are the cards which were turned (in order)

#1, #20, #38, #11, #29, #25, #33, #31, #35, #36, #37 = 11 cards total.

Remark: The minimum number of turns by this method is 8 which would have turned the cards

#1, #20, #38, #11, #6, #4, #3, #2 = 8 cards total.

If it's known the cards are numbered 1 — 55 then we could terminate at any time by happening upon 55.